Three New Models for Statistical Language Modelling

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Statistical language modelling

- Goal: Model the joint distribution of words in a sentence.
- Most statistical language models are based on the Markov assumption: the distribution of the next word depends on only *n* words that immediately precede it.
- *N*-gram models are the most widely used statistical language models.
  - Conditional probability tables ($P\left(w_N \middle| w_{1:N-1}\right)$) estimated by counting *n*-tuples of words and smoothing the estimates.
  - Curse of dimensionality: lots of data is needed if *n* is large.

Conditional Restricted Boltzmann Machine for language modelling

- We propose using Restricted Boltzmann Machines for modelling the distribution of the next word.
- An RBM is an undirected graphical model with fast exact inference and efficient approximate learning.
  - Two types of variables / units: visible and hidden
  - Bipartite structure: direct interactions are allowed only between units of different types.
- An RBM is typically defined using an energy function that assigns an energy value to every joint setting of the visible and hidden units.
  - Probabilities are obtained by exponentiating negative energies and normalizing.

Conditional RBM for language modelling

- We model words using multinomial variables that can take on as many values as there are words (D).
  - Each word is encoded as a D-bit vector using 1-of-D encoding.
- The energy function for an RBM with N input words and M binary hidden units can be written as

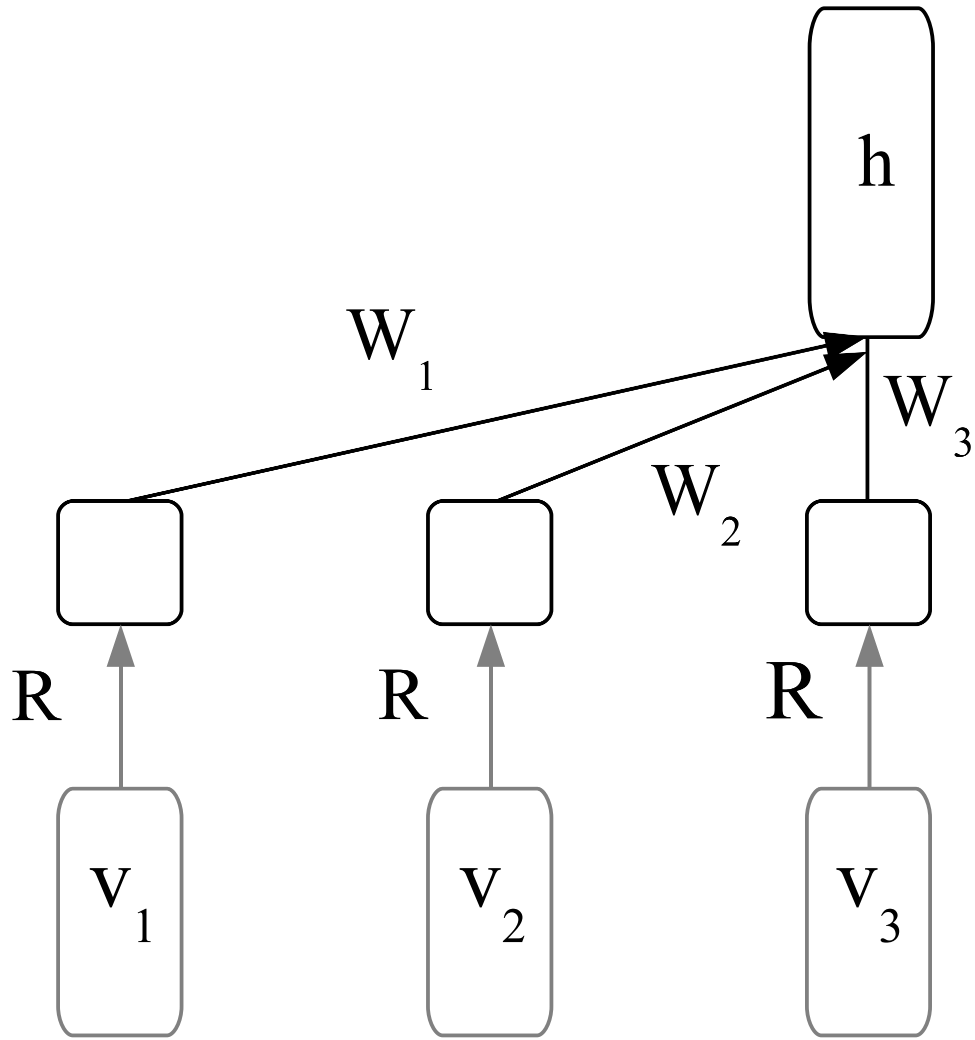
$E(w_{1:N}, h) = -\sum w_i^T J_i h$ where each J_i is a $ND \times M$ matrix.

- This parameterization can have too many parameters when D or M is large.
  - It also does not separate the position-independent word parameters (i.e. word "identity") from the position-dependent ones.

Factored (conditional) RBM

- To reduce the number of model parameters, we represent each word using an F-dimensional (feature) vector of real numbers.
- We stack these vectors for all words in the dictionary to obtain a word feature matrix R and express J_i as a product of R and another low-rank matrix W_i .
  - W_i is an interaction matrix between the feature vector for the word in position *i* and the hidden units.
  - The energy function becomes $E(w_{1:N}, h) = -\sum w_i^T R W_i h$
- This parameterization decouples the position-independent word identity parameters (R) and the position-dependent interaction parameters (W_i).

Factored RBM



Learning and inference in FRBMs

- Exact ML learning is possible but is too slow.
  - We use Contrastive Divergence learning instead.
- The learning rules for R and W_i are minor variations on the standard CD learning rule. E.g.:

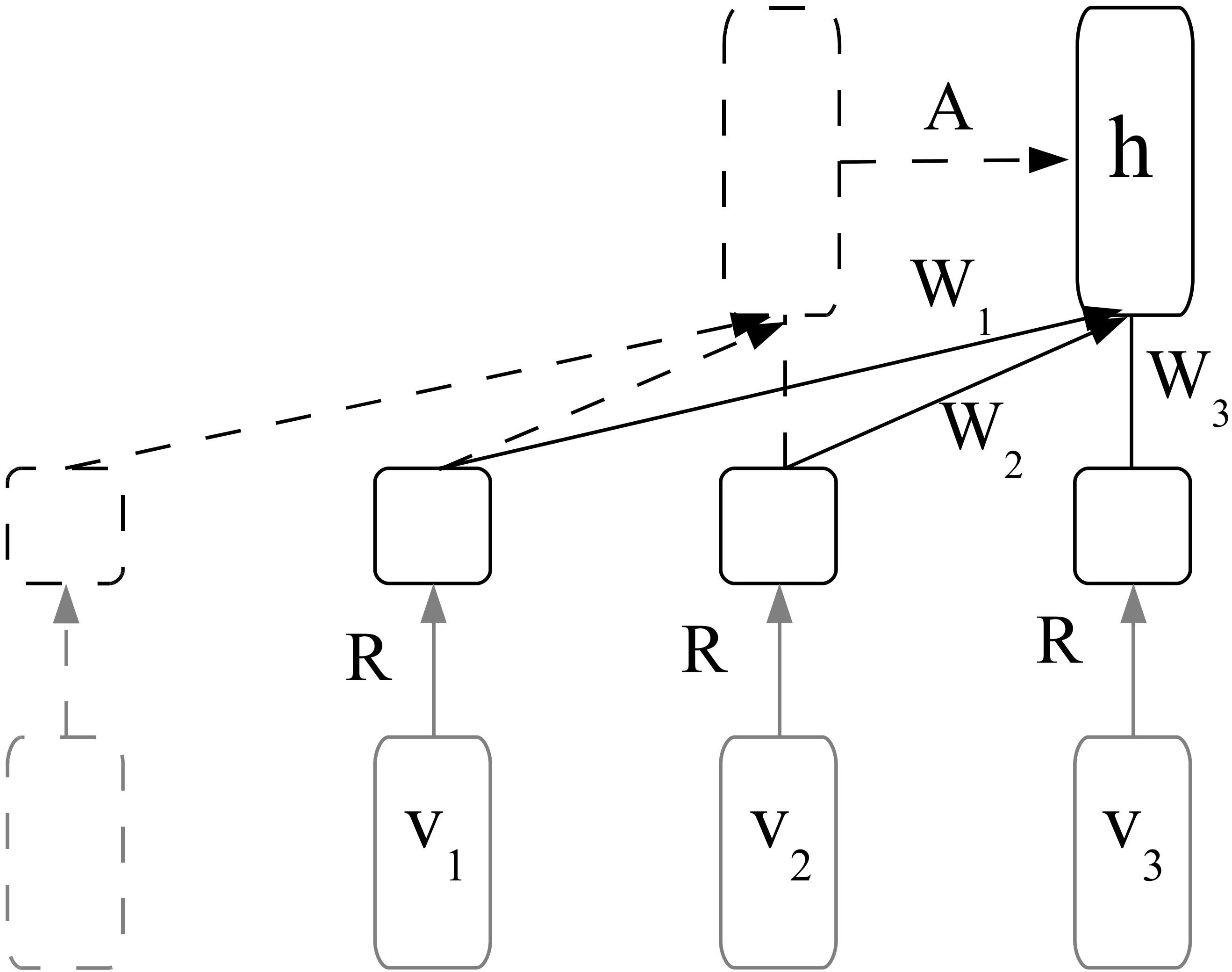
$$\Delta W_i = \left\langle R^T w_i h^T \right\rangle_{data} - \left\langle R^T w_i h^T \right\rangle_{reconstruction}$$

- Computing the posterior distribution over the hidden units is easy.
- Making predictions using this model is tractable.
  - It takes time linear in the number of hidden units and words in the dictionary.

Temporal Factored RBM

- Would like to take advantage of indefinitely large contexts without needing a very large number of parameters.
- Turn FRBM into a temporal model:
  - Given a sequence $w_{1:t}$, apply an instance of the FRBM to each of the *n*-tuples in the sequence in succession.
  - Make the hidden units of the n^{th} instance depend on the hidden units of the $n-1^{st}$ instance by making the hidden biases of the n^{th} instance a linear function of the hidden states on the $n-1^{st}$ instance.
  - Make predictions as before, but use the new “shifted” biases.

Temporal Factored RBM



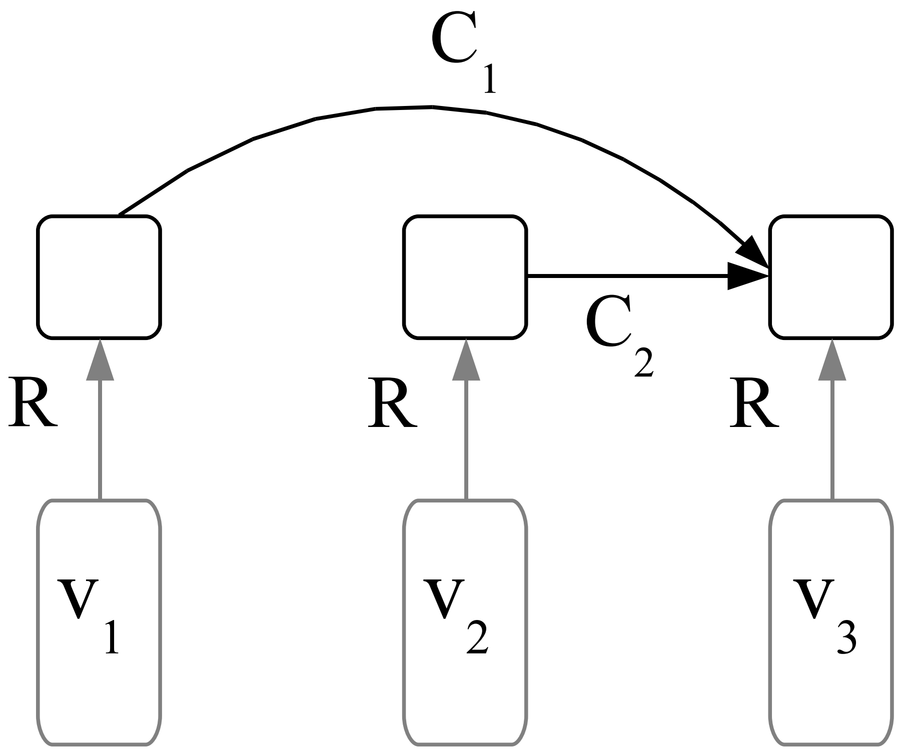
Inference and learning in TFRBM

- Exact inference in TFRBM is intractable due to explaining away.
  - even filtering is intractable
- We perform approximate filtering by using the mean field approximation to the previous hidden state distribution when shifting the biases.
- Temporal connections are learned greedily by treating the previous hidden state as a constant input and using the CD learning rule.
- The non-temporal parameters are learned as before.

Log-bilinear model

- It might be easier to learn direct interactions between the context words and the next word and leave out the hidden units altogether.
- We define these interactions on word feature vectors to keep the number of model parameters manageable.
- The resulting model can be viewed both as a feed-forward network and as a FRBM with visible-to-visible connections but without hidden units.
- Energy function: $E(w_{1:n}) = -\sum_{i=1}^{n-1} w_i^T R C_i R^T w_n$

Log-bilinear model



Dataset and evaluation

- The dataset is a collection of Associated Press news stories (16 million words).
- Preprocessing (Yoshua Bengio):
  - convert all words to lower case
  - map all rare words and proper nouns to special symbols
  - Result: just under 18000 unique words.
- Models are compared based on the perplexity they assign to a test set.
  - Perplexity is the geometric mean of $\frac{1}{P\left(w_n \middle| w_{1:n-1}\right)}$.

Experiments (I)

Preliminary comparison: 10M training set, 0.5M validation set, 0.5M test set

- Feature-based models have 100D feature vectors.
- Models with hidden units have 1000 hidden units.

Model type	Context size	Model test perplexity	Mixture test perplexity
FRBM	2	169.4	110.6
Temporal FRBM	2	127.3	95.6
Log-bilinear	2	132.9	102.2
Log-bilinear	5	124.7	96.5
Back-off KN3	2	124.3	
Back-off KN6	5	116.2	

Experiments (II)

Final comparison: 14M training set, 1M validation set, 1M test set

- Feature-based models have 100D feature vectors.
- Models with hidden units have 1000 hidden units.

Model type	Context size	Model test perplexity	Mixture test perplexity
Log-bilinear	5	117.0	97.3
Log-bilinear	10	107.8	92.1
Back-off KN3	2	129.8	
Back-off KN5	4	123.2	
Back-off KN6	5	123.5	
Back-off KN9	8	124.6	

Summary

- Log-bilinear models outperform FRBM-based models as well as the best *n*-gram models and are easier to train than models with hidden units.
- Adding temporal connections to the FRBM model makes it perform much better.
- Averaging the predictions of any network model with a good *n*-gram model results in better predictions than using any model on its own.
- Future work: training models that have hidden units as well as direct connections; using FRBMs to train deep networks.

The End

FRBM details

- Energy function: $E(w_{1:N}, h) = -\sum w_i^T R W_i h$

- Joint probability of the next word and a hidden state:

$$P(w_N, h | w_{1:N-1}) = \frac{1}{Z} \exp(-E(w_{1:N}, h))$$

$$Z = \sum_{w_n} \exp(-E(w_{1:N}, h))$$

- Probability of the next word:

$$P(w_N | w_{1:N-1}) = \frac{1}{Z} \sum_h \exp(-E(w_{1:N}, h))$$