

Consider the following argument about the existence of Superman.

If Superman were able and willing to prevent evil, he would do so.

If Superman were unable to prevent evil he would be impotent.

If Superman were unwilling to prevent evil, he would be malevolent.

Superman does not prevent evil.

If Superman exists, he is neither impotent nor malevolent.

Therefore, Superman does not exist.

An impromptu dictionary:

- Impotent: powerless.
- Malevolent: malicious, hostile.

Questions:

1. Translate the premises and the conclusion into propositional logic formulas.
Define your own propositions.

Proposition definitions:

Premises:

Conclusion:

Check your answer with us before proceeding to the following questions.

If you proved that the semantic entailment holds, proceed to question 4. Otherwise, if you proved that the semantic entailment does not hold, proceed to question 5.

4. If you proved that the semantic entailment holds, give a natural deduction proof to show that you can derive the conclusion from the premises.

5. If you proved that the semantic entailment does not hold...
 - a. Could you modify the argument to make the semantic entailment hold? Hint: try removing a part of a premise or the conclusion. If you can, prove that the semantic entailment holds in the modified argument.
 - b. Give a natural deduction proof to show that you can derive the conclusion from the premises in the modified argument.

You may use the additional rules including Modus Tollens, De Morgan's law, and equivalence.