Examples of The Arc Consistency Algorithm

Alice Gao

Fall 2018

1 Forward Checking

The starting domains: $x_0 = 0, dom(x_1) = \{0, 1, 2, 3\}, dom(x_2) = \{0, 1, 2, 3\}, dom(x_3) = \{0, 1, 2, 3\}$.

We have just assigned $x_0 = 0$. This automatically eliminates all other values in $dom(x_0)$. In forward checking, we never add new constraints to the set.

Since we've just assigned a value to x_0, we will add every constraint that involves x_0 to the set for the other variable in the constraint.

The starting set of constraints:

$(x_1, x_0 \neq x_1), (x_1, |x_0-x_1| \neq 1), (x_2, x_0 \neq x_2), (x_2, |x_0-x_2| \neq 2), (x_3, x_0 \neq x_3), (x_3, |x_0-x_3| \neq 3)$

1. Remove $(x_1, x_0 \neq x_1)$.

   Remove 0 from $dom(x_1)$.

   The resulting domains are $x_0 = 0, dom(x_1) = \{1, 2, 3\}, dom(x_2) = \{0, 1, 2, 3\}, dom(x_3) = \{0, 1, 2, 3\}$.

2. Remove $(x_1, |x_0 - x_1| \neq 1)$.

   Remove 1 from $dom(x_1)$.

   The resulting domains are $x_0 = 0, dom(x_1) = \{2, 3\}, dom(x_2) = \{0, 1, 2, 3\}, dom(x_3) = \{0, 1, 2, 3\}$.

3. Remove $(x_2, x_0 \neq x_2)$.

   Remove 0 from $dom(x_2)$.

   The resulting domains are $x_0 = 0, dom(x_1) = \{2,3\}, dom(x_2) = \{1,2,3\}, dom(x_3) = \{0,1,2,3\}$.

4. Remove $(x_2, |x_0 - x_2| \neq 2)$.

   Remove 2 from $dom(x_2)$.

   The resulting domains are $x_0 = 0, dom(x_1) = \{2,3\}, dom(x_2) = \{1,3\}, dom(x_3) = \{0,1,2,3\}$.

5. Remove $((x_3, x_0 \neq x_3)$.

   Remove 0 from $dom(x_3)$.

   The resulting domains are $x_0 = 0, dom(x_1) = \{2,3\}, dom(x_2) = \{1,3\}, dom(x_3) = \{1,2,3\}$.

6. Remove $(x_3, |x_0 - x_3| \neq 3)$.

   Remove 3 from $dom(x_3)$.

   The resulting domains are $x_0 = 0, dom(x_1) = \{2,3\}, dom(x_2) = \{1,3\}, dom(x_3) = \{1,2\}$.

2 Maintaining Arc Consistency

Maintaining Arc Consistency is essentially running the AC-3 algorithm on the CSP after the assignment. Consider the same example again.

The starting domains: $x_0 = 0, dom(x_1) = \{0, 1, 2, 3\}, dom(x_2) = \{0, 1, 2, 3\}, dom(x_3) = \{0, 1, 2, 3\}$.

We have just assigned $x_0 = 0$. This automatically eliminates all other values in $dom(x_0)$.

Again, we start by adding every constraint that involves x_0 to the set for the other variable in the constraint.

The starting set of constraints:

$(x_1, x_0 \neq x_1), (x_1, |x_0 - x_1| \neq 1), (x_2, x_0 \neq x_2), (x_2, |x_0 - x_2| \neq 2), (x_3, x_0 \neq x_3), (x_3, |x_0 - x_3| \neq 3)$

1. Remove $(x_1, x_0 \neq x_1)$.

   Remove 0 from $dom(x_1)$.

   Add the following constraints to the set.

   $(x_2, x_1 \neq x_2), (x_2, |x_1 - x_2| \neq 1, (x_3, x_1 \neq x_3), (x_3, |x_1 - x_3| \neq 2)$

   The resulting domains are $x_0 = 0, dom(x_1) = \{1, 2, 3\}, dom(x_2) = \{0, 1, 2, 3\}, dom(x_3) = \{0, 1, 2, 3\}$.

   The resulting set of constraints is $(x_1, |x_0 - x_1| \neq 1), (x_2, x_0 \neq x_2), (x_2, |x_0 - x_2| \neq 2), (x_3, x_0 \neq x_3), (x_3, |x_0 - x_3| \neq 3), (x_2, x_1 \neq x_2), (x_2, |x_1 - x_2| \neq 1, (x_3, x_1 \neq x_3), (x_3, |x_1 - x_3| \neq 2)$

   (Technically, we should add back constraints for x_0 as well. However, since we've just assigned a value to x_0, we will omit all such constraints.)

2. Remove $(x_1, |x_0 - x_1| \neq 1)$.

   Remove 1 from $dom(x_1)$.

   Add no constraint to the set.

   The resulting domains are $x_0 = 0, dom(x_1) = \{2, 3\}, dom(x_2) = \{0, 1, 2, 3\}, dom(x_3) = \{0, 1, 2, 3\}$.

   The resulting set of constraints is $(x_2, x_0 \neq x_2), (x_2, |x_0 - x_2| \neq 2), (x_3, x_0 \neq x_3), (x_3, |x_0 - x_3| \neq 3), (x_2, x_1 \neq x_2), (x_2, |x_1 - x_2| \neq 1, (x_3, x_1 \neq x_3), (x_3, |x_1 - x_3| \neq 2)$

3. Remove $(x_2, x_0 \neq x_2)$.

   Remove 0 from $dom(x_2)$.

   Add the following constraints to the set.

   $(x_1, x_1 \neq x_2), (x_1, |x_1 - x_2| \neq 1), (x_3, x_2 \neq x_3), (x_3, |x_2 - x_3| \neq 1)$

   The resulting domains $x_0 = 0, dom(x_1) = \{2, 3\}, dom(x_2) = \{1, 2, 3\}, dom(x_3) = \{0, 1, 2, 3\}$.

   The resulting set of constraints is $(x_2, |x_0 - x_2| \neq 2), (x_3, x_0 \neq x_3), (x_3, |x_0 - x_3| \neq 3), (x_2, x_1 \neq x_2), (x_2, |x_1 - x_2| \neq 1, (x_3, x_1 \neq x_3), (x_3, |x_1 - x_3| \neq 2), (x_1, x_1 \neq x_2), (x_1, |x_1 - x_2| \neq 1), (x_3, x_2 \neq x_3), (x_3, |x_2 - x_3| \neq 1)$

4. Remove $(x_2, |x_0 - x_2| \neq 2)$.

   Remove 2 from $dom(x_2)$.

   Add no constraint to the set.

   The resulting domains are $x_0 = 0, dom(x_1) = \{2, 3\}, dom(x_2) = \{1, 3\}, dom(x_3) = \{0, 1, 2, 3\}$.

   The resulting set of constraints is $(x_3, x_0 \neq x_3), (x_3, |x_0 - x_3| \neq 3), (x_2, x_1 \neq x_2), (x_2, |x_1 - x_2| \neq 1, (x_3, x_1 \neq x_3), (x_3, |x_1 - x_3| \neq 2), (x_1, x_1 \neq x_2), (x_1, |x_1 - x_2| \neq 1), (x_3, x_2 \neq x_3), (x_3, |x_2 - x_3| \neq 1)$

5. Remove $((x_3, x_0 \neq x_3)$.

   Remove 0 from $dom(x_3)$.

   Add the following constraints to the set. $(x_1, x_1 \neq x_3), (x_1, |x_1 - x_3| \neq 2), (x_2, x_2 \neq x_3), (x_2, |x_2 - x_3| \neq 1)$

   The resulting domains are $x_0 = 0, dom(x_1) = \{2, 3\}, dom(x_2) = \{1, 3\}, dom(x_3) = \{1, 2, 3\}$.

   The resulting set of constraints is $(x_3, |x_0 - x_3| \neq 3), (x_2, x_1 \neq x_2), (x_2, |x_1 - x_2| \neq 1, (x_3, x_1 \neq x_3), (x_3, |x_1 - x_3| \neq 2), (x_1, x_1 \neq x_2), (x_1, |x_1 - x_2| \neq 1), (x_3, x_2 \neq x_3), (x_3, |x_2 - x_3| \neq 1), (x_1, x_1 \neq x_3), (x_1, |x_1 - x_3| \neq 2), (x_2, x_2 \neq x_3), (x_2, |x_2 - x_3| \neq 1)$

6. Remove $(x_3, |x_0 - x_3| \neq 3)$.

   Remove 3 from $dom(x_3)$.

   Add no constraint to the set.

   The resulting domains are $x_0 = 0, dom(x_1) = \{2,3\}, dom(x_2) = \{1,3\}, dom(x_3) = \{1,2\}$.

   The resulting set of constraints is $(x_2, x_1 \neq x_2), (x_2, |x_1 - x_2| \neq 1, (x_3, x_1 \neq x_3), (x_3, |x_1 - x_3| \neq 2), (x_1, x_1 \neq x_2), (x_1, |x_1 - x_2| \neq 1), (x_3, x_2 \neq x_3), (x_3, |x_2 - x_3| \neq 1), (x_1, x_1 \neq x_3), (x_1, |x_1 - x_3| \neq 2), (x_2, x_2 \neq x_3), (x_2, |x_2 - x_3| \neq 1)$

7. Remove $(x_2, x_1 \neq x_2)$.

   Do nothing.

   The resulting domains are $x_0 = 0, dom(x_1) = \{2,3\}, dom(x_2) = \{1,3\}, dom(x_3) = \{1,2\}$.

   The resulting set of constraints is $(x_2, |x_1 - x_2| \neq 1, (x_3, x_1 \neq x_3), (x_3, |x_1 - x_3| \neq 2), (x_1, x_1 \neq x_2), (x_1, |x_1 - x_2| \neq 1), (x_3, x_2 \neq x_3), (x_3, |x_2 - x_3| \neq 1), (x_1, x_1 \neq x_3), (x_1, |x_1 - x_3| \neq 2), (x_2, x_2 \neq x_3), (x_2, |x_2 - x_3| \neq 1)$

8. Remove $(x_2, |x_1 - x_2| \neq 1$.

   Do nothing.

   The resulting domains are $x_0 = 0, dom(x_1) = \{2,3\}, dom(x_2) = \{1,3\}, dom(x_3) = \{1,2\}$.

   The resulting set of constraints is $(x_3, x_1 \neq x_3), (x_3, |x_1 - x_3| \neq 2), (x_1, x_1 \neq x_2), (x_1, |x_1 - x_2| \neq 1), (x_3, x_2 \neq x_3), (x_3, |x_2 - x_3| \neq 1), (x_1, x_1 \neq x_3), (x_1, |x_1 - x_3| \neq 2), (x_2, x_2 \neq x_3), (x_2, |x_2 - x_3| \neq 1)$

9. Remove $(x_3, x_1 \neq x_3)$.

   Do nothing.

   The resulting domains are $x_0 = 0, dom(x_1) = \{2,3\}, dom(x_2) = \{1,3\}, dom(x_3) = \{1,2\}$.

   The resulting set of constraints is $(x_3, |x_1 - x_3| \neq 2), (x_1, x_1 \neq x_2), (x_1, |x_1 - x_2| \neq 1), (x_3, x_2 \neq x_3), (x_3, |x_2 - x_3| \neq 1), (x_1, x_1 \neq x_3), (x_1, |x_1 - x_3| \neq 2), (x_2, x_2 \neq x_3), (x_2, |x_2 - x_3| \neq 1)$

10. Remove $(x_3, |x_1 - x_3| \neq 2)$.

    Do nothing.

    The resulting domains are $x_0 = 0, dom(x_1) = \{2, 3\}, dom(x_2) = \{1, 3\}, dom(x_3) = \{1, 2\}$.

    The resulting set of constraints is $(x_1, x_1 \neq x_2), (x_1, |x_1 - x_2| \neq 1), (x_3, x_2 \neq x_3), (x_3, |x_2 - x_3| \neq 1), (x_1, x_1 \neq x_3), (x_1, |x_1 - x_3| \neq 2), (x_2, x_2 \neq x_3), (x_2, |x_2 - x_3| \neq 1)$

11. Remove $(x_1, x_1 \neq x_2)$.

    Do nothing.

    The resulting domains are $x_0 = 0, dom(x_1) = \{2, 3\}, dom(x_2) = \{1, 3\}, dom(x_3) = \{1, 2\}$.

    The resulting set of constraints is $(x_1, |x_1 - x_2| \neq 1), (x_3, x_2 \neq x_3), (x_3, |x_2 - x_3| \neq 1), (x_1, x_1 \neq x_3), (x_1, |x_1 - x_3| \neq 2), (x_2, x_2 \neq x_3), (x_2, |x_2 - x_3| \neq 1)$

12. Remove $(x_1, |x_1 - x_2| \neq 1)$.

    Remove 2 from $dom(x_1)$.

    Add the following constraints to the set.

    $(x_2, x_1 \neq x_2), (x_3, x_1 \neq x_3), (x_3, |x_1 - x_3| \neq 2)$.

    The resulting domains are $x_0 = 0, dom(x_1) = \{3\}, dom(x_2) = \{1, 3\}, dom(x_3) = \{1, 2\}$.

    The resulting set of constraints is $(x_3, x_2 \neq x_3), (x_3, |x_2 - x_3| \neq 1), (x_1, x_1 \neq x_3), (x_1, |x_1 - x_3| \neq 2), (x_2, x_2 \neq x_3), (x_2, |x_2 - x_3| \neq 1), (x_2, x_1 \neq x_2), (x_3, x_1 \neq x_3), (x_3, |x_1 - x_3| \neq 2)$.

13. Remove $(x_3, x_2 \neq x_3)$.

    Do nothing.

    The resulting domains are $x_0 = 0, dom(x_1) = \{3\}, dom(x_2) = \{1, 3\}, dom(x_3) = \{1, 2\}$.

    The resulting set of constraints is $(x_3, |x_2 - x_3| \neq 1), (x_1, x_1 \neq x_3), (x_1, |x_1 - x_3| \neq 2), (x_2, x_2 \neq x_3), (x_2, |x_2 - x_3| \neq 1), (x_2, x_1 \neq x_2), (x_3, x_1 \neq x_3), (x_3, |x_1 - x_3| \neq 2)$.

14. Remove $(x_3, |x_2 - x_3| \neq 1)$.

    Remove 2 from $dom(x_3)$.

    Add the following constraints to the set.

    $(x_2, x_2 \neq x_3)$

    The resulting domains are $x_0 = 0, dom(x_1) = \{3\}, dom(x_2) = \{1, 3\}, dom(x_3) = \{1\}$.

    The resulting set of constraints is $(x_1, x_1 \neq x_3), (x_1, |x_1 - x_3| \neq 2), (x_2, x_2 \neq x_3), (x_2, |x_2 - x_3| \neq 1), (x_2, x_1 \neq x_2), (x_3, x_1 \neq x_3), (x_3, |x_1 - x_3| \neq 2), (x_2, x_2 \neq x_3)$.

15. Remove $(x_1, x_1 \neq x_3)$.

    Do nothing.

    The resulting domains are $x_0 = 0, dom(x_1) = \{3\}, dom(x_2) = \{1, 3\}, dom(x_3) = \{1\}$.

    The resulting set of constraints is $(x_1, |x_1 - x_3| \neq 2), (x_2, x_2 \neq x_3), (x_2, |x_2 - x_3| \neq 1), (x_2, x_1 \neq x_2), (x_3, x_1 \neq x_3), (x_3, |x_1 - x_3| \neq 2), (x_2, x_2 \neq x_3)$.

16. Remove $(x_1, |x_1 - x_3| \neq 2)$.

    Remove 3 from $dom(x_1)$.

    The resulting domains are $x_0 = 0, dom(x_1) = \{\}, dom(x_2) = \{1, 3\}, dom(x_3) = \{1\}$.

    The domain of x_1 is empty. No solution! Backtrack!