Predicate Logic: Logical Consequence

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Lecture 15

Outline

Learning goals

By the end of this lecture, you should be able to:

- Define logical consequence for predicate logic.
- Prove that a logical consequence holds.
- Prove that a logical consequence does not hold.

Definition of Logical Consequence

Define the symbols.

- Σ is a set of predicate formulas.
- A is a predicate formula.

$\Sigma \vDash A$
(Σ logically implies A)
(A is a logical consequence of Σ)
iff for every valuation v, if $\Sigma^v = 1$, then $A^v = 1$.

Prove a logical consequence

Consider the logical consequence $\Sigma \models A$.
To prove that the logical consequence holds, we need to consider

(A) Every valuation v such that $\Sigma^v = 1$.

(B) Every valuation v such that $\Sigma^v = 0$.

(C) One valuation v such that $\Sigma^v = 1$.

(D) One valuation v such that $\Sigma^v = 0$.

Disprove a logical consequence

Consider the logical consequence $\Sigma \models A$.
To prove that the logical consequence does NOT hold, we need to consider

(A) Every valuation v such that $\Sigma^v = 1$ and $A^v = 1$.

(B) Every valuation v such that $\Sigma^v = 1$ and $A^v = 0$.

(C) One valuation v such that $\Sigma^v = 1$ and $A^v = 1$.

(D) One valuation v such that $\Sigma^v = 1$ and $A^v = 0$.

Example: Prove/Disprove a logical consequence

Consider the logical consequence below.

$$\forall x \, \neg A(x) \vDash \neg(\exists x \, A(x)).$$

If the logical consequence holds, prove it.
If it does not hold, provide a counterexample.

Example: Prove/Disprove a logical consequence

Consider the logical consequence below.

$$\forall x(A(x) \to B(x)) \vDash \forall x A(x) \to \forall x B(x)$$

If the logical consequence holds, prove it.
If it does not hold, provide a counterexample.

Example: Prove/Disprove a logical consequence

Consider the logical consequence below.

$$\forall x A(x) \to \forall x B(x) \vDash \forall x (A(x) \to B(x))$$

If the logical consequence holds, prove it.
If it does not hold, provide a counterexample.

Example: Prove/Disprove the logical consequence

Prove the following

$$\exists x(A(x) \wedge B(x)) \vDash \exists x A(x) \wedge \exists x B(x).$$

Example: Prove/Disprove the logical consequence

Prove the following

$$\exists x A(x) \land \exists x B(x) \not\models \exists x (A(x) \land B(x)).$$

Revisiting the learning goals

By the end of this lecture, you should be able to:

- Define logical consequence for predicate logic.
- Prove that a logical consequence holds.
- Prove that a logical consequence does not hold.