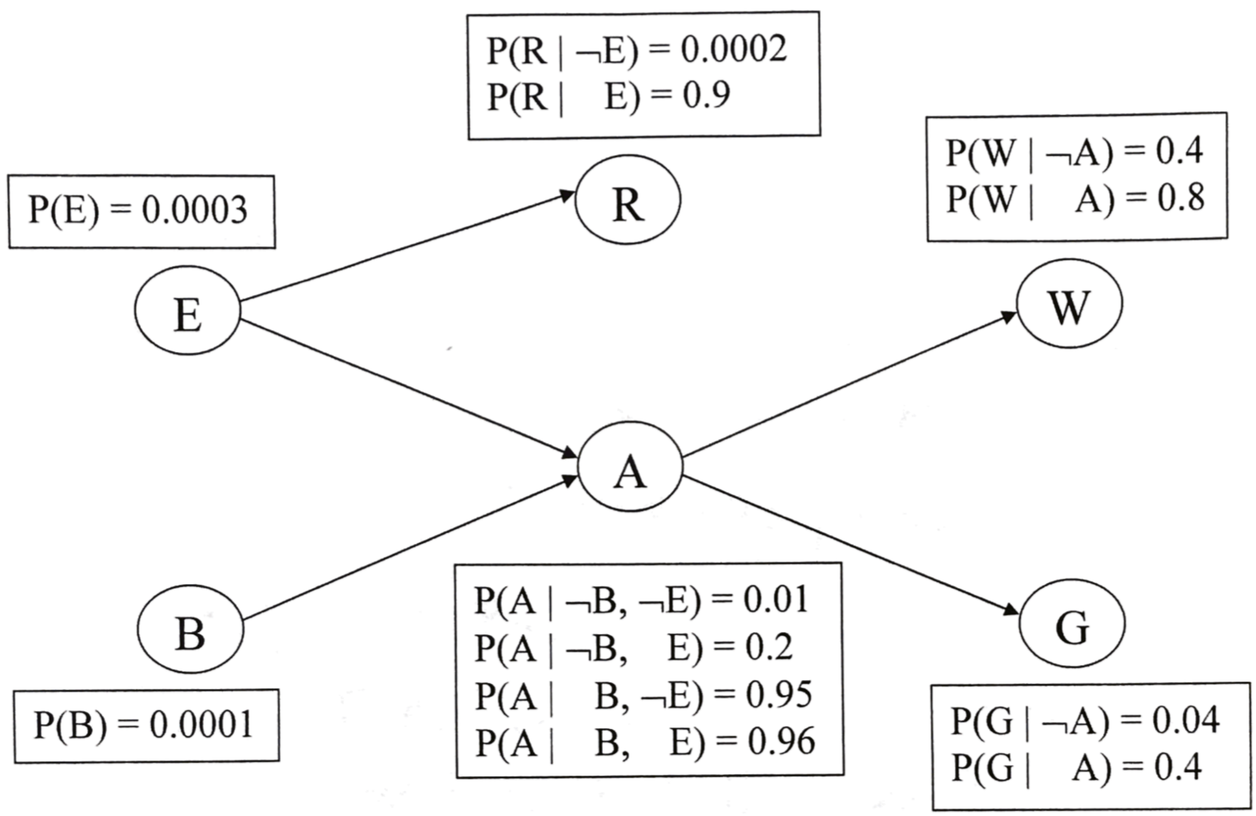
The Holmes scenario

Mr. Holmes lives in a high crime area and therefore has installed a burglar alarm. He relies on his neighbors to phone him when they hear the alarm sound. Mr. Holmes has two neighbors, Dr. Watson and Mrs. Gibbon.

Unfortunately, his neighbors are not entirely reliable. Dr. Watson is known to be a tasteless practical joker and Mrs. Gibbon, while more reliable in general, has occasional drinking problems.

Mr. Holmes also knows from reading the instruction manual of his alarm system that the device is sensitive to earthquakes and can be triggered by one accidentally. He realizes that if an earthquake has occurred, it would surely be on the radio news.



Calculating a joint probability given a Bayes Net

What is the probability that

- NEITHER a burglary NOR an earthquake has occurred,
- The alarm has NOT sounded,
- NEITHER of Watson and Gibbon is calling, and
- There is NO radio report of an earthquake?

$$
\begin{aligned}
&P(\neg B)P(\neg E)P(\neg A|\neg B \wedge \neg E)P(\neg W|\neg A)P(\neg G|\neg A)P(\neg R|\neg E) \\
&= (1-0.0001)(1-0.0003)(1-0.01)(1-0.4)(1-0.04)(1-0.0002) \\
&\approx 0.57
\end{aligned}
$$

Identify conditional/unconditional independence assumptions

1. Is Radio conditionally independent of Gibbon given Earthquake?

   Answer: Yes.

   To verify this, notice that earthquake is the parent of Radio. Let's choose an ordering of the nodes where Gibbon come before Earthquake. For example, consider the order E, B, A, G, W, R. Given this order, we can derive the following equation.

   $$P(R|E \wedge B \wedge A \wedge G \wedge W) = P(RjE)$$

   Given this equation, we know that Radio is independent of Burglary, Alarm, and Gibbon given Earthquake.

   This is a basic property of a Bayesian network. Once we choose a particular ordering of the variables, each node is conditionally independent of all its predecessors in the ordering given the node's parents.

2. Is Radio conditionally independent of Burglary given Alarm?

   Answer: No.

   Burglary and Earthquake are independent of each other. However, given the value of Alarm, Burglary and Earthquake become dependent – they are independent causes of Alarm. If the probability of one increases, then the probability of the other one decreases.

   Given Alarm, if the probability of Burglary changes, then the probability of Earthquake changes, which means that the probability of Radio changes. Thus, given Alarm, Burglary and Radio are NOT independent.