Social and Economic Networks - Matthew O. Jackson

Sections 5.4 - 6.2

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5.4.1 - Diameter

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Proposition 5.1 (Bollobás and Riordan)

In a preferential attachment model in which each newborn node forms m $\geq$ 2 links, as n grows the resulting network consists of a single component with diameter proportional to $\frac{\log(n)}{\log\log(n)}$ almost surely.

Does this give a bound for the diameter of a hybrid model?

5.4.2 - Positive Assortivity and Degree Correlation

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Proposition 5.2 (Jackson and Rogers)

Consider a growing hybrid random-network formation process as described in section 5.3. Under the mean-field estimate, a node i's degree is larger than a node j's degree at time t after both are born if and only if i is older than j. In that case, if $\alpha > 0$, then the estimated distribution of i's neighbours' degrees strictly first-order stochastically dominates that of j's at each time $t > j$ relative to younger nodes; that is, $F_i^t(d) < F_j^t(d)$ for all $d < d_j(t)$.

Proof of Proposition 5.2

The degree of a node is:

$$d_i(t) = \left(m + \frac{2\alpha m}{1-\alpha}\right)\left(\frac{t}{i}\right)^{(1-\alpha)/2} - \frac{2\alpha m}{1-\alpha}$$

This implies that if $d_i(t) > d_j(t)$, then $i < j < t$.

$$F_i^t(d) = 1 - \frac{d_i(t^*(d,t)}{d_i(t))}$$

$t^*(d,t)$ is the date of birth of a node with degree d at time t. We want to show that for $i < j < t' < t$:

$$\frac{d_i(t')}{d_i(t)} > \frac{d_j(t')}{d_j(t)}$$

$$\frac{d_i(t')}{d_i(t)} = \frac{\left(m + \frac{2\alpha m}{1-\alpha}\right)(t')^{(1-\alpha)/2} - \left(\frac{2\alpha m}{1-\alpha}\right) i^{(1-\alpha)/2}}{\left(m + \frac{2\alpha m}{1-\alpha}\right) t^{(1-\alpha)/2} - \left(\frac{2\alpha m}{1-\alpha}\right) i^{(1-\alpha)/2}}$$

5.4.5 - Clustering

- Transitive Triples:

$$Cl^{TT}(g) = \frac{\sum_{i;j\neq i;k\neq j} g_{ij}g_{jk}g_{ik}}{\sum_{i;j\neq i;k\neq j} g_{ij}g_{jk}}$$

Proposition 5.3 (Jackson and Rogers)

Under a mean-field approximation the fraction of transitive triples, Cl^{TT}, tends to

$$\begin{cases} \frac{1}{(r+1)m} & r \geq 1 \\ \frac{(m-1)r}{m(m-1)(1+r)r-m(1-r)} & r < 1 \end{cases}$$

Where $m = m_r + m_n$ and $r = \frac{m_r}{m_n}$.

6.1 - Pairwise Stability

. A network is pairwise stable if, where $u_i(g)$ is the utility of a network g for node i:

1. for all $ij \in g, u_i(g) \geq u_i(g - ij)$ and $u_j(g) \geq u_j(g - ij)$
2. for all $ij \notin g$, if $u_i(g + ij) > u_i(g)$ then $u_j(g + ij) < u_j(g)$

6.2 - Efficient Networks

Efficiency: g is efficient relative to the utility functions $u_1, ..., u_n$ if $\sum_i u_i(g) \geq \sum_i u_i(g')$ for all $g' \in G(N)$.

Pareto Efficiency: g is Pareto efficient if there does not exist any $g' \in G$ such that $u_i(g') \geq u_i(g)$ for all i, with strict inequality for some i.

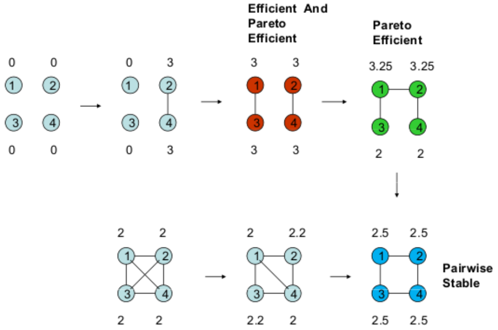

Figure: 6.1