Designing Samplers is Easy: The Boon of Testers

Mate Soos[1]

(Substitute Presenter: Kuldeep S. Meel[1])

Joint work with Sourav Chakraborty[2] and Priyanka Golia[1]

[1] National University of Singapore
[2] Indian Statistical Institute, Kolkata

(Relevant Publications: AAAI-19, FMCAD-21, CP-22)

Input A CNF Formula F and tolerance parameter ε
Output $\sigma \in Sol(F)$ such that

$$\frac{1}{(1+\varepsilon)|Sol(F)|} \leq \Pr[\mathcal{A}(F) = \sigma] \leq \frac{1+\varepsilon}{|Sol(F)|}$$

Motivation: Fundamental problem in CS (theory) and applications in hardware and software testing (practice)

Snapshot from early 2010's

Scalability WES04,NRJK+06, KK07

Guarantees JVV86, BGP00, YAPA04

- Core Idea: Use 3-wise independence (random XORs) to partition the solution space
- Makes $\mathcal{O}(\log n)$ calls to SAT oracle
- Theoretical guarantees

$$\frac{1}{(1+\varepsilon)|Sol(F)|} \leq \Pr[\mathcal{A}(\mathcal{F}) = y] \leq \frac{1+\varepsilon}{|Sol(F)|}$$

- Scalability: CryptoMiniSat (A specialized solver for CNF+XOR)

How do you test a sampler is uniform?

Input: A reference sampler $\mathcal{U}$, a test sampler $\mathcal{A}$, and a formula F
Approach: Run both samplers and plot their distributions

- Eyeball the distributions
- Run statistical tests (KL divergence, chi-square)

Caveat Requires number of samples $>>$ number of solutions

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What if you try to draw conclusions based on fewer samples?

DLBS18: Efficient Sampling of SAT Solutions for Testing

"We can see that SearchTreeSampler and UniGen2 are more uniform, but **QuickSampler** is still close to uniform on most benchmarks. However, this result should be taken with care, since the uniformity test is not very reliable on benchmarks where QuickSampler completed a small number of epochs or when the number of produced samples is too low."

Input: A reference sampler $\mathcal{U}$, a test sampler $\mathcal{A}$, and a formula F
Problem: Return Yes if the distribution of $\mathcal{U}(F)$ (known to be uniform) and $\mathcal{A}(F)$ are close, else return No
Approach II: Just keep sampling and stop the first time you see a collision

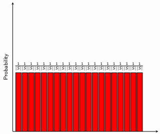


Figure: $\mathcal{U}$: Reference Distribution

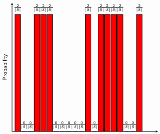


Figure: $\mathcal{A}$: far from uniform

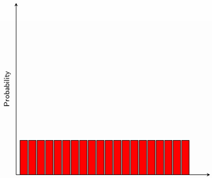
No collisions until you have generated at least $\sqrt{|Sol(F)|}$ solutions!
BFRSW98 $\implies$ The above technique is *optimal* (i.e., if we are only allowed to look at samples)

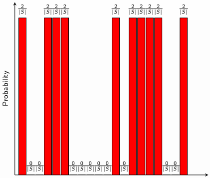
Definition (Conditional Sampling)

Given a distribution $\mathcal{D}$ on S; allow one to specify a set $T \subseteq S$ and draw samples from $\mathcal{A}$ conditioned on T

$$\Pr[\sigma \text{is generated}] = \begin{cases} 0 & \text{if } \sigma \notin T \\ \frac{\mathcal{D}(\sigma)}{\sum_{\sigma \in T} D(\sigma)} & \text{otherwise} \end{cases}$$

Conditional sampling is at least as powerful as drawing normal samples but is it more powerful?





- Draw σ_1 uniformly at random from the domain and draw σ_2 according to the distribution $\mathcal{A}$. Let $T = \{\sigma_1, \sigma_2\}$.
- In the case of the "far" distribution, with constant probability, σ_1 will have "low" probability and σ_2 will have "high" probibility.
- We will be able to distinguish the far distribution from the uniform distribution using constant number of samples from $\mathcal{A}|_T$.
- The constant depend on the farness parameter.

The above algorithm works for all cases

Constrained Sampling

- Input formula: F over variables X
- Challenge: Conditional Sampling over $T = \{\sigma_1, \sigma_2\}$.
- Construct $G = F \wedge (X = \sigma_1 \vee X = \sigma_2)$
- Most of the samplers will just enumerate all the solutions when the number of solutions is very small
- Need way to construct formulas whose solution space is large but every solution can be mapped to either σ_1 or σ_2.

Input: A Boolean formula φ, two assignments σ_1 and σ_2, and desired number of solutions τ
Output: Formula $\hat{\varphi}$

- $\tau = |Sol(\hat{\varphi})|$
- $z \in Sol(\hat{\varphi}) \implies z_{\downarrow Supp(\varphi)} \in \{\sigma_1, \sigma_2\}$
- $|\{z \in Sol(\hat{\varphi}) \mid z_{\downarrow Supp(\varphi)} = \sigma_1\}| = |\{z \in Sol(\hat{\varphi}) \mid z_{\downarrow Supp(\varphi)} = \sigma_2\}|$
- φ and $\hat{\varphi}$ has "*similar*" structure

Kernel

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- φ and $\hat{\varphi}$ has "*similar*" structure

Definition

The **non-adversarial sampler assumption** states that the distribution of the projection of samples obtained from $\mathcal{A}(\hat{\varphi})$ to variables of φ is same as the conditional distribution of $\mathcal{A}(\varphi)$ restricted to either σ_1 or σ_2

- If $\mathcal{A}$ is a uniform sampler for all the input formulas, it satisfies non-adversarial sampler assumption
- If $\mathcal{A}$ is not a uniform sampler for all the input formulas, it may not necessarily satisfy non-adversarial sampler assumption

Input: A sampler under test $\mathcal{A}$, a reference uniform sampler $\mathcal{U}$, a tolerance parameter $\varepsilon > 0$, an intolerance parmaeter $\eta > \varepsilon$, a guarantee parameter δ and a CNF formula φ
Output: ACCEPT or REJECT with the following guarantees:

- if the generator $\mathcal{A}$ is an ε-additive almost-uniform generator then Barbarik ACCEPTS with probability at least $(1 - \delta)$.
- if $\mathcal{A}(\varphi, .)$ is η-far from a uniform generator and if non-adversarial sampler assumption holds then Barbarik REJECTS with probability at least $1 - \delta$.
- Barbarik needs at most $K = \widetilde{O}(\frac{1}{(\eta-\varepsilon)^4})$ samples.

- Samplers without guarantees (Uniform-like Samplers):
    - STS (Ermon, Gomes, Sabharwal, Selman,2012)
    - QuickSampler (Dutra, Laeufer, Bachrach, Sen, 2018)
- Sampler with guarantees:
    - UniGen3

	QuickSampler	STS	UniGen3
ACCEPTs	0	14	50
REJECTs	50	36	0

To ACCEPT, we needed 10^6 samples but we could reject with just 250 samples

How can we use the availability of Barbarik to design a good sampler? Is it even possible ?

Wishlist

- Sampler should be at least as fast as STS and QuickSampler.
- Sampler should pass the Barbarik test.
- Sampler should perform well on real world applications.

CMSGen

- Exploits the flexibility of CryptoMiniSat.
- Pick polarities and branch on variables at random.
  - To explore the search space as evenly as possible.
  - To have samples over all the solution space.
- Turn off all pre and inprocessing.
  - Processing techniques: bounded variable elimination, local search, or symmetry breaking, and many more.
  - Can change solution space of instances.
- Restart at static intervals.
  - Helps to generate samples which are very hard to find.

```
./cryptominisat5 --maxsol $1 --nobansol --restart fixed --maple 0 ---verb 0 --scc 1 --n 1
--presimp 0 --polar rnd --freq 0.9999 --fixedconfl $2 --random $3 --dumpresult $4 [CNFFILE]
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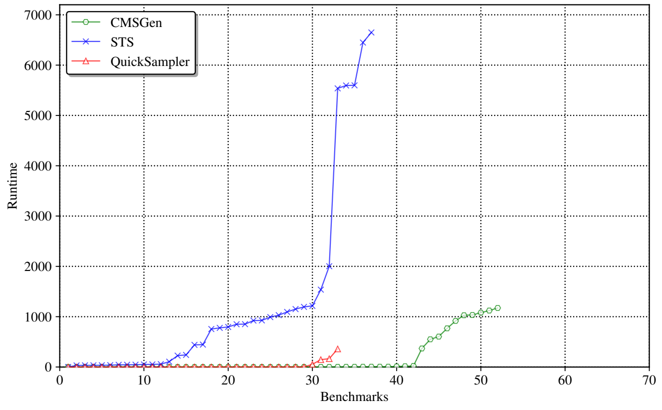
- Parameters of CMSGen are decided iteratively with the help of Barbarik
- Lack of theoretical analysis.

CMSGen vs. Other State-of-the-Art Samplers (I)



QuickSampler	STS	CMSGen
33	37	52

Testing of Samplers

- Samplers without guarantees (Uniform-like Samplers):
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- Sampler should pass the Barbarik test. ✓
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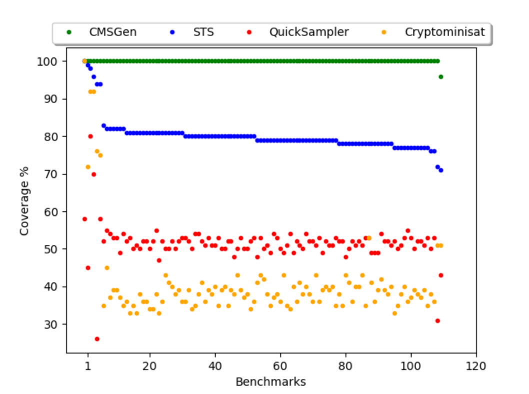
Combinatorial Testing

- A powerful paradigm for testing configurable system.
- Challenge: To generate test suites that maximizes t-wise coverage.

$$\text{t-wise coverage:} = \frac{\#\text{ of t-sized combinations in test suite}}{\text{all possible valid t-sized combinations}}$$

- To generate the test suites use constraint samplers.
- Uniform sampling to have high t-wise coverage (Plazar, Acher, Perrouin et al., 2019).
- Experimental Evaluations:
  - Generate 1000 samples (test cases).
  - 110 Benchmarks, Timeout: 3600 seconds
  - 2-wise coverage $t = 2$.

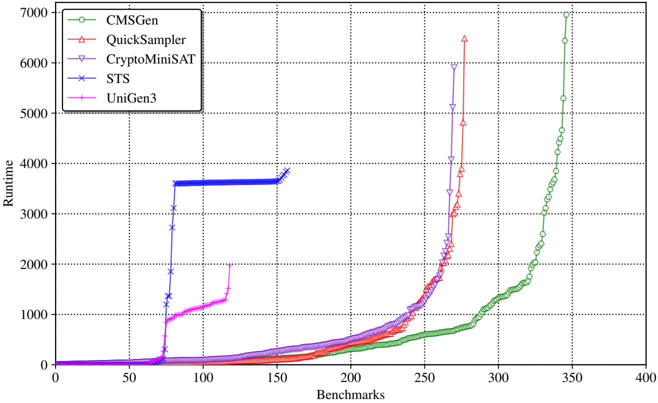
Higher is better



	QuickSampler	STS	CMSGen
Avg. Coverage	51.5%	80.15%	$\sim$ 100%

Remark: UniGen3 could sample for only 6 benchmarks

State of the art approach (Manthan): Sampling + Machine Learning + Counter-example guided repair



Where do we go from here?

Summary Design of a practically efficient sampler via test-driven development that works well in real-world applications

Practice A Virtuous cycle: Improve Barbarik so that it can reject CMSGen and then improve CMSGen

- Trade-off between runtime performance and quality
- Frequent restarts degrade solution quality

Theory Explain why CMSGen works well

- Perhaps CDCL with randomization is all you need in practice?
- Perhaps, you don't really need uniformity in most cases. What do we really need?

Theory and Practice And a testing methodology independent of non-adversarial assumption