Array Assignment

Tue Nov 21

A is an array of n integers $A[1], A[2], \ldots, A[n]$.

$\{\ ???\ \}$ — if $x = y$, ??? should be $1 = 0$.

$A[x] = 1;$ — if $x \neq y$, ??? should be $A[y] = 0$.

$\{\ A[y] = 0\ \}$

When we use variables as indices into arrays, we need to account for multiple cases for many possible values that the variables can take.

Solutions: Write down the sequence of changes first and resolve them when we need to prove any implied conditions.

$$\{\ Q[\underbrace{A\{e_1 \leftarrow e_2\}}_{\text{new array}} / A]\ \}$$

original array

$A[e_1] = e_2;$

$\{\ Q\ \}$ — array assignment.

For an assignment to an array value $A[e_1] = e_2$, assume that the assignment produced a new array $A\{e_1 \leftarrow e_2\}$.

input: array A — index, value

output: array $A\{e_1 \leftarrow e_2\}$, which is identical to A except the e_i^{th} element is changed to have the value e_2.

$A\{1 \leftarrow 7\}\{2 \leftarrow 14\}[2] = ?$ 14

$A\{1 \leftarrow 2\}\{1 \leftarrow 7\}[1] = ?$ 7

$A\{1 \leftarrow 2\}\{1 \leftarrow 7\}[2] = ?$ $A[2]$

We apply assignments from left to right.

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Prove that the following program satisfies the given triple under partial correctness.

$(\!| (A[x] = x_0) \wedge (A[y] = y_0) |\!)$

$(\!| ((A\{x \leftarrow A[y]\}\{y \leftarrow A[x]\}[x] = y_0) \wedge$
$(A\{x \leftarrow A[y]\}\{y \leftarrow A[x]\}[y] = x_0)) |\!)$ implied

assignment

$t = A[x];$

$(\!| ((A\{x \leftarrow A[y]\}\{y \leftarrow t\}[x] = y_0) \wedge$
$(A\{x \leftarrow A[y]\}\{y \leftarrow t\}[y] = x_0)) |\!)$ assignment

array assignment $A[x] = A[y];$ $Q[A\{x \leftarrow A[y]\}/A]$

$(\!| ((A\{y \leftarrow t\}[x] = y_0) \wedge (A\{y \leftarrow t\}[y] = x_0)) |\!)$ array assignment

array assignment $A[y] = t;$ $Q[A\{y \leftarrow t\}/A]$

$(\!| \underbrace{((A[x] = y_0) \wedge (A[y] = x_0))}_{Q} |\!)$ array assignment

To prove the implied condition, we need to prove the following:

① $A\{x \leftarrow A[y]\}\{y \leftarrow A[x]\}[x] = A[y]$, and

② $A\{x \leftarrow A[y]\}\{y \leftarrow A[x]\}[y] = A[x]$

Proof of ②: The first assignment $x \leftarrow A[y]$ does not matter because the second assignment changes the y^{th} element of A to $A[x]$. This is we we want to show. QED

Proof of ①: Consider 2 cases:

① $x = y$. The second assignment can be rewritten as $x \leftarrow A[y]$, which is the same as the first assignment. Thus, the x^{th} element of A is $A[y]$ after both assignments.

② $x \neq y$. The second assignment does not change the x^{th} element of A. Therefore, the x^{th} element of A is $A[y]$ after both assignments.

QED

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Prove that the following program satisfies the given triple under partial correctness.

$(\!|\ (A[x] = x_0) \land (A[y] = y_0)\ |\!)$

$(\!|\ (A\{x \leftarrow A[y]\}\{y \leftarrow A[x]\}[x] = y_0)$
$\land\ (A\{x \leftarrow A[y]\}\{y \leftarrow A[x]\}[y] = x_0)\ |\!)$ implied

$t = A[x];$

$(\!|\ (A\{x \leftarrow A[y]\}\{y \leftarrow t\}[x] = y_0)$
$\land\ (A\{x \leftarrow A[y]\}\{y \leftarrow t\}[y] = x_0)\ |\!)$ assignment

$A[x] = A[y];$

$(\!|\ ((A\{y \leftarrow t\}[x] = y_0) \land (A\{y \leftarrow t\}[y] = x_0))\ |\!)$ array assignment

$A[y] = t;$

$(\!|\ ((A[x] = y_0) \land (A[y] = x_0))\ |\!)$ array assignment.

To prove the "implied" condition, we need to prove the following:

① $A\{x \leftarrow A[y]\}\{y \leftarrow A[x]\}[x] = A[y]$, and

② $A\{x \leftarrow A[y]\}\{y \leftarrow A[x]\}[y] = A[x]$.

Proof of ①: The first assignment "$x \leftarrow A[y]$" assigns $A[y]$ to the x^{th} element of A. Consider 2 cases for y.

(1) If $y \neq x$, then the second assignment does not change the x^{th} element of A. Thus, the x^{th} element of A is $A[y]$ after the assignments.

(2) If $y = x$, the second assignment can be rewritten as $x \leftarrow A[y]$, which is the same as the first assignment. Thus, the x^{th} element of A is $A[y]$ after the assignments.

Proof of ②: The first assignment does not matter because the second assignment assigns $A[x]$ to the y^{th} element of A, and this is the desired result.

- solutions