Sudoku

	0	1	2	3	4	5	6	7	8
0									
1									
2									
3									
4									
5									
6									
7									
8									

Variables: x_{ij} is the value in row i and column j where i is in {0, …, 8} and j is in {0, …, 8}.

Domains: If the initial value of x_{ij} is k, then dom(x_{ij}) = {k}. Otherwise, dom(x_{ij}) = {1, …, 9}.

Also could encode the initial value as a constraint. (Talk about River Crossing.)

Constraints:

- All the numbers in each row are different.

The constraint ``All numbers in row 0 are different" can be expressed as follows.

$$\text{alldifferent}(x_{00}, x_{01}, x_{02}, x_{03}, x_{04}, x_{05}, x_{06}, x_{07}, x_{08})$$

- All the numbers in each column are different.

The constraint ``All numbers in column 0 are different" can be expressed as follows.

$$\text{alldifferent}(x_{00}, x_{10}, x_{20}, x_{30}, x_{40}, x_{50}, x_{60}, x_{70}, x_{80})$$

- All the numbers in each sub-grid are different.

The constraint ``All numbers in the top left sub-grid are different" can be expressed as follows.

$$\text{alldifferent}(x_{00}, x_{01}, x_{02}, x_{10}, x_{11}, x_{12}, x_{20}, x_{21}, x_{22})$$

Convert a row constraint to binary constraints:
$x_{0a} \neq x_{0b}$, where a and b are in {0, …, 8} and $a \neq b$.

Convert a row constraint to tertiary constraints:
alldifferent(x_{0a}, x_{0b}, x_{0c}) where a, b, c are in {0, …, 8} and a, b, and c are all different.

4-Queens Problem

	0	1	2	3
0				
1				
2				
3				

Variables: x_i is the row position of the queen in column i, where i is in {0, 1, 2, 3}.

Domains: $\mathrm{dom}(x_i) = \{0, 1, 2, 3\}$ for all x_i.

Example of a state: 3 2 0 1

- The first queen is in column 0 and row 3.

	0	1	2	3
0			Q	
1				Q
2		Q		
3	Q			

Constraints: No pair of queens are in the same row or the same diagonal.

As a propositional formula:
$((x_0 \neq x_2) \wedge (|x_0 - x_2|=2))$

General formula:
$(\forall i\ (\forall j\ ((i \neq j) \rightarrow ((x_i \neq x_j) \wedge (|x_i - x_j|=|i - j|)))))$
If i and j are two different columns, then the row positions of the two queens are different and they are not in the same diagonal.

As a table: x_0 and x_2 are not in the same row nor in the same diagonal.

x_0	x_2
0	1
0	3
1	0
1	2
2	1
2	3
3	0
3	2