Voting with Preference Intensities

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- Voting is a way to aggregate agents' preferences
 - Political elections
 - Movie night
 - Choose a representative committee
 - Recommender systems

Voting









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How to collect the preferences?





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Top votes







- How to collect the preferences?
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 - Show of hands







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 - Ranked ballots
 - Approval ballots







- We have a surplus of 4000\$ in our budget.
- What should we do with that?
- We can buy a copier, a set of chairs, or go
- out for lunch for a week? Let's decide.





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Voting rule

























































































Total utility (social welfare)



Can we make sure that the winner is close to optimal?







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- Unit-sum assumption: $\sum_{c \in C} u_i(c) = 1$.



Distortion



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- rule

$$sw(\clubsuit) = \sum_{i \in V} u_i (\bigstar)$$

$$opt = \operatorname{argmax} sw(c)$$

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$$Apx(\bigstar) = \frac{sw(opt)}{sw(\bigstar)}$$

Distortion: worst-case approximation ratio of the winner determined by a voting



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$$(\vec{\sigma})) = \max_{\vec{u} \, \triangleright \, \vec{\sigma}} \mathbb{E}_{c \sim f(\vec{\sigma})}[\mathsf{Apx}(c)]$$







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I'm taking everyone out for lunch today. Pizza, Chinese, Steak, or Falafel? Let's decide.











I prefer Steak, then Chinese and then Falafel. I don't really like Pizza.



I'm a vegetarian, so I don't eat steak. Among other options I prefer Falafel, Pizza and then Chinese.



Thanks Michael! I prefer Steak.

I'm taking everyone out for lunch today. Pizza, Chinese, Steak, or Falafel? Let's decide.

You're not invited Toby!











I prefer Pizza and then Steak. I don't really like the two other options but I prefer Chinese to Falafel.



All options seem good to me. But if I have to vote I say Falafel, Pizza, Chinese and then Steak.



The answer is Pizza, and then by far Steak, Chinese and Falafel.



OK. I swallowed all your ideas. I'm going to digest them and see what comes out the other end.





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$$\Rightarrow u_i(c) \ge u_i(c'),$$

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• Extreme cases: $\alpha \simeq 1$, $\alpha = 0$

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 $\alpha = \frac{1}{\alpha}$

0.27





Deterministic

Ramdomized











Special Cases





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 $\mathsf{POII}((\vec{\sigma}, \vec{\pi}), \alpha) = \min_{x \in \Lambda(C)}$ $x \in \Delta(C)$



$$\operatorname{dist}_{\alpha}\left(x,\left(\vec{\sigma},\vec{\pi}\right)\right)$$
$$\operatorname{dist}_{\alpha}\left(\operatorname{opt}_{\alpha}^{\operatorname{aw}}\left(\left(\vec{\sigma},\vec{\pi}\right)\right),\left(\vec{\sigma},\vec{\pi}\right)\right)$$



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- Price of Ignoring Intensities (POII)

 $POII((\vec{\sigma}, \vec{\pi}), \alpha) = \min$ $x \in \Delta(C)$

> $POII(\alpha) = \max POII((\vec{\sigma}, \vec{\pi}), \alpha)$ $(\vec{\sigma},\vec{\pi})$



$$dist_{\alpha} \left(x, (\vec{\sigma}, \vec{\pi}) \right)$$
$$dist_{\alpha} \left(opt_{\alpha}^{aw} \left((\vec{\sigma}, \vec{\pi}) \right), (\vec{\sigma}, \vec{\pi}) \right)$$







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 $POII(\alpha) \in \Omega$



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Deterministic:

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Mandatory Reporting







- $c \succ_i c' \Rightarrow u_i(c) \ge u_i(c') \ge \alpha u_i(c)$.





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 - Voter's distributions
- Decisive preferences in other settings

