Probabilities and Independence

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Lecture 10

Based on work by K. Leyton-Brown, K. Larson, and P. van Beek

Outline

Learning Goals

By the end of the lecture, you should be able to

- Given a description of a domain or a probabilistic model for the domain, determine whether two variables are independent.
- Given a description of a domain or a probabilistic model for the domain, determine whether two variables are conditionally independent given a third variable.

The Holmes Scenario

Mr. Holmes lives in a high crime area and therefore has installed a burglar alarm. He relies on his neighbors to phone him when they hear the alarm sound. Mr. Holmes has two neighbors, Dr. Watson and Mrs. Gibbon.

Unfortunately, his neighbors are not entirely reliable. Dr. Watson is known to be a tasteless practical joker and Mrs. Gibbon, while more reliable in general, has occasional drinking problems.

Mr. Holmes also knows from reading the instruction manual of his alarm system that the device is sensitive to earthquakes and can be triggered by one accidentally. He realizes that if an earthquake has occurred, it would surely be on the radio news.

(Unconditional) Independence

Definition ((unconditional) independence)

Random variable X is independent of random variable Y if,

$$P(X|Y) = P(X)$$

In other words, $\forall x_i \in dom(X)$, $\forall y_j \in dom(Y)$ and $\forall y_k \in dom(Y)$,

$$\begin{aligned} & P(X = x_i | Y = y_j) \\ = \quad & P(X = x_i | Y = y_k) \\ = \quad & P(X = x_i). \end{aligned}$$

Knowing Y's value doesn't affect your belief in the value of X.

Conditional Independence

Definition (conditional independence)

Random variable X is conditionally independent of random variable Y given random variable Z if

$$P(X|Y,Z) = P(X|Z).$$

In other words, $\forall x_i \in dom(X)$, $\forall y_j \in dom(Y)$, $\forall y_k \in dom(Y)$ and $\forall z_m \in dom(Z)$,

$$\begin{aligned} & P(X = x_i | Y = y_j \wedge Z = z_m) \\ = \quad & P(X = x_i | Y = y_k \wedge Z = z_m) \\ = \quad & P(X = x_i | Z = z_m). \end{aligned}$$

Knowing Y's value doesn't affect your belief in the value of X, given a value of Z.

Burglary, Alarm and Watson

$P(B) = 0.1$ $\quad$ $P(A|B) = 0.9$ $\quad$ $P(A|\neg B) = 0.1$

$P(W|B \wedge A) = 0.8$ $\quad$ $P(W|B \wedge \neg A) = 0.4$

$P(W|\neg B \wedge A) = 0.8$ $\quad$ $P(W|\neg B \wedge \neg A) = 0.4$

CQ Unconditional Independence

CQ: Is Burglary independent of Watson?

(A) Yes

(B) No

(C) I don't know.

CQ: Conditional Independence

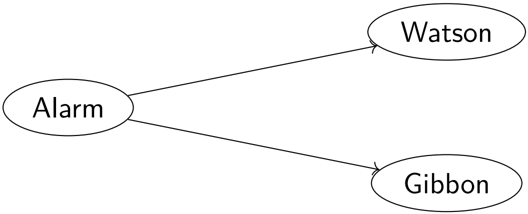
CQ: Is Burglary conditionally independent of Watson given Alarm?

(A) Yes

(B) No

(C) I don't know.

Alarm, Watson and Gibbon



$P(A) = 0.1$ $\quad$ $P(W|A) = 0.8$ $\quad$ $P(W|\neg A) = 0.4$

$P(G|W \wedge A) = 0.4$ $\quad$ $P(G|W \wedge \neg A) = 0.1$

$P(G|\neg W \wedge A) = 0.4$ $\quad$ $P(G|\neg W \wedge \neg A) = 0.1$

CQ Unconditional Independence

CQ: Is Watson independent of Gibbon?

(A) Yes

(B) No

(C) I don't know.

CQ Conditional Independence

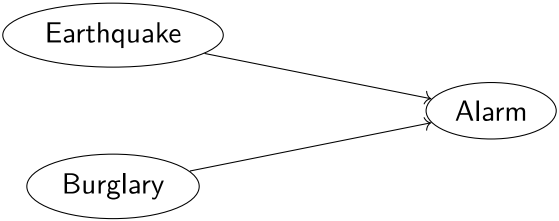
CQ: Is Watson conditionally independent of Gibbon given Alarm?

(A) Yes

(B) No

(C) I don't know.

Earthquake, Burglary, and Alarm



$P(E) = 0.1$ $P(B|E) = 0.2$ $P(B|\neg E) = 0.2$

$P(A|B \wedge E) = 0.9$ $P(A|B \wedge \neg E) = 0.8$
$P(A|\neg B \wedge E) = 0.2$ $P(A|\neg B \wedge \neg E) = 0.1$

CQ Unconditional Independence

CQ: Is Earthquake independent of Burglary?

(A) Yes

(B) No

(C) I don't know.

CQ: Conditional Independence

CQ: Is Earthquake conditionally independent of Burglary given Alarm?

(A) Yes

(B) No

(C) I don't know.

CQ: Calculating a probability

CQ: What is probability of Earthquake given Burglary and Alarm $P(E|B \wedge A)$?

(A) $0 \leq p \leq 0.2$

(B) $0.2 < p \leq 0.4$

(C) $0.4 < p \leq 0.6$

(D) $0.6 < p \leq 0.8$

(E) $0.8 < p \leq 1$

CQ: Calculating a probability

CQ: What is probability of Earthquake given NO Burglary and Alarm $P(E|\neg B \wedge A)$?

(A) $P(E|\neg B \wedge A) > P(E|B \wedge A)$

(B) $P(E|\neg B \wedge A) = P(E|B \wedge A)$

(C) $P(E|\neg B \wedge A) < P(E|B \wedge A)$

CQ: Conditional Independence

CQ: Is Earthquake conditionally independent of Burglary given Alarm?

(A) Yes

(B) No

(C) I don't know.

Revisiting the Learning Goals

By the end of the lecture, you should be able to

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