UNIVERSITY OF TORONTO
Faculty of Arts & Science

October 2026 Midterm Examination

CSC 304 H1F
(Algorithmic Game Theory and Mechanism Design)
Instructor(s): N. Shah

Duration: 50 minutes

Aids Allowed: One *8.5"×11"* sheet of *hand*written notes on *one* side.

*Do **not** turn this page until you have received the signal to start.*

Write your name and student number below **(do this now)**, then read the instructions.

STUDENTS MUST HAND IN ALL EXAM MATERIALS AT THE END

Crowdmark Automated Matching Area

*Any content within this area (and a small margin around it) will be **overwritten** by Crowdmark.*

- As a student, you help create a fair and inclusive writing environment. If you possess an unauthorized aid during the exam, you may be charged with an academic offence.
- Turn off and place all cell phones, smart watches, electronic devices, and unauthorized study materials in your bag under your desk. If it is left in your pocket, it may be an academic offence.
- When you are done your exam, raise your hand for someone to come and collect your exam. *Do not collect your bag and jacket before your exam is handed in.* If you are feeling ill and unable to finish your exam, *please bring it to the attention of the Exam Facilitator* so it can be recorded before leaving the room. In the event of a fire alarm, do **not** check your cell phone when escorted outside.
- *Please use a **dark** pen or pencil* to ensure your answers are legible (even when scanned).
- Answer each question directly on the exam paper, in the space provided. Use "blank" pages for rough work, or for answers that did not fit elsewhere— *indicate clearly the pages that should be marked.*
- This exam contains 3 (+1 bonus) questions on 10 pages (including 3 "blank" pages), printed on both sides of the paper. *When you start, please make sure that your copy is complete.*

Q1	Q2	Q3	Bonus	Total
15	20	15	10	50

Good Luck!

Question 1. [15 marks] Nash Equilibria

Consider the following two normal-form games between Alice (the row player) and Bob (the column player), who each choose between H and T. In each cell, the first number is Alice's reward and the second number is Bob's reward. The two games differ only in Alice's reward when both players choose H. You are guaranteed that each game has exactly one Nash equilibrium, and in that equilibrium, both players strictly randomize (play each action with a positive probability). So, in your calculations below, you do not need to consider and rule out any other cases.

Game 1

	H	T
H	$(1, -1)$	$(-1, 1)$
T	$(-1, 1)$	$(1, -1)$

Game 2

	H	T
H	$(\mathbf{3}, -1)$	$(-1, 1)$
T	$(-1, 1)$	$(1, -1)$

Part (a) [5 marks] Find the Nash equilibrium of Game 1 using the indifference principle. Show your calculation for full marks.

Midterm 1 Examination

CSC 304 H1F

October 2026 Midterm

Part (b) [5 marks] Find the Nash equilibrium of Game 2 using the indifference principle. Show your calculation for full marks.

Part (c) [5 marks] Compared to Game 1, which player's strategy changed in the Nash equilibrium of Game 2? Why did the other player's strategy not change? Justify your answer.

Question 2. [20 marks] Price of Stability

Consider a congestion game with n players and m resources. Each player i chooses a set of resources P_i. If $n_j(P)$ players use resource j under the pure strategy profile $P = (P_1, \ldots, P_n)$, each of them pays $f_j(n_j(P))$ for it, and a player's cost is the sum of what she pays for the resources she uses.

Suppose all cost functions are linear, i.e., $f_j(k) = a_j \cdot k$ for some constant $a_j \geqslant 0$. Let $C(P)$ denote the social cost (the sum of the costs of all players), and recall the potential function $\Phi(P) = \sum_{j=1}^{m} \sum_{k=1}^{n_j(P)} f_j(k)$.

Part (a) [10 marks] Show that $C(P) = \sum_{j=1}^{m} a_j \cdot n_j(P)^2$ and $\Phi(P) = \sum_{j=1}^{m} a_j \cdot \frac{n_j(P)\,(n_j(P)+1)}{2}$.

Midterm 1 Examination

CSC 304 H1F

October 2026 Midterm

Part (b) [5 marks] Use part (a) to show that $\Phi(P) \leqslant C(P) \leqslant 2\,\Phi(P)$ for every pure strategy profile P. You can solve this part even if you did not solve part (a).

Part (c) [5 marks] Use part (b) to prove that the price of stability of this game with respect to pure Nash equilibria is at most 2, i.e., there is a pure Nash equilibrium P^* with $C(P^*) \leqslant 2 \cdot C(P)$ for every pure strategy profile P. You can solve this part even if you did not solve part (b).

Question 3. [15 marks] Stackelberg Games

Consider the following 2×2 game between Alice (row player) and Bob (column player). Each cell shows Alice's reward, then Bob's reward.

	L	R
T	$(0, 1)$	$(3, 0)$
B	$(4, 1)$	$(1, 3)$

Part (a) [5 marks] Using the indifference principle, find the Nash equilibrium of this game and Alice's expected reward in it. Show your calculation for full marks. You are guaranteed that this game has exactly one Nash equilibrium, in which both players strictly randomize (play each action with a positive probability), so you do not need to rule out any other cases.

Part (b) [10 marks] Suppose now that Alice can commit to a *pure* strategy, and Bob then plays a best response to it. Which pure strategy should Alice commit to, and what would her reward be? Is this reward lower or higher than her reward in the Nash equilibrium from part (a)? Justify your answer.

Midterm 1 Examination

CSC 304 H1F

October 2026 Midterm

Bonus. [10 marks] Congestion Game Returns

Consider the congestion game with linear cost functions from Question 2. Design an example game of this form in which no social optimum is a Nash equilibrium, proving that the price of stability with respect to pure Nash equilibria can be strictly greater than 1. Show your calculations for full marks.

[Hint: There is such a game with two players $\{1, 2\}$ and two resources $\{x, y\}$, in which each player chooses either $\{x\}$ or $\{y\}$. In every social optimum, one player uses x and the other uses y, but then the player using y can reduce her own cost by switching to x. Your task is to find the two cost function parameters, a_x and a_y, which will make this happen.]

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Midterm 1 Examination

CSC 304 H1F

October 2026 Midterm

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You may detach this sheet but ***you must hand it in with your exam****.*

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