

Propositional Logic: The basic rules of natural deduction.

conjunction
rules

$\wedge$

introduction

$$\frac{a \quad b}{(a \wedge b)} \wedge i$$

elimination

$$\frac{(a \wedge b)}{a} \wedge e \quad \frac{(a \wedge b)}{b} \wedge e$$

disjunction
rules

$\vee$

$$\frac{a}{(a \vee b)} \vee i \quad \frac{b}{(a \vee b)} \vee i$$

$$\frac{(a \vee b) \quad \begin{array}{|c|} \hline a \\ \hline \vdots \\ \hline c \\ \hline \end{array} \quad \begin{array}{|c|} \hline b \\ \hline \vdots \\ \hline c \\ \hline \end{array}}{c} \vee e$$

implication
rules

$\rightarrow$

$$\frac{\begin{array}{|c|} \hline a \\ \hline \vdots \\ \hline b \\ \hline \end{array}}{(a \rightarrow b)} \rightarrow i$$

$$\frac{a \quad (a \rightarrow b)}{b} \rightarrow e$$

negation
rules

$\neg$

$$\frac{\begin{array}{|c|} \hline a \\ \hline \vdots \\ \hline \perp \\ \hline \end{array}}{(\neg a)} \neg i$$

$$\frac{a \quad (\neg a)}{\perp} \neg e / \perp i$$

contradiction
rules

$\perp$

$$\frac{a \quad (\neg a)}{\perp} \perp i / \neg e$$

$$\frac{\perp}{a} \perp e$$

double negation
rules

$\neg \neg$

$$\frac{(\neg(\neg a))}{a} \neg \neg e$$

Some useful derived rules

$$\frac{(a \rightarrow b) \quad (\neg b)}{(\neg a)} \text{ MT (modus tollens)}$$

$$\frac{a}{(\neg(\neg a))} \neg\neg\text{-i}$$

$$\frac{\boxed{\begin{array}{c} (\neg a) \\ \vdots \\ \perp \end{array}}}{a} \text{ PBC (proof by contradiction).}$$

$$\overline{(a \vee (\neg a))} \text{ LEM (law of excluded middle)}$$