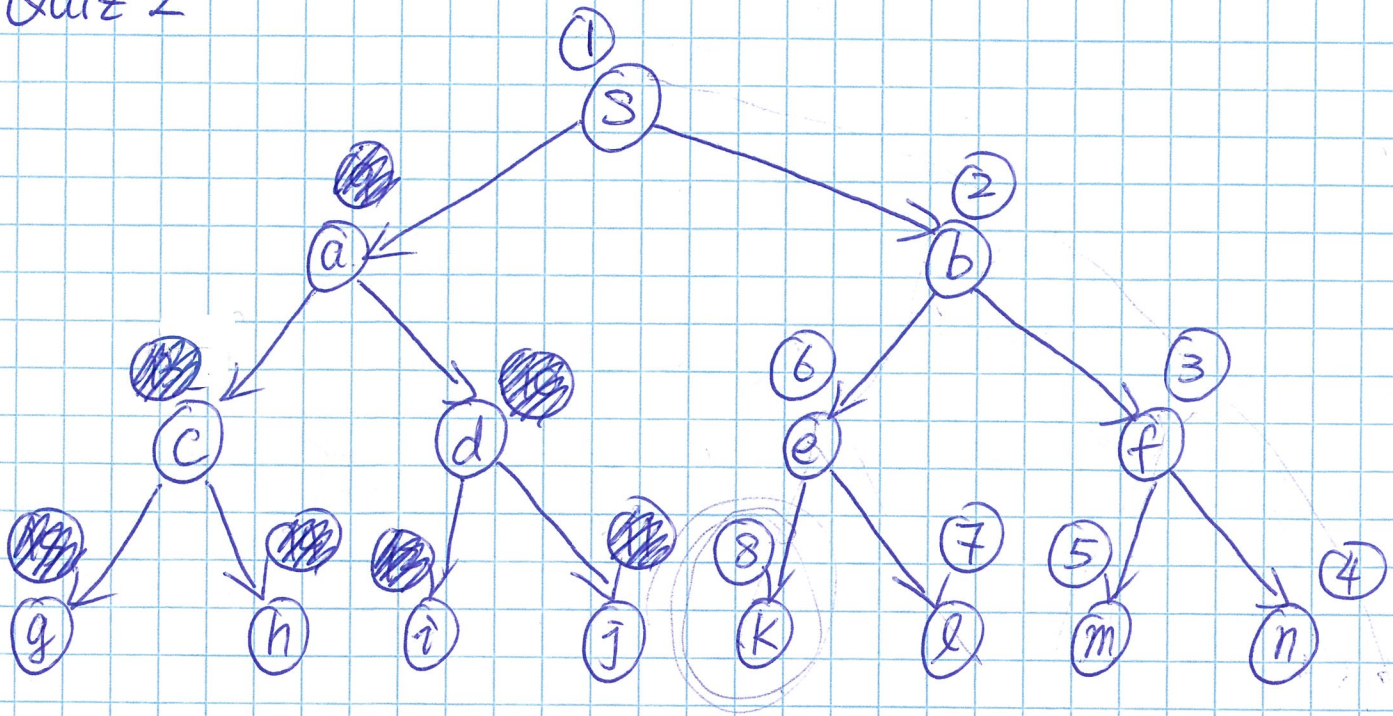
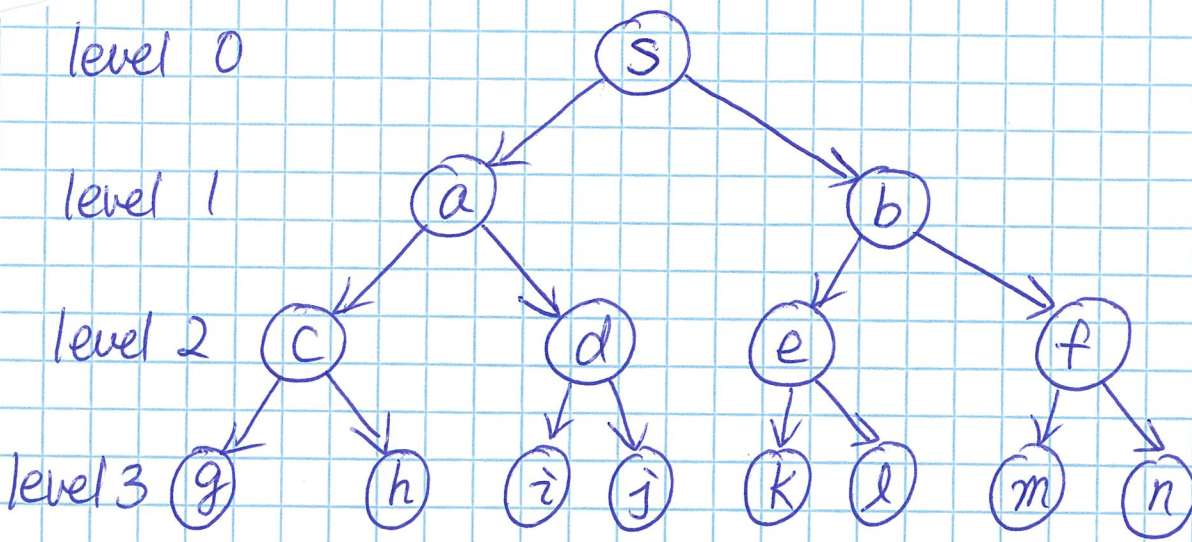


# Quiz 2

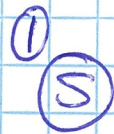


The nodes expanded by Depth-First Search.

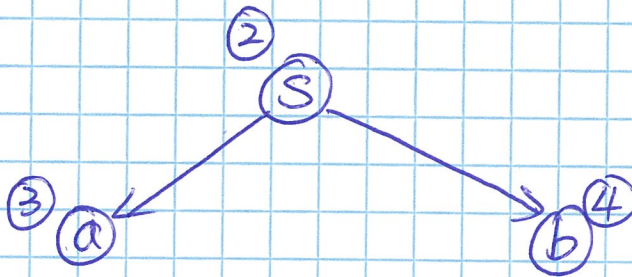
We stop at **(k)** because it's a goal node.



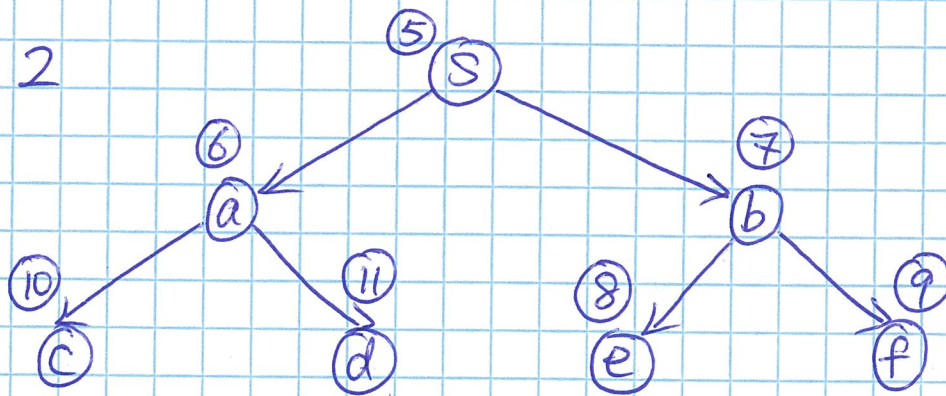
depth limit = 0



depth limit = 1



depth limit = 2

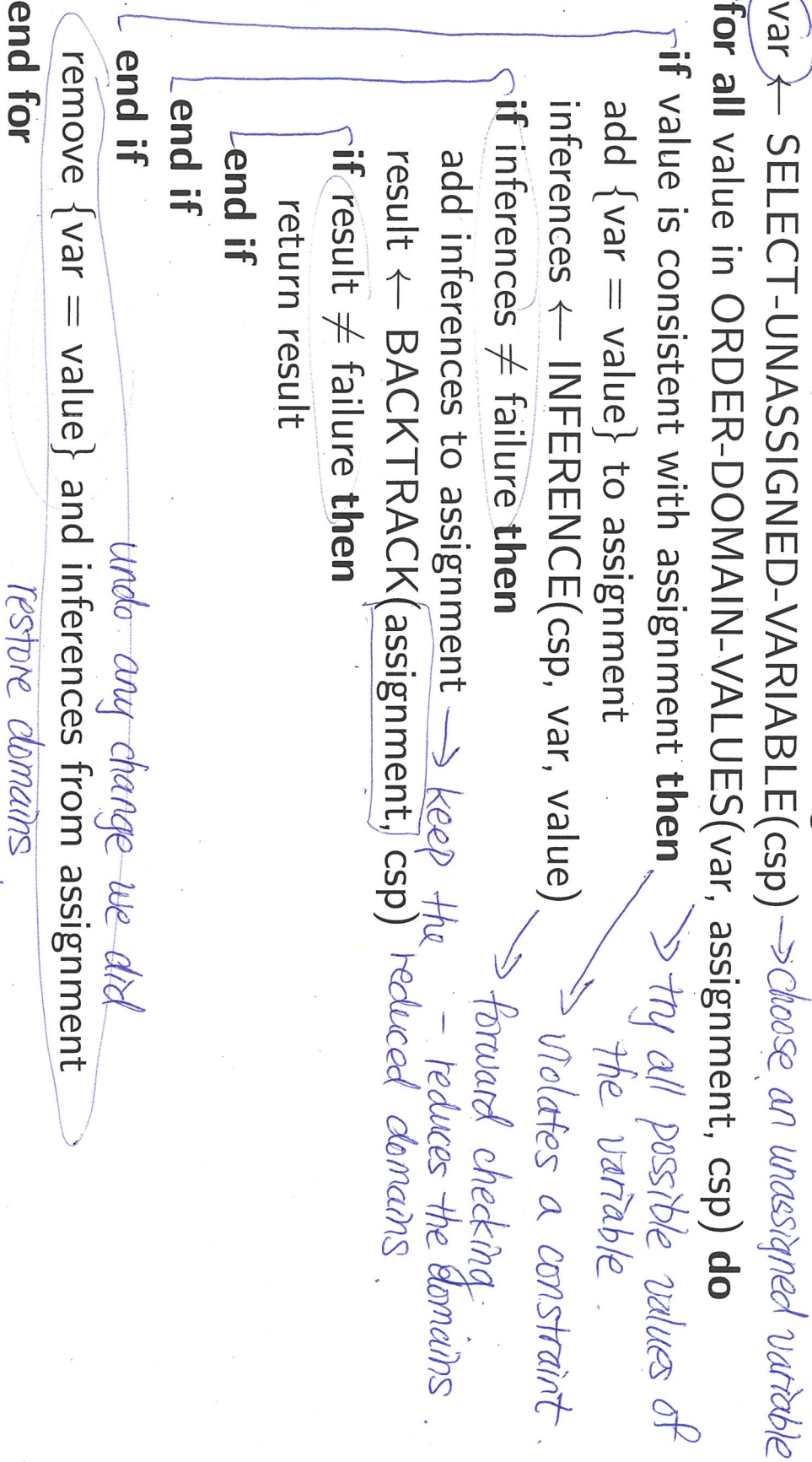


The nodes added to the frontier by Iterative-Deepening Search

# The Backtracking Search Algorithm

## Algorithm 1 BACKTRACK(assignment, csp)

```
1: if assignment is complete then return assignment
2: var ← SELECT-UNASSIGNED-VARIABLE(csp)
3: for all value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
4:   if value is consistent with assignment then
5:     add {var = value} to assignment
6:     inferences ← INFERENCE(csp, var, value)
7:     if inferences ≠ failure then
8:       add inferences to assignment
9:       result ← BACKTRACK(assignment, csp)
10:      if result ≠ failure then
11:        return result
12:      end if
13:    end if
14:  end if
15:  remove {var = value} and inferences from assignment
16: end for
17: return false
```



# Backtracking Search 4 Queens Problem Lecture 7

current assignment  $\{y\}$

variable  $x_0$  value 0

$\{x_0 = 0\}$

forward checking.

remove 1, 2, 3 from  $\text{dom}(x_0)$

remove 2, 3 from  $\text{dom}(x_1)$

remove 0, 2 from  $\text{dom}(x_2)$

remove 0, 3 from  $\text{dom}(x_3)$

	$x_0$	$x_1$	$x_2$	$x_3$
0	Q	X	X	X
1	X	X		
2	X		X	
3	X			X

current assignment  $\{x_0 = 0\}$

$x_1 \in \{2, 3\}$   $x_2 \in \{1, 3\}$   $x_3 \in \{1, 2\}$

variable  $x_1$  value 3 (least-constraining-value heuristic)

$\{x_1 = 3\}$

forward checking.

remove 2 from  $\text{dom}(x_1)$

remove 3 from  $\text{dom}(x_2)$

remove 1 from  $\text{dom}(x_3)$

	$x_0$	$x_1$	$x_2$	$x_3$
0	Q	X	X	X
1	X	X		X
2	X	X	X	
3	X	Q	X	X

current assignment  $\{x_0 = 0, x_1 = 3\}$

$x_2 \in \{1\}$   $x_3 \in \{2\}$

variable  $x_2$  value 1

$\{x_2 = 1\}$

forward checking.

remove 2 from  $\text{dom}(x_3)$

$\text{dom}(x_3)$  is empty. failure!

no other value of  $x_2$  to try.

backtrack to  $x_1$ .

	$x_0$	$x_1$	$x_2$	$x_3$
0	Q	X	X	X
1	X	X	Q	X
2	X	X	X	X
3	X	Q	X	X

# Backtracking Search 4-Queens Problem

before trying another value of  $x_1$

current assignment  $\{x_0 = 0\}$

$x_1 \in \{2, 3\}, x_2 \in \{1, 3\}, x_3 \in \{1, 2\}$

	$x_0$	$x_1$	$x_2$	$x_3$
0	Q	X	X	X
1	X	X		
2	X		X	
3	X			X

Variable  $x_1$  value 2

$\{x_1 = 2\}$

forward checking.

remove 3 from  $\text{dom}(x_1)$

remove 1, 3 from  $\text{dom}(x_2)$

remove 2 from  $\text{dom}(x_3)$

	$x_0$	$x_1$	$x_2$	$x_3$
0	Q	X	X	X
1	X	X	X	
2	X	Q	X	X
3	X	X	X	X

$\text{dom}(x_2)$  is empty. failure!

no other value of  $x_1$  to try.

backtrack to  $x_0$ .

before trying another value of  $x_0$

current assignment  $\{\}$

$x_0 \in \{0, \dots, 3\} \quad x_1 \in \{0, \dots, 3\}$

$x_2 \in \{0, \dots, 3\} \quad x_3 \in \{0, \dots, 3\}$

	$x_0$	$x_1$	$x_2$	$x_3$
0				
1				
2				
3				

Variable  $x_0$  value 1

$\{x_0 = 1\}$

forward checking.

remove 0, 2, 3 from  $\text{dom}(x_0)$

remove 0, 1, 2 from  $\text{dom}(x_1)$

remove 1, 3 from  $\text{dom}(x_2)$

remove 1 from  $\text{dom}(x_3)$

	$x_0$	$x_1$	$x_2$	$x_3$
0	X	X		
1	Q	X	X	X
2	X	X		
3	X		X	

current assignment  $\{x_0 = 1\}$

$x_1 \in \{3\}, x_2 \in \{0, 2\}, x_3 \in \{0, 2, 3\}$

# Backtracking Search 4-Queens Problem

variable  $x_1$  value 3 (minimum-remaining-values heuristic)

$$\{x_1 = 3\}$$

forward checking

remove 2 from dom( $x_2$ )

remove 3 from dom( $x_3$ )

	$x_0$	$x_1$	$x_2$	$x_3$
0	X	X		
1	Q	X	X	X
2	X	X	X	
3	X	Q	X	X

current assignment  $\{x_0 = 1, x_1 = 3\}$

$$x_2 \in \{0\} \quad x_3 \in \{0, 2\}$$

variable  $x_2$  value 0 (minimum-remaining-values heuristic)

$$\{x_2 = 0\}$$

forward checking

remove 0 from dom( $x_3$ )

	$x_0$	$x_1$	$x_2$	$x_3$
0	X	X	Q	X
1	Q	X	X	X
2	X	X	X	
3	X	Q	X	X

current assignment  $\{x_0 = 1, x_1 = 3, x_2 = 0\}$

$$x_3 \in \{2\}$$

variable  $x_3$  value 2

$$\{x_3 = 2\}$$

	$x_0$	$x_1$	$x_2$	$x_3$
0	X	X	Q	X
1	Q	X	X	X
2	X	X	X	Q
3	X	Q	X	X

solution:  $\{x_0 = 1, x_1 = 3, x_2 = 0, x_3 = 2\}$

# 4Queens Problem Local Search

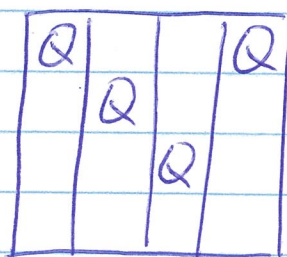
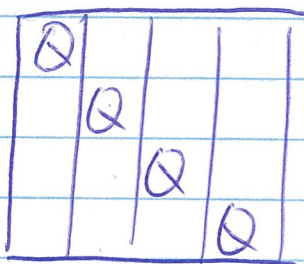
Variables:  $x_0, x_1, x_2, x_3$  where  $x_i$  is the row position of the queen in column  $i$ . One queen per column.

Domain:  $\{0, 1, 2, 3\}$ .

Constraints:  $\leftarrow$

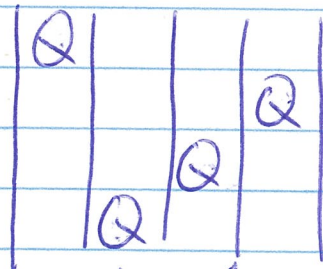
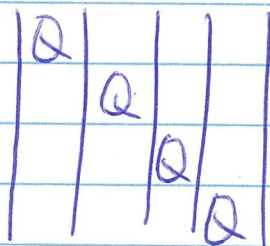
Neighbour relation:

Version A: move a single queen to another square in the same column.



$$3 \times 4 = 12$$

Version B: swap the row positions of two queens.



$$\binom{4}{2} = \frac{4 \times 3}{2 \times 1} = 6$$

$$\text{cost} = 4$$

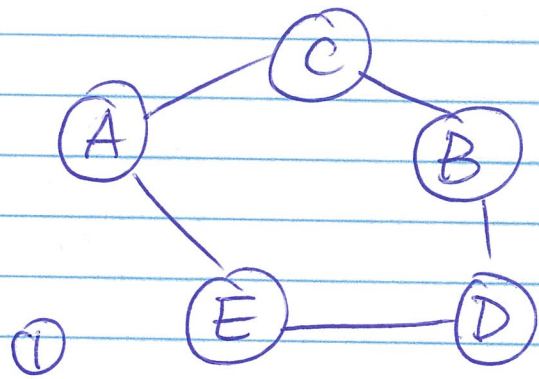
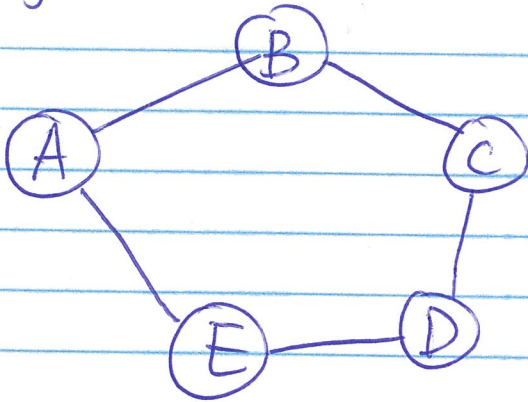
Cost function: The number of pairs of queens attacking each other directly or indirectly.

# Traveling Salesperson Problem

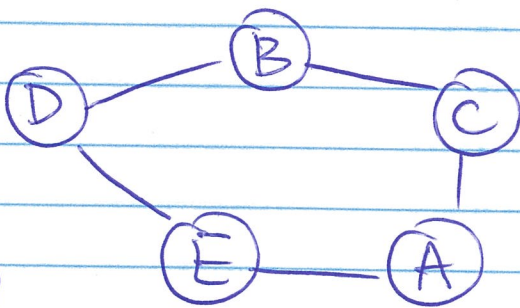
State: a complete tour of all the cities starting from city A and ending with city A. The tour visits each city exactly once.

Cost function: The sum of the costs of all the edges on the tour.

Neighbour relation:

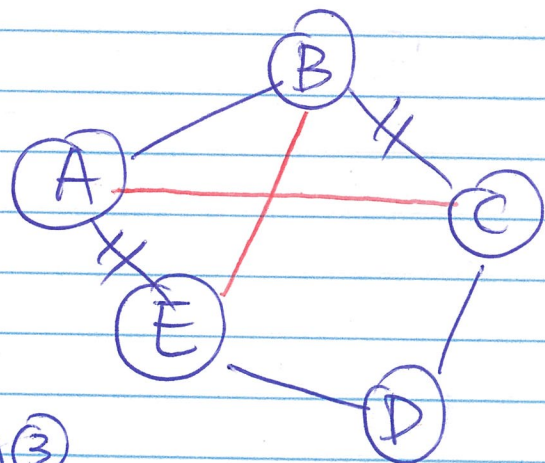


① swap neighbouring nodes



②

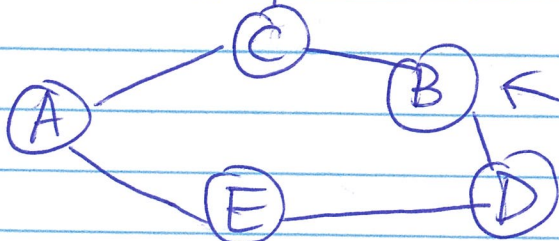
swap any 2 nodes



④ move one node to a different position in the tour

③

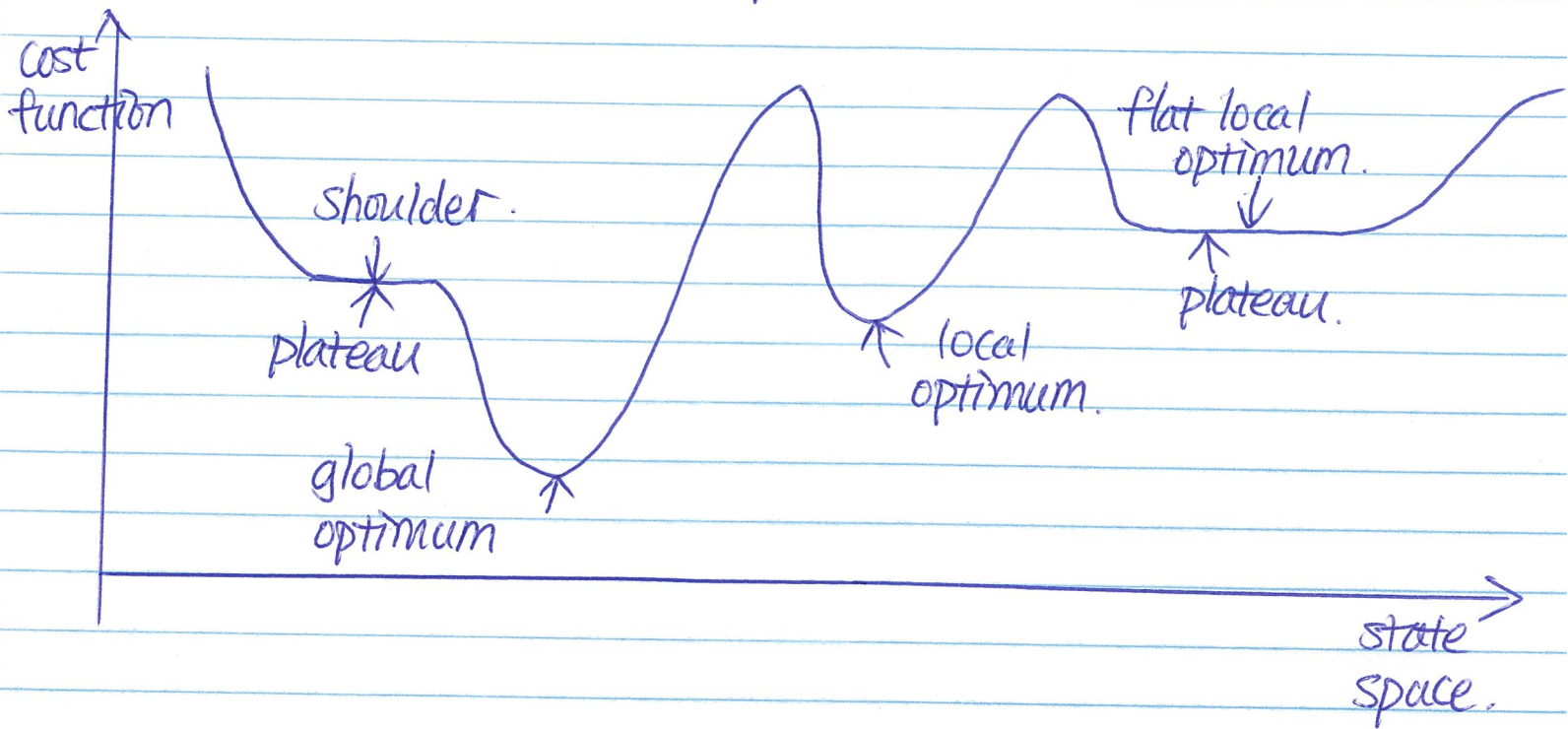
2-swap. reconnect 2 edges



← moved B from between A & C to between C & D.



## A one-dimensional state space.



Local optima: A state  $s^*$  is locally optimal if  $c(s^*) \leq c(s)$  for every neighbour  $s$  of  $s^*$ .

A Plateau is a local optimum.