Nonlinear Dynamics & Vision

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Outline

- Whirlwind tour of nonlinear dynamics
- Overview of higher form vision areas
- Marroquin illusion
- Detailed analysis of competitive networks in rivalry

Nonlinear Dynamics: Equilibria & Linearization

- Coupled, first order equations
- Solve for steady states or equilibrium points
- Compute Jacobian matrix
- Evaluate at each steady state
- Determine eigenvalues

Linearized Stability Analysis

$$J = \begin{pmatrix} \dfrac{\partial F}{\partial x} & \dfrac{\partial F}{\partial y} \\ \dfrac{\partial G}{\partial x} & \dfrac{\partial G}{\partial y} \end{pmatrix}_{x,y=Equilibrium}$$

- All real(eig) < 0, asymptotic stability
- Any real(eig) > 0, unstable
- Pure imaginary eig: theorem does not apply

Nonlinear Oscillations

- Limit cycles: cannot exist in linear systems
- Only one general theorem: Hopf Bifurcation Theorem
- Conservative oscillations (analogous to linear systems) can exist, but not in neural systems
- Chaos can occur in > 2 dimensions
- All neural oscillations are limit cycles!

Hopf Bifurcation Theorem

$$\frac{d\vec{X}}{dt} = \vec{F}\left(\vec{X}, \beta\right)$$

- Equilibrium is asymptotically stable for b < a
- Pair of pure imaginary eig for b = a
- For all other eig, real(eig) < 0
- Equilibrium point unstable for b > a
- Asymptotically stable limit cycle for b > a; or unstable for b < a

Hopf Addendum

$$\frac{d\vec{X}}{dt} = \vec{F}\left(\vec{X},\beta\right)$$

- Limit cycle emerges with infinitesimal amplitude
- Frequency = Im(eig)/2pi
- Hodgkin Huxley Euations exhibit Hopf bifurcation

Nonlinear Oscillations Caveats

- Not all limit cycles emerge via Hopf bifurcations
- Example: Mammalian cortical neurons
- Conservative oscillations (analogous to linear systems) can exist, but not in neural systems
- Chaos can occur in > 2 dimensions
- All neural oscillations are limit cycles!

Conduction & Spiking Dynamics

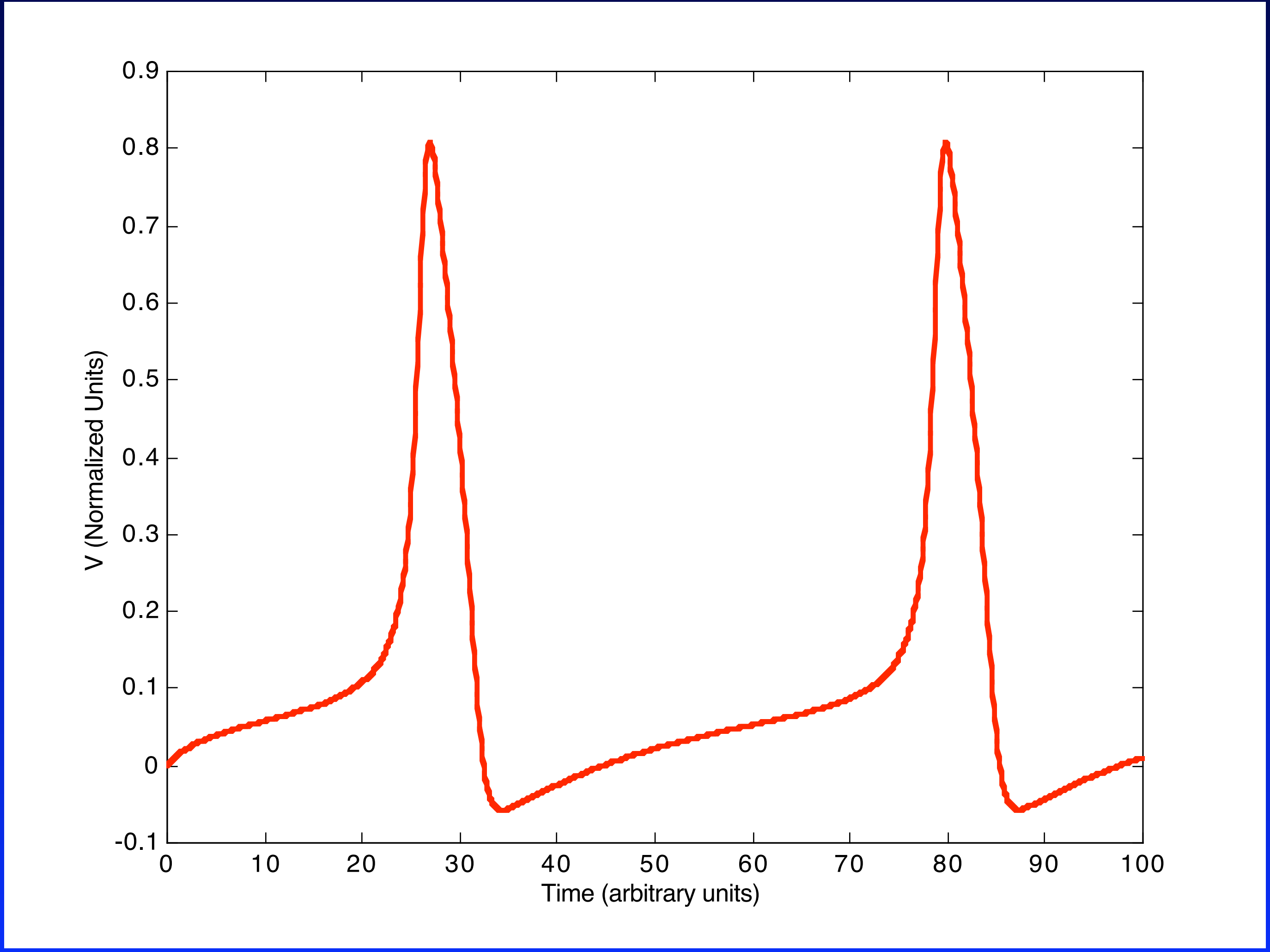
- Two-compartment model (Rinzel et al)
- Excitatory neurons: slow AHP currents
- Simple but accurate cubic model (Wilson, 1999)

(conductance)x(potential)

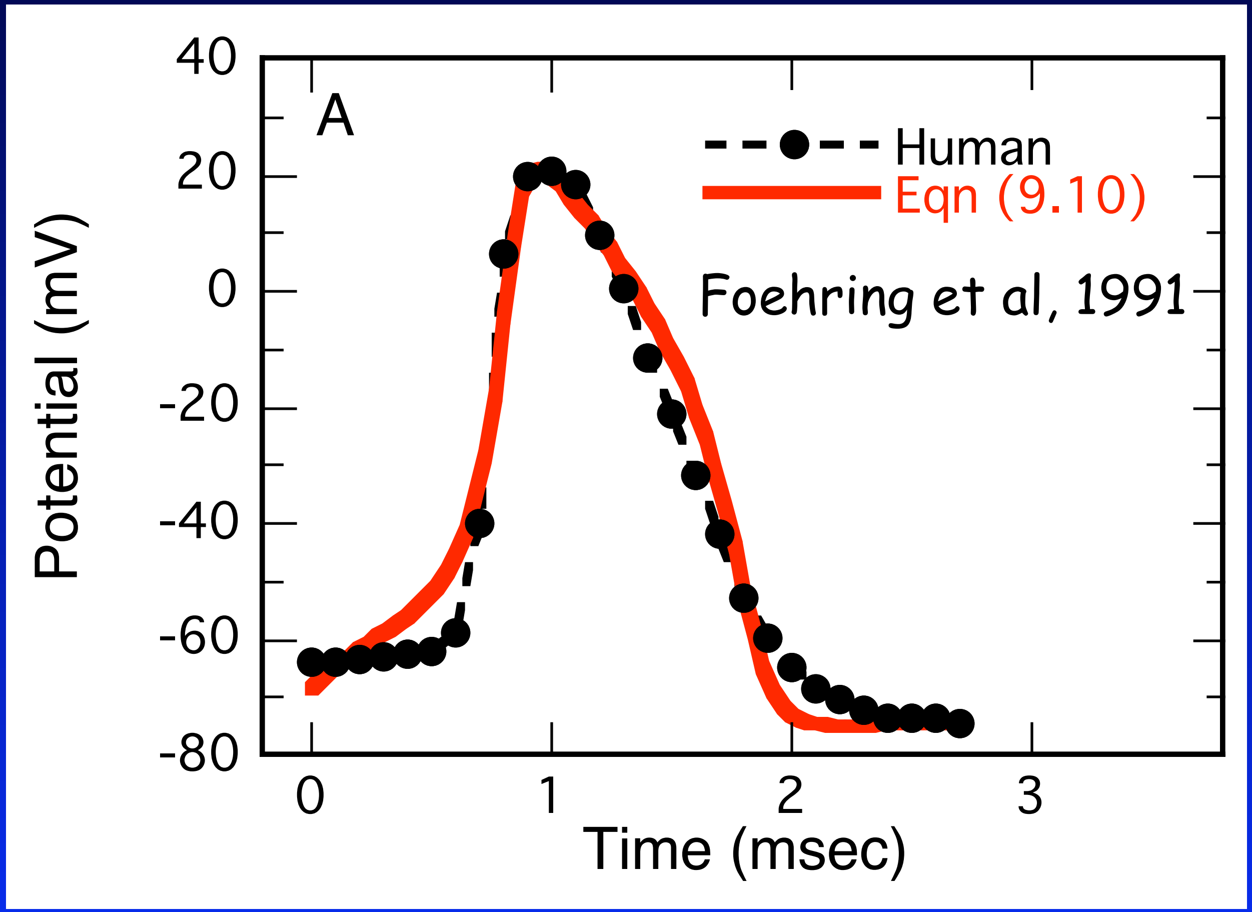
$$\frac{dV}{dt} = -a\left(V^2 - bV\right)\left(V - E_{Na}\right) - R\left(V - E_K\right) + I_{input}$$

$$\tau_R \frac{dR}{dt} = -R + cV^2$$

Phase Plane & Spike Generation



Fit to Human Action Potentials



Spike Rate Adaptation

- Human excitatory cortical neurons: slow hyperpolarizing current
- Causes spike rate adaptation

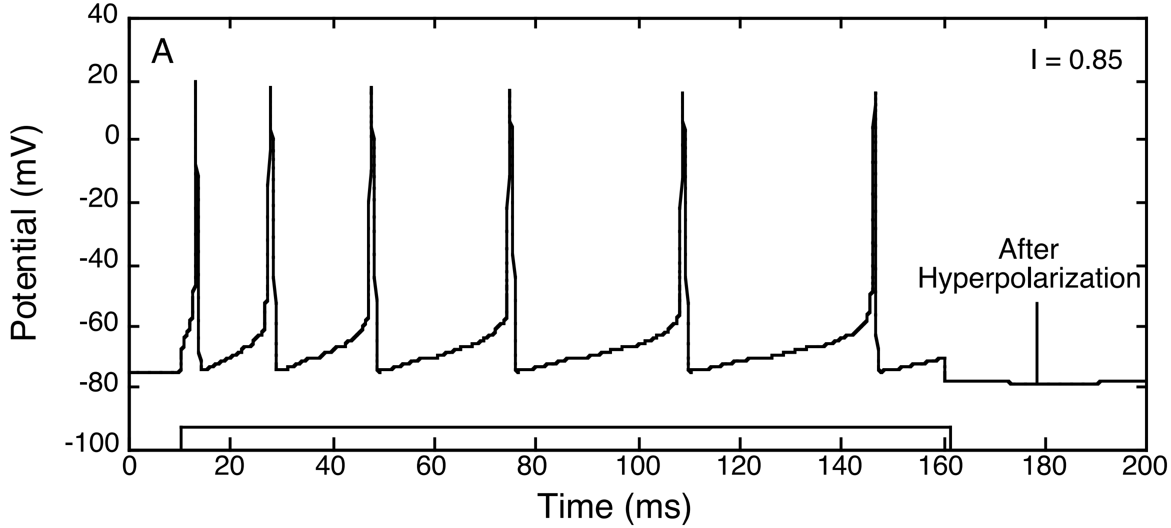
$$\frac{dV}{dt} = -a\left(V^2 - bV\right)\left(V - E_{Na}\right) - R\left(V - E_K\right) - H\left(V - E_K\right) + I$$

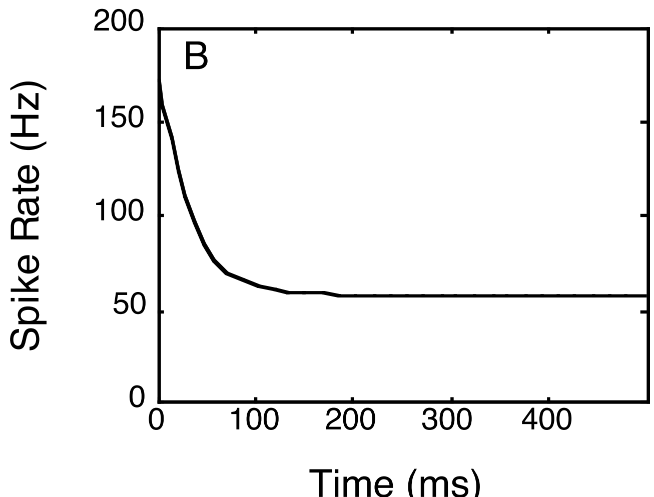
$$\tau_R \frac{dR}{dt} = -R + cV^2$$

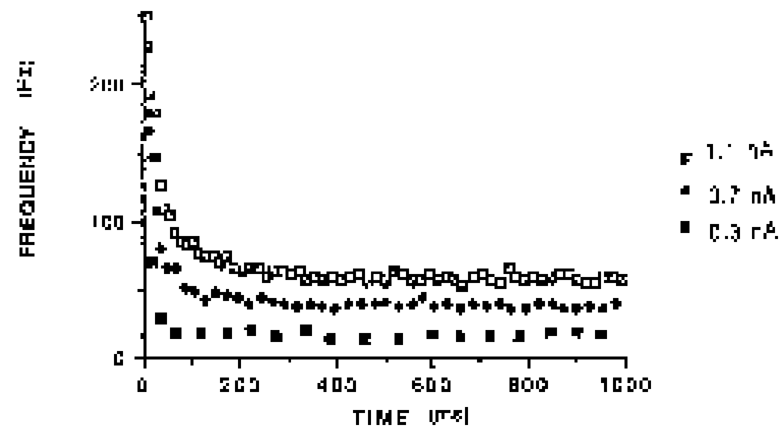
$$\tau_H \frac{dH}{dt} = -H + gV^2$$

Very slow K^+ current

Spike Frequency Adaptation







Controlled by neuromodulators (dopamine, serotonin)

Lyapunov Functions & Memory

$$\frac{dU}{dt} = \sum_i \frac{\partial U}{\partial x_i} \frac{dx_i}{dt}$$

- Positive definite function U(t) around an equilibrium
- dU/dt < 0 along trajectories in a region surrounding equilibrium
- Then equilibrium is asymptotically stable
- Lyapunov fcns. always exist, but not unique

Lyapunov Functions & Memory

- Apply where linearization fails
- Permit estimate of domain of attraction

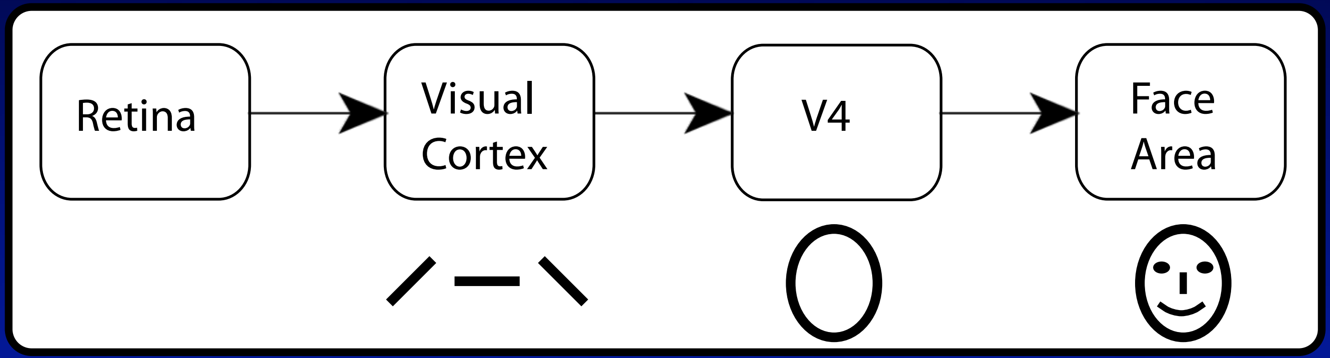
$$\frac{dx}{dt} = -y - x^3$$

$$\frac{dy}{dt} = x - y^3$$

$$U(x,y) = \frac{x^2 + y^2}{2}$$

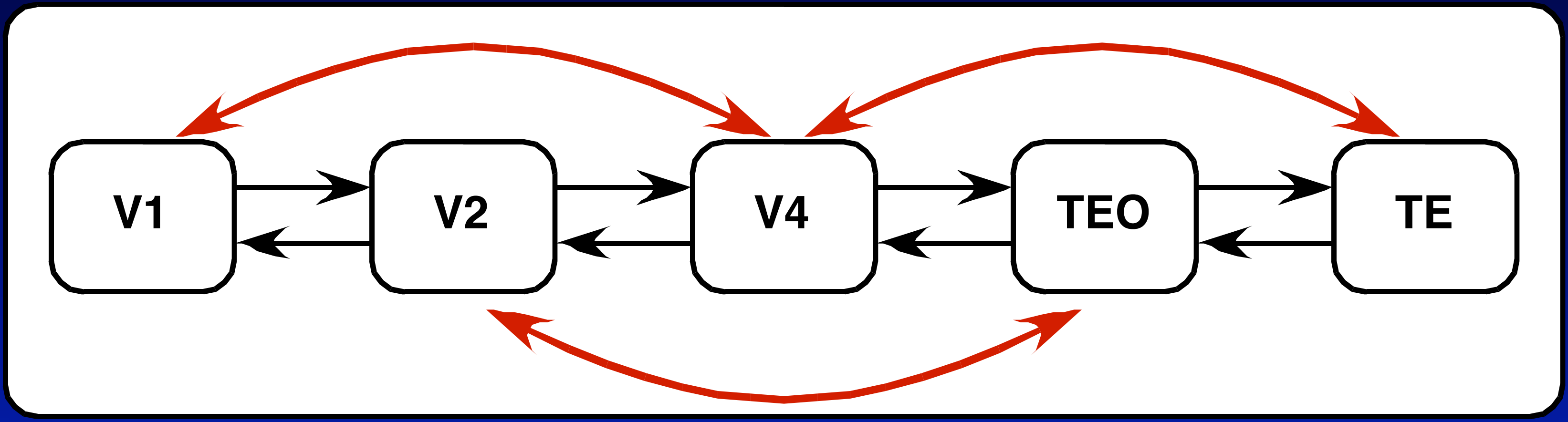
$$\frac{dU}{dt} = -x^4 - y^4$$

Form Pathway Connections



Retina & LGN: Local contrast differences
V1: Contour & edge orientations
V2: Curvature, angles
V4: Elliptical object shapes, T & Y junctions
Higher Areas: Combines V4 info to represent faces & objects

Form Pathway Connections

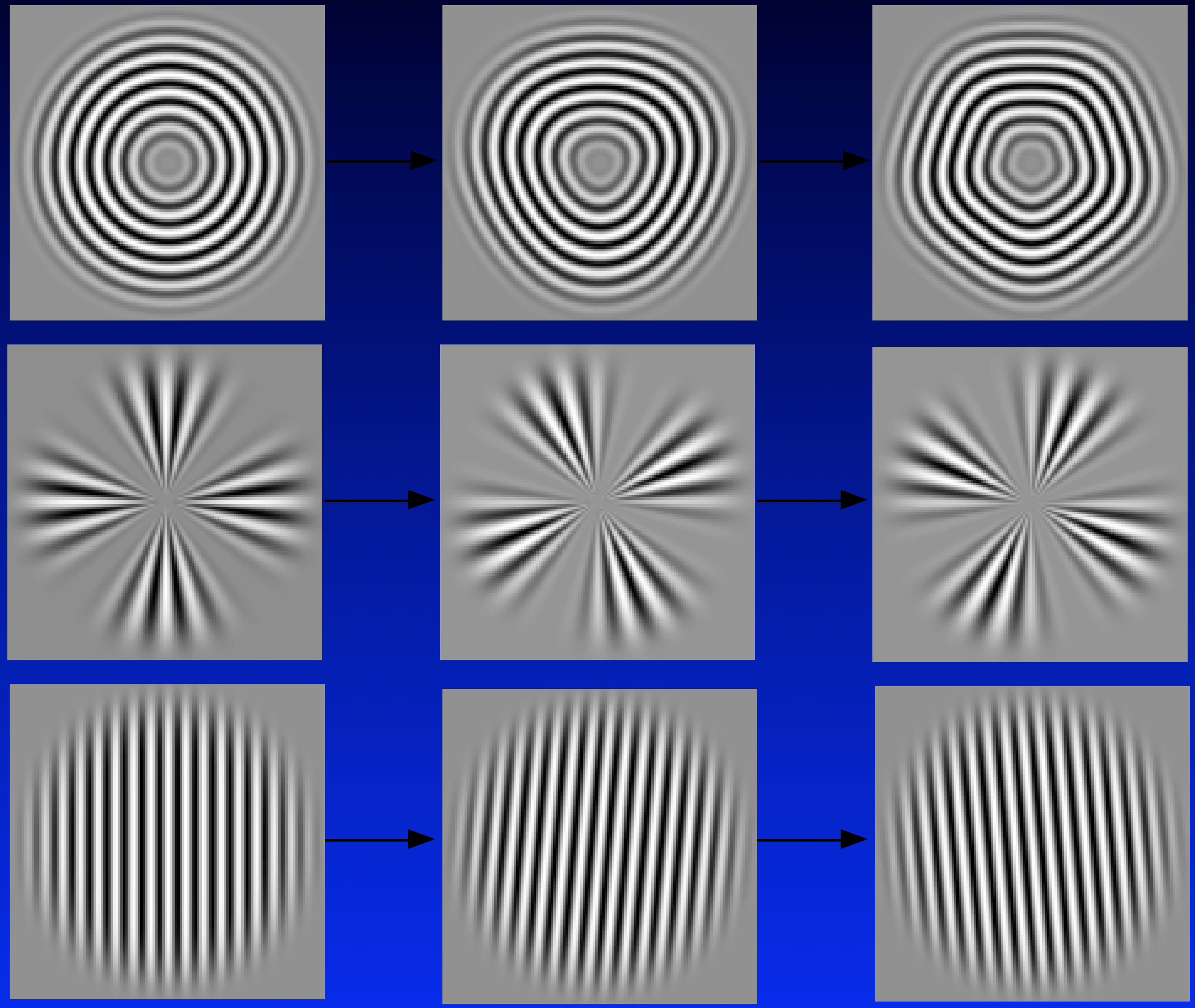


Area to area feedback connections

Skipping connections

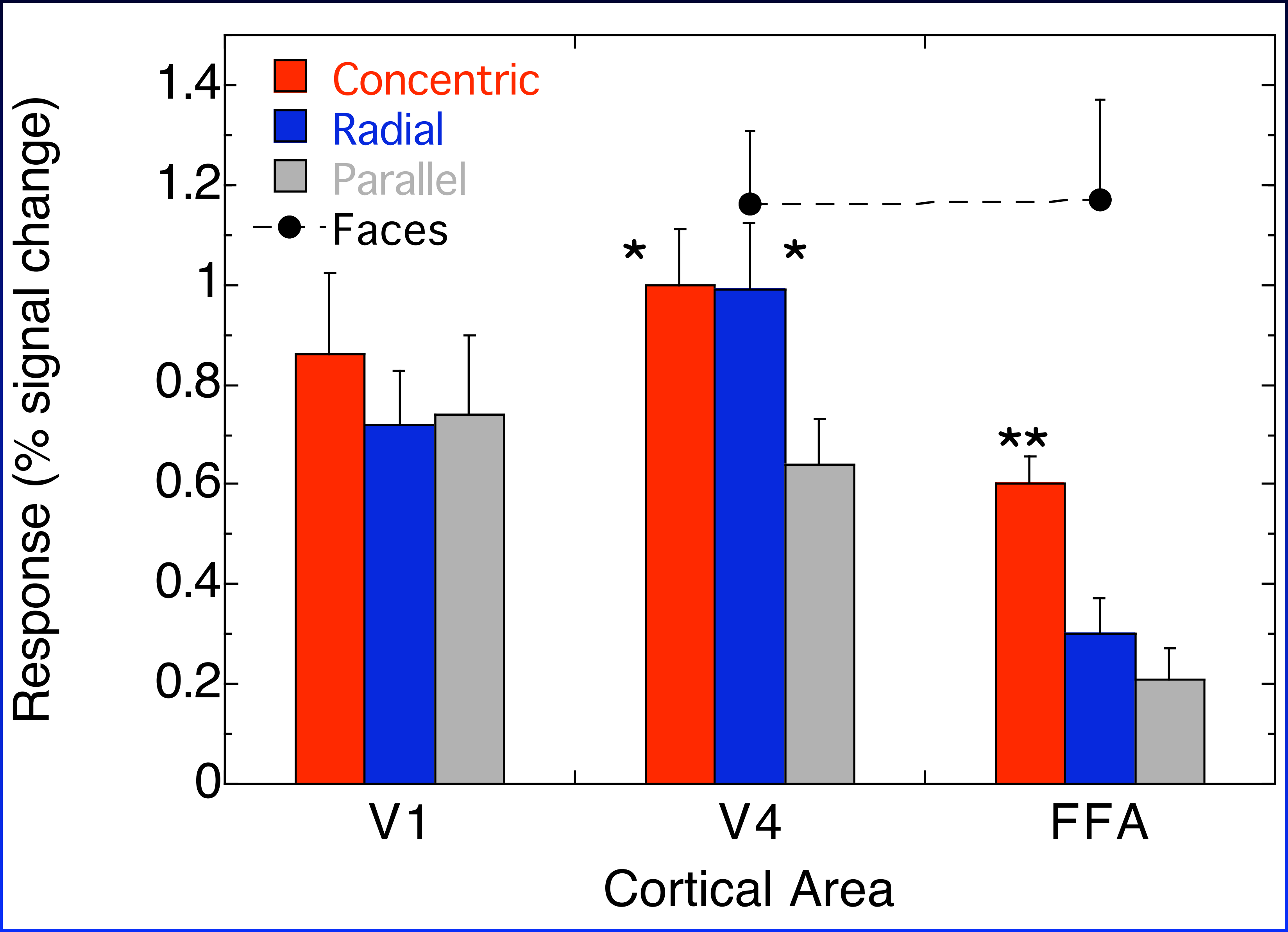
Feedback local but patchy

fMRI of V4

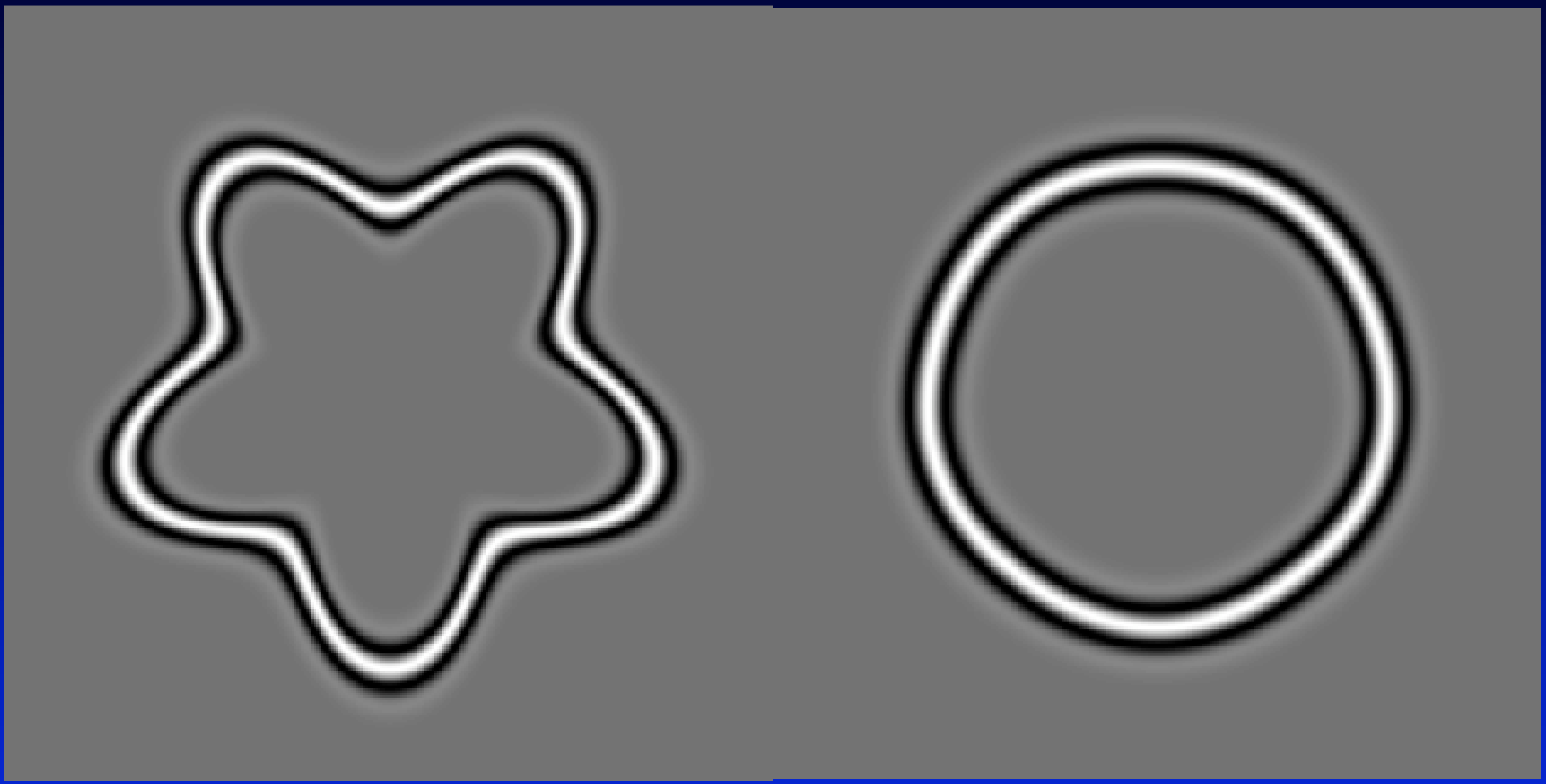


Wilkinson et al, Current Biology (2000)

V4 & FFA Activation



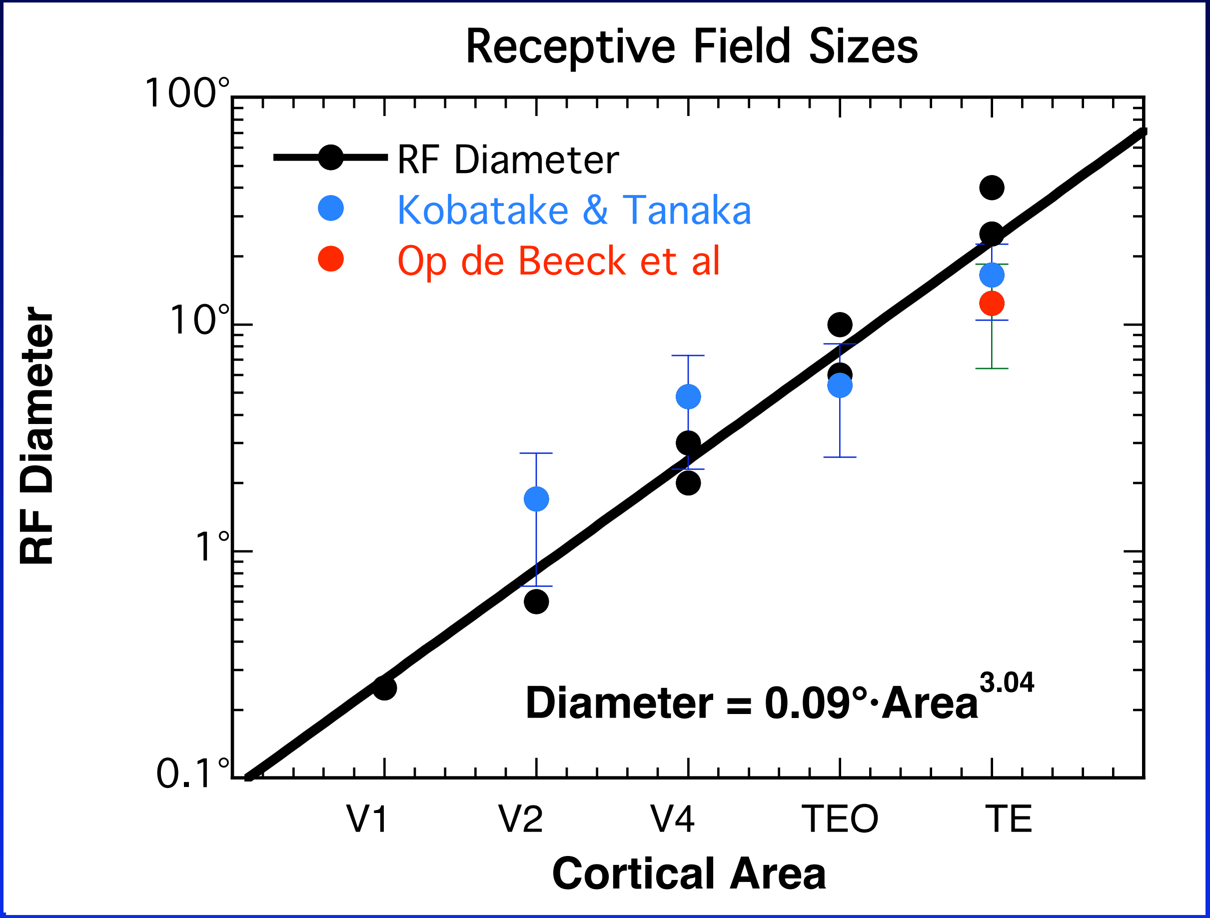
Gallant et al, 2000: V4 damage



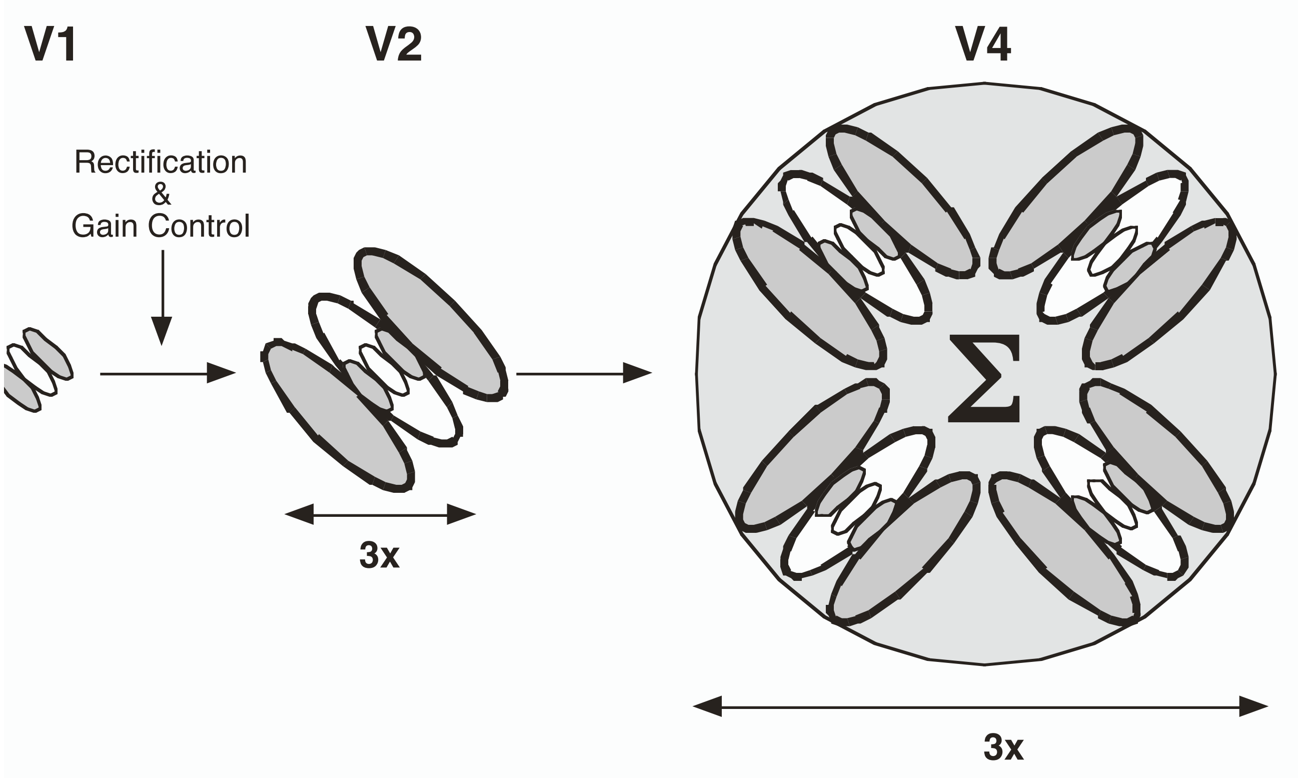
Damaged V4 Normal V4

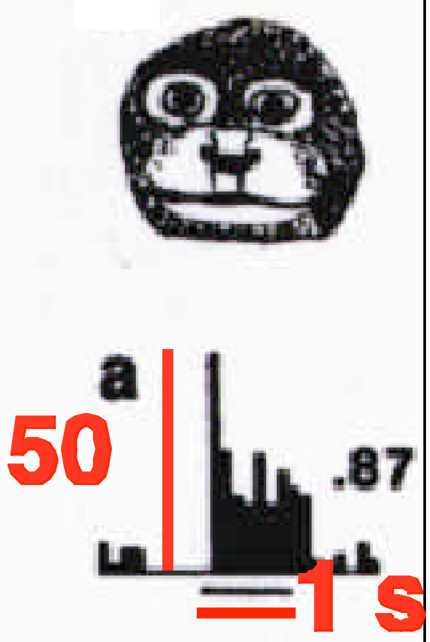
But: Orientation Discrimination normal (normal V1)

Receptive Field Size Increases



Construction of V4 Units

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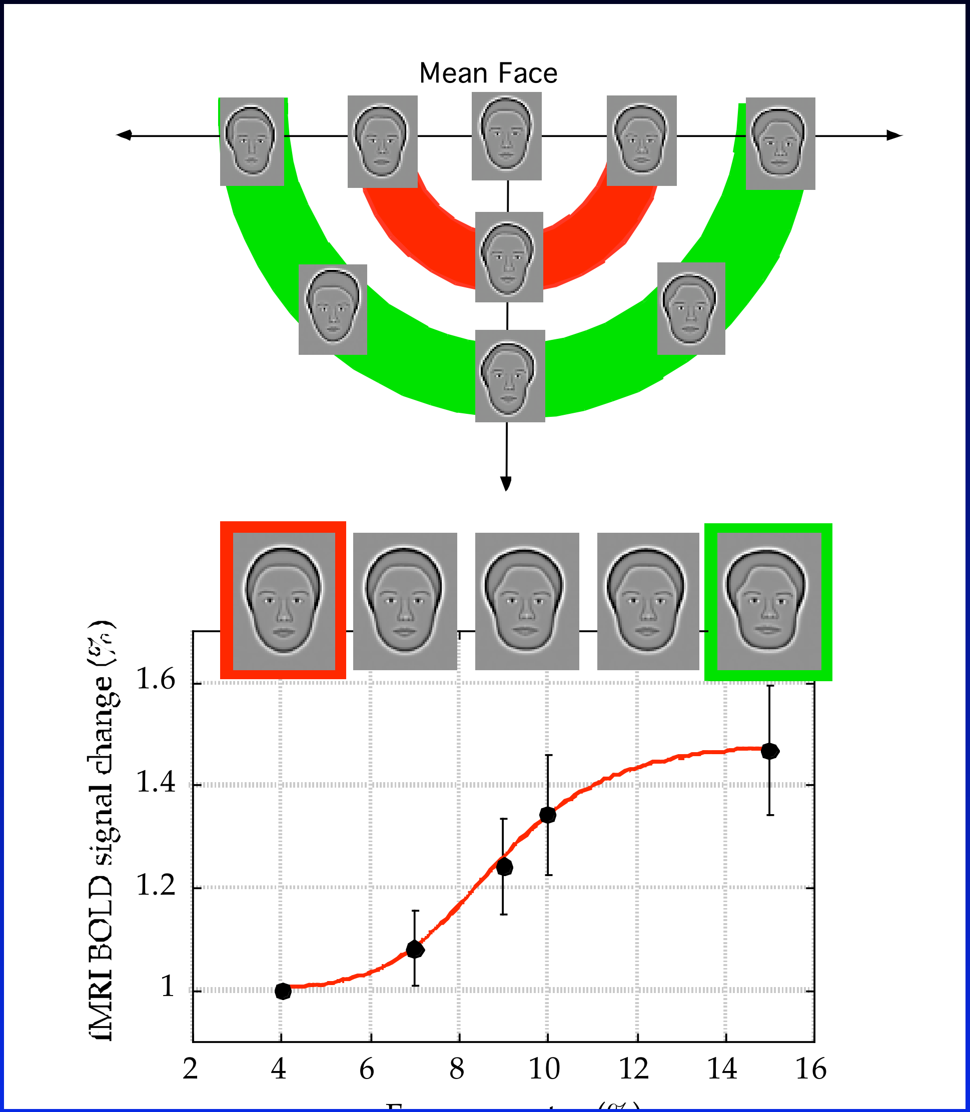
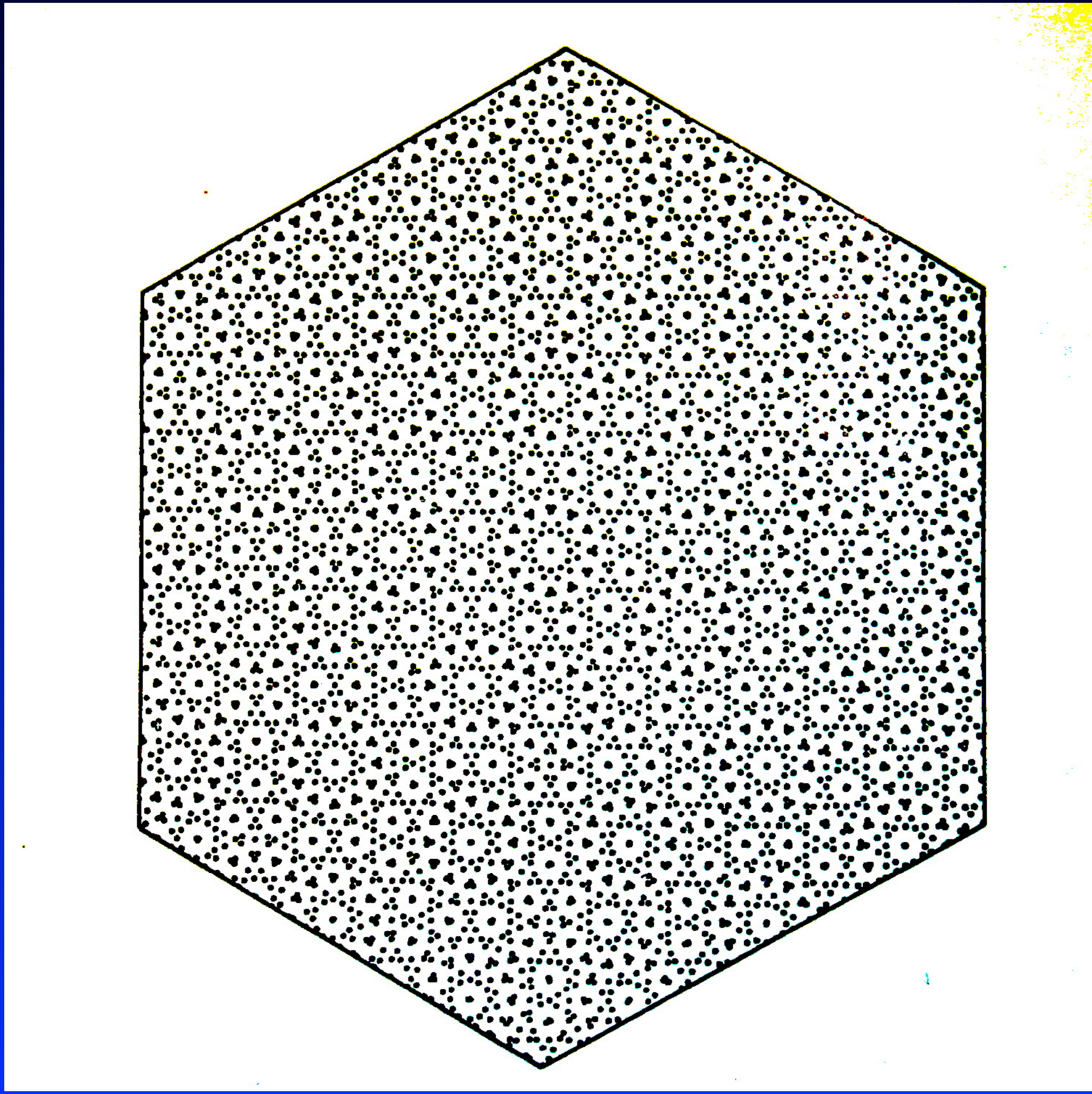
fMRI & Distance from Mean



FFA neurons increase firing with distance from mean face

Nature Neuroscience, October, 2005

Marroquin Illusion (1976)



Competitive Marroquin Model

Describe neurons by spike rate equations

$$\tau_E \frac{dE_n}{dt} = -E_n + \frac{100\,P_+^2}{\left(10 + H_n\right)^2 + P_+^2}$$

Sigmoid Nonlinearity

$$\text{where} \quad P = S_{Marroquin} - 0.6 \sum_{n \neq k} I_k \exp\left(-R_{nk}{}^5/\sigma_5\right)$$

Spatial Competitive Inhibition

$$\tau_I \frac{dI_n}{dt} = -I_n + E_n$$

Slow adapt (400-900 ms)

$$\tau_H \frac{dH_n}{dt} = -H_n + g\,E_n$$

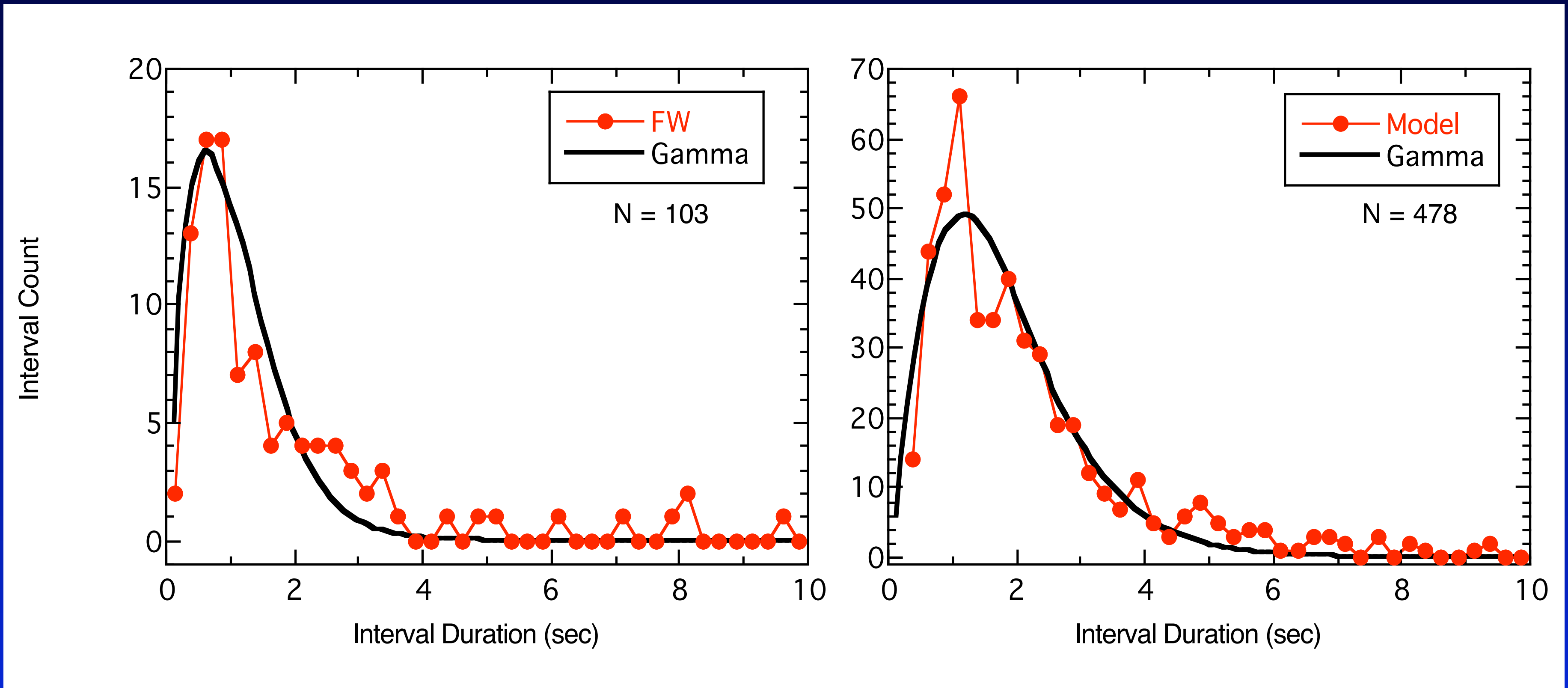
Ca^{++} - K^{+} potential

McCormick, Avoli

Competitive Model (demo)

- Spatially Regional Winner Take All
- Winner slowly adapts, so new winners emerge
- Model generates gamma distribution

Gamma Distribution



Too many long intervals for true Gamma!