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Structural induction for well-formed formulas

Problem: Prove that every well-formed formula φ has property P.

Define $P(\varphi)$ to be the property in the problem.

Theorem: For every well-formed formula φ, $P(\varphi)$ holds.

Proof by structural induction:

<u>Base case</u>: φ is a propositional variable. We need to prove that $P(\varphi)$ holds.

<u>Induction step</u>:

Case 1: φ is a well-formed formula of the form $(\neg \mathrm{x})$ where x is a well-formed formula.

Induction hypothesis: Assume $P(\mathrm{x})$ holds.
Prove that $P((\neg \mathrm{x}))$ holds.

Case 2: φ is a well-formed formula of the form $(\mathrm{x} * \mathrm{y})$ where x and y are well-formed formulas and * is one of the four binary connectives ($\wedge$, $\vee$, $\rightarrow$, and $\leftrightarrow$).

Induction hypothesis: Assume $P(\mathrm{x})$ and $P(\mathrm{y})$ hold.
Prove that $P((\mathrm{x} * \mathrm{y}))$ holds.

By the principle of structural induction, $P(\varphi)$ holds for every well-formed formula φ.

QED