

Counting, Sampling, and Synthesis: The Quest for Scalability

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Computing: The Story of an Endless Quest for Scalability

Watson, 1940s: “I think there is a world market for about five computers.”

Gates & Allen, 1970s: “A computer on every desk and in every home”

2020: 22 billion IoT connected devices

Automated Reasoning



```
PC1 [char[] s, char[] t] {  
  for (int i=0; i<t.length; i++) {  
    }  
  }  
  return Yes;  
}
```



satisfies



Automated Reasoning



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Central Question Is it always the case that $\mathcal{M} \models \mathcal{P}$?

Equivalently, can it be the case that $\mathcal{M} \wedge \neg \mathcal{P}$?

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Boolean Satisfiability (SAT): Given a Boolean formula, is there a solution, i.e., an assignment of 0's and 1's to the variables that makes the formula equal 1?

Example: $(X_1 \vee \neg X_2 \vee \neg X_3) \wedge (X_2 \vee \neg X_3)$ $X_1 = 1, X_2 = 1, X_3 = 1$

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[Circa 2012]: Now that SAT is “easy”, it is time to look beyond satisfiability

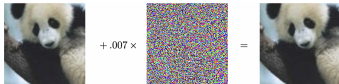
Beyond SAT I: Quantification

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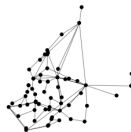
Information Leakage



Fairness



Robustness



Critical Infrastructure

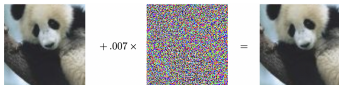
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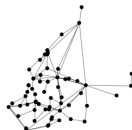
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Critical Infrastructure

Quantification: How often does \mathcal{M} satisfy \mathcal{P} ?

Counting

Beyond SAT II: Sampling



Constrained-Random Verification



Configuration Testing

- System is simulated with test vectors
- Constraints represent *relevant* verification scenarios
- **Test vectors**: random solutions of constraints

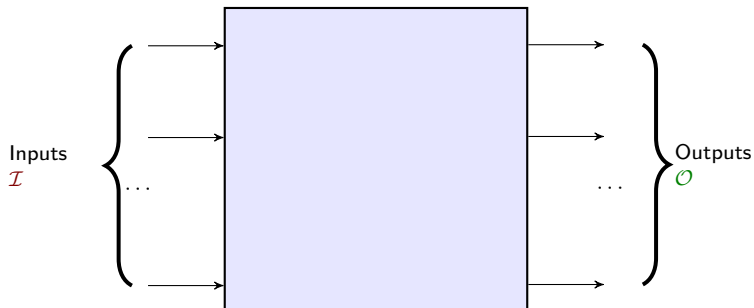
Sampling

Beyond SAT III: Automated Synthesis

$$\begin{aligned}f(u, v) &\geq u; \\f(u, v) &\geq v; \\f(u, v) &= u \vee v \\f(u, v) &= v\end{aligned}$$

Specification: $\mathcal{P}(\mathcal{I}, \mathcal{O})$

age	25
capital-gain	4000
occupation	coach



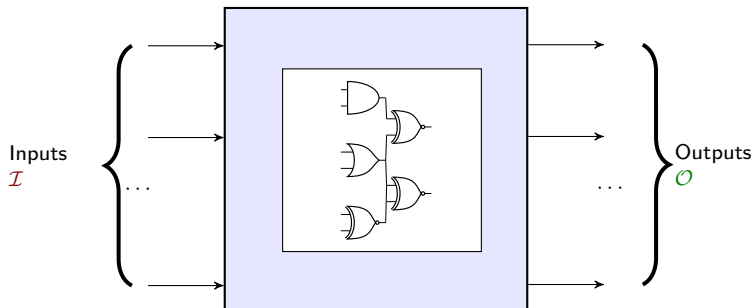
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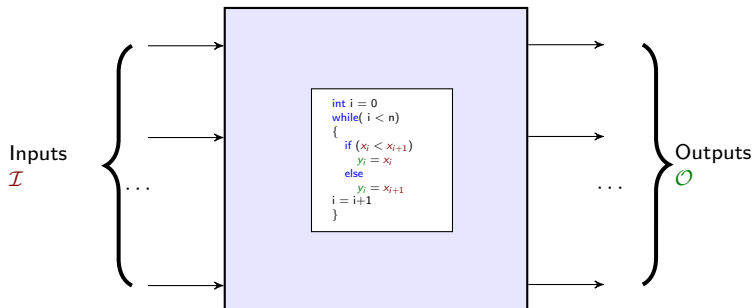
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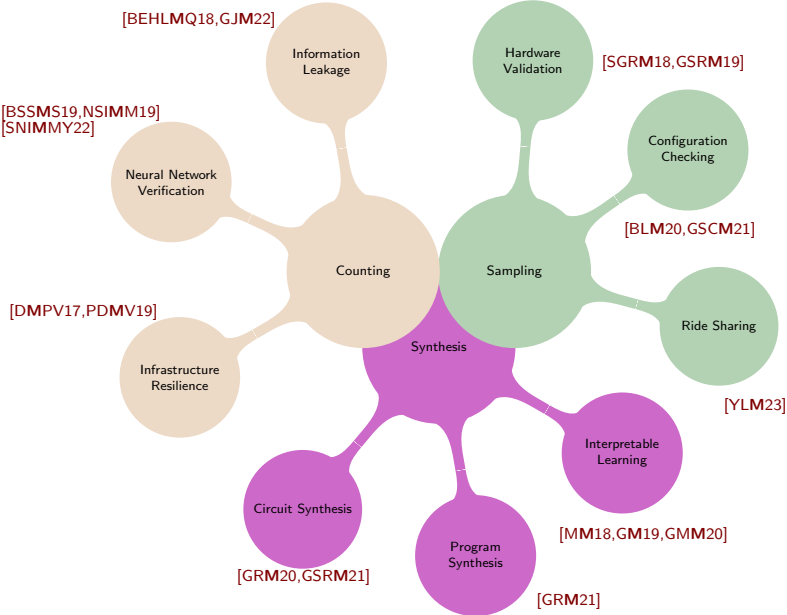
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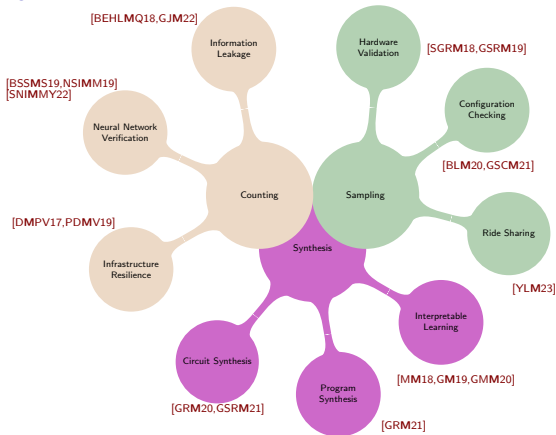


Synthesis

Research Overview



Research Overview



Artificial Intelligence AAI:17×, IJCAI:9×, NeurIPS: 6×, SAT:5×, CP:8×, KR:1×

Formal Methods CAV:6×, TACAS: 3×, ICCAD: 2×, DATE:2×, DAC: 1×

Logic/Databases LICS:2×, LPAR:2×, PODS:3×

Software Engineering ICSE:2×, FSE: 2×, CCS:1×

Today's Talk: Counting

Counting

- **Given:** A Boolean formula F over X_1, X_2, \dots, X_n
- $\text{Sol}(F) = \{ \text{solutions of } F \}$
- **SAT:** Determine if $\text{Sol}(F)$ is non-empty
- **Counting:** Determine $|\text{Sol}(F)|$

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- Example: $F := (X_1 \vee X_2)$
 - $\text{Sol}(F) = \{(0, 1), (1, 0), (1, 1)\}$
 - $|\text{Sol}(F)| = 3$

Counting

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 - $|\text{Sol}(F)| = 3$
- Generalization to arbitrary weights
 - Given weight function (implicitly represented) $W: \{0, 1\}^n \mapsto [0, 1]$
 - $W(F) = \sum_{y \in \text{Sol}(F)} W(y)$
 - **(Weighted) Counting:** Determine $W(F)$

Today's talk: We focus on unweighted case, i.e., $|\text{Sol}(F)|$

Today's Menu

Appetizer Applications

- Critical Infrastructure Resilience
- Quantitative Analysis of AI Systems

Main Course ApproxMC: A Scalable Counting Framework

Dessert Future Outlook

Resilience of Critical Infrastructure Networks

[DMPV17,PDMV19]



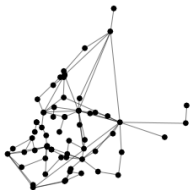
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Resilience of Critical Infrastructure Networks

[DMPV17,PDMV19]



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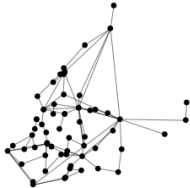
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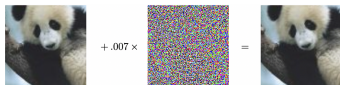


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Constrained Counting

Key Idea: Encode disconnectedness using constraints

Impact: The first theoretically sound estimates of resilience in power transmission networks of ten medium sized cities in US



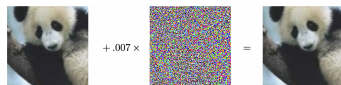
Robustness Quantification

$$\left| \{x : \mathcal{N}(x + \varepsilon) \neq \mathcal{N}(x)\} \right|$$

Quantitative Analysis of AI Systems

Our Focus: Binarized Neural Networks

[BSSMS19,NSMIS19,NSMISV22]



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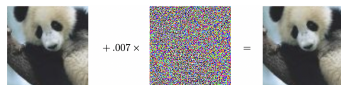
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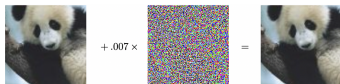
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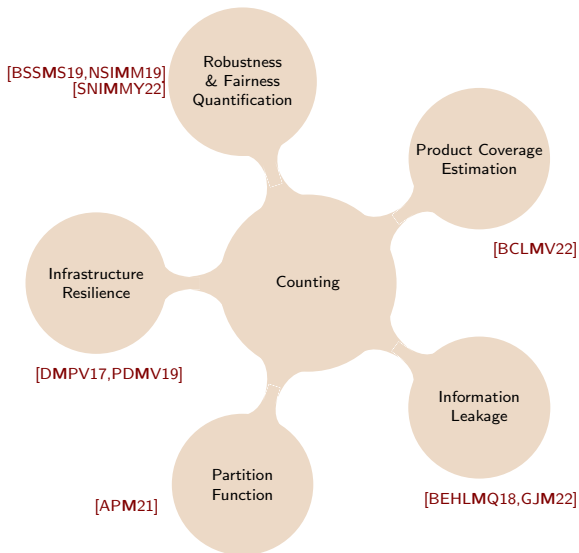
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Constrained Counting

Impact: The first scalable technique for rigorous quantification of robustness and fairness of Binarized Neural Networks

Applications across Computer Science



Impact: Counting-based approach is now the state of the art for all these applications

So Fundamental Yet So Hard

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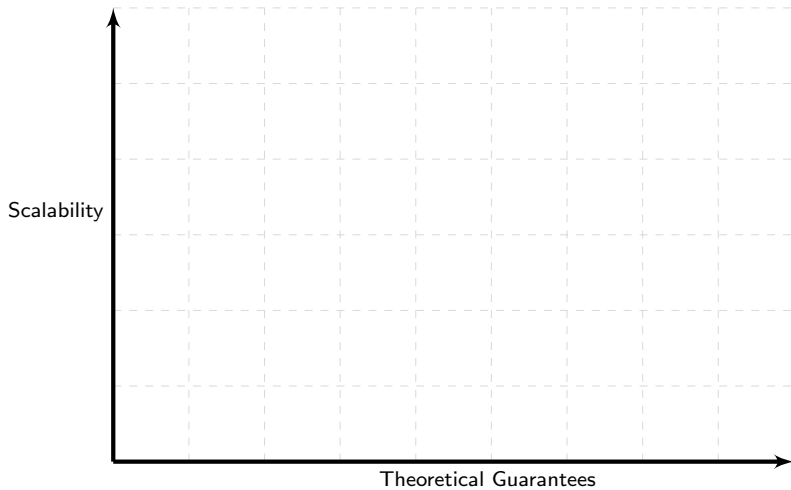
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- Not practical

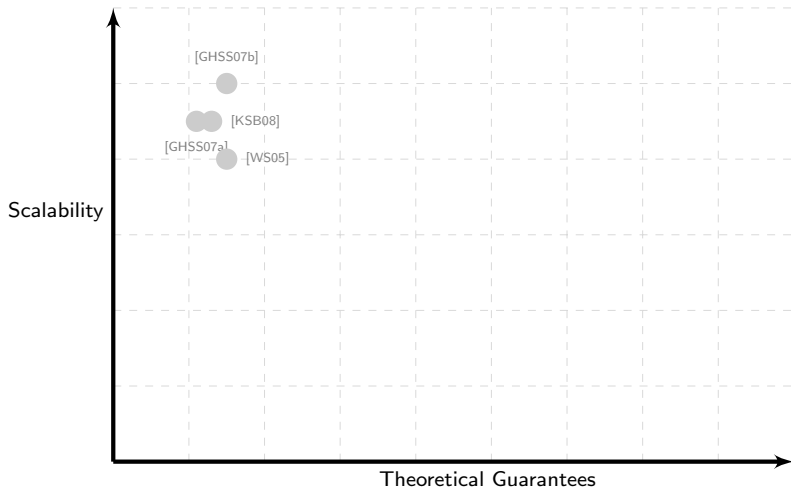
SAT Solver \neq SAT Oracle

Performance of state of the art SAT solvers depends on the formulas

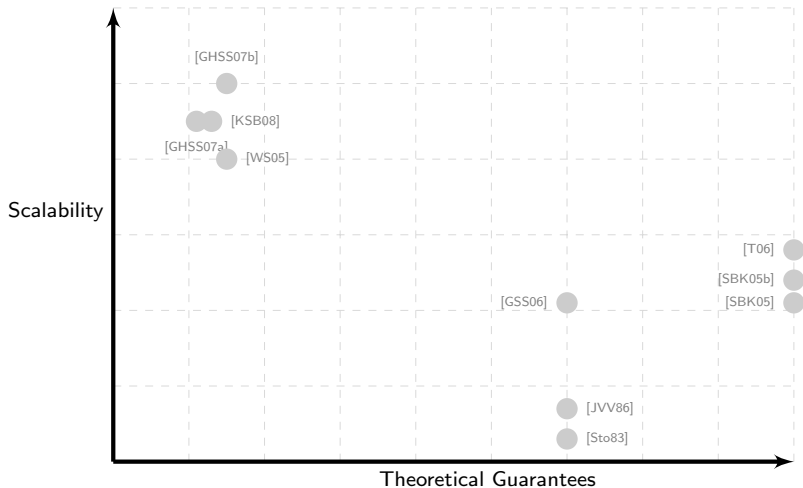
Snapshot from 2012



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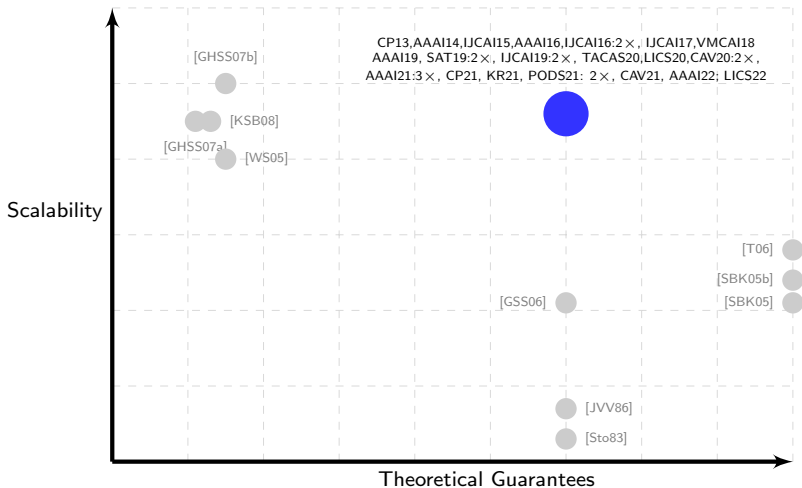


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Can we bridge the gap between theory and practice?



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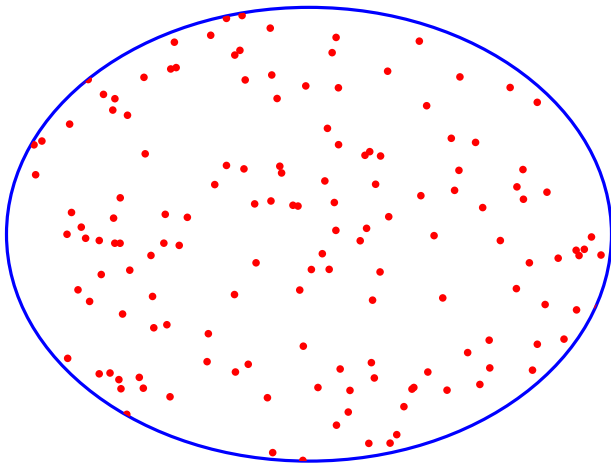
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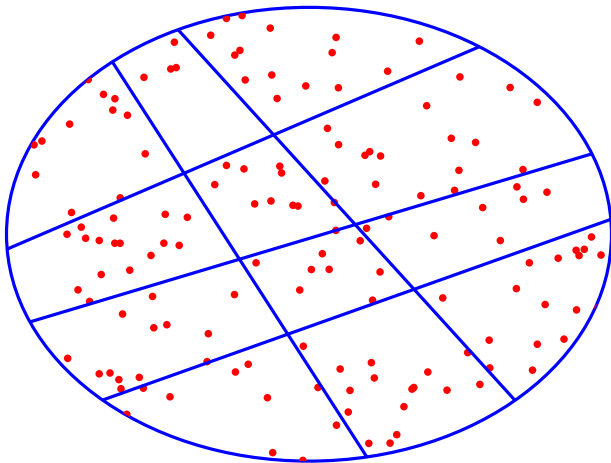
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 - Potentially 2^n queries

Can we do with lesser # of SAT queries – $\mathcal{O}(n)$ or $\mathcal{O}(\log n)$?

As Simple as Counting Dots

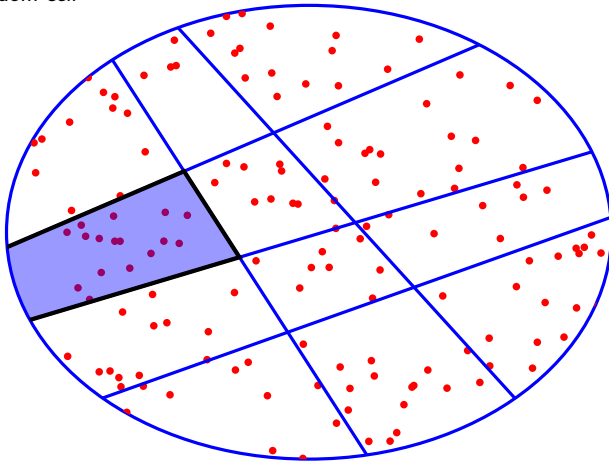


As Simple as Counting Dots



As Simple as Counting Dots

Pick a random cell



Estimate = Number of solutions in a cell \times Number of cells

Challenges

Challenge 1 How to partition into **roughly equal small** cells of solutions without knowing the distribution of solutions?

Challenge 2 How many cells?

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- Designing function h : assignments \rightarrow cells (hashing)
- Deterministic h unlikely to work
- Choose h randomly from a large family H of hash functions

2-wise Independent Hashing

[CW77]

2-wise Independent Hash Functions

- To construct $h : \{0, 1\}^n \rightarrow \{0, 1\}^m$, choose m random XORs
- Pick every X_i with prob. $\frac{1}{2}$ and XOR them
 - ▶ $X_1 \oplus X_3 \oplus X_6 \cdots \oplus X_{n-2}$

2-wise Independent Hash Functions

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- Pick every X_i with prob. $\frac{1}{2}$ and XOR them
 - ▶ $X_1 \oplus X_3 \oplus X_6 \cdots \oplus X_{n-2}$
- To choose $\alpha \in \{0, 1\}^m$, set every XOR equation to 0 or 1 randomly

$$X_1 \oplus X_3 \oplus X_6 \cdots \oplus X_{n-2} = 0 \quad (Q_1)$$

$$X_2 \oplus X_5 \oplus X_6 \cdots \oplus X_{n-1} = 1 \quad (Q_2)$$

$$\cdots \quad (\cdots)$$

$$X_1 \oplus X_2 \oplus X_5 \cdots \oplus X_{n-2} = 1 \quad (Q_m)$$

- Solutions in a cell: $F \wedge Q_1 \cdots \wedge Q_m$

Challenges

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Random XOR-based Hash Functions

[CW77]

Challenge 2 How many cells?

Challenge 2: How many cells?

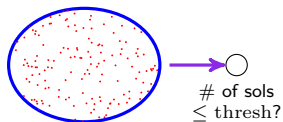
[CMV13,CMV16]

- A cell is small if it has \approx thresh = $5(1 + \frac{1}{\epsilon})^2$ solutions
- Many solutions \implies Many cells & Fewer solutions \implies Fewer cells

Challenge 2: How many cells?

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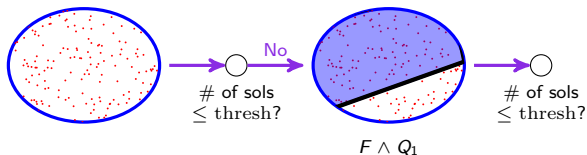
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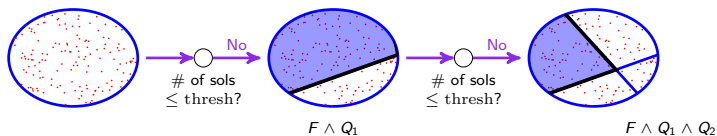
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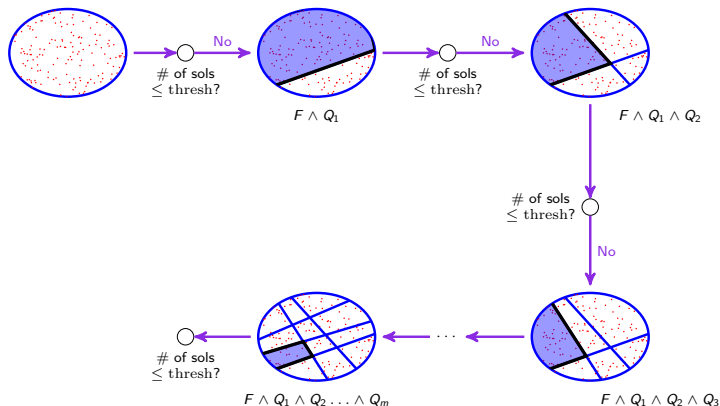
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- Many solutions \implies Many cells & Fewer solutions \implies Fewer cells



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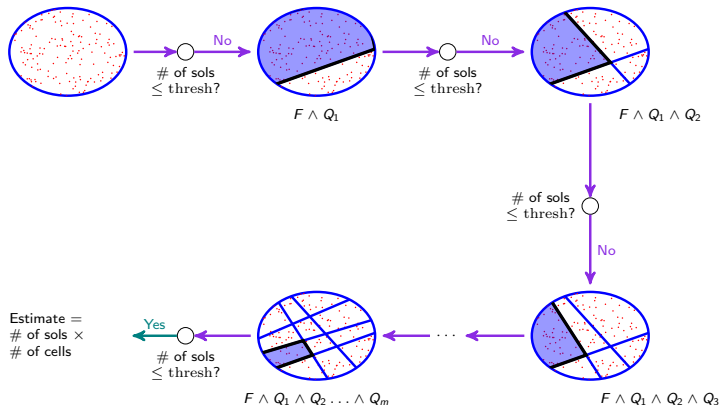
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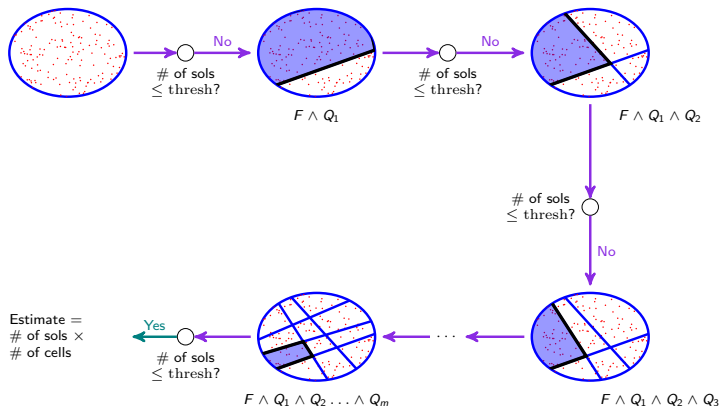
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Theorem: $\Pr \left[\frac{|\text{Sol}(F)|}{1+\epsilon} \leq \text{ApproxMC}(F, \epsilon, \delta) \leq |\text{Sol}(F)|(1+\epsilon) \right] \geq 1 - \delta$

ApproxMC: Early Years (2013-16)

Handle **reasonable** formulas: **reasonable** grids, **reasonable** programs

2019: CP-13 paper selected as one of the 25 papers across 25 years of CP conference

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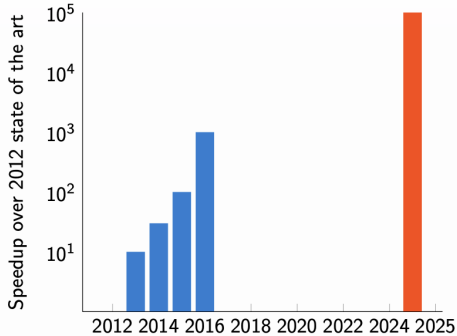
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B. Cook: Virtuous cycle: application areas drives more investment in foundational tools, while improvements in the foundational tools drive further applications. Around and around.

The definition of “**reasonable**” changes after every iteration of the cycle

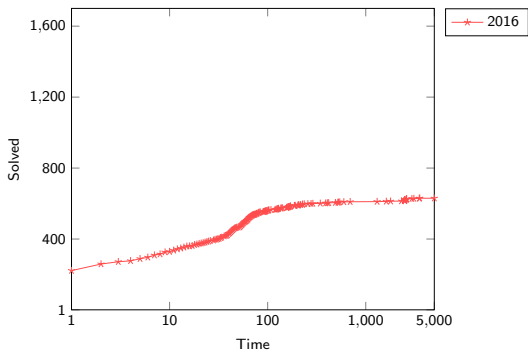
Mission 2025: Constrained Counting and Sampling Revolution



Requires combinations of ideas from theory, statistics and systems

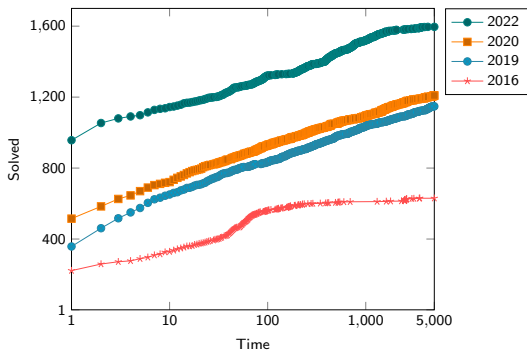
2025 Target: $100\times$ speedup over 2016

ApproxMC: In Pursuit of Scalability



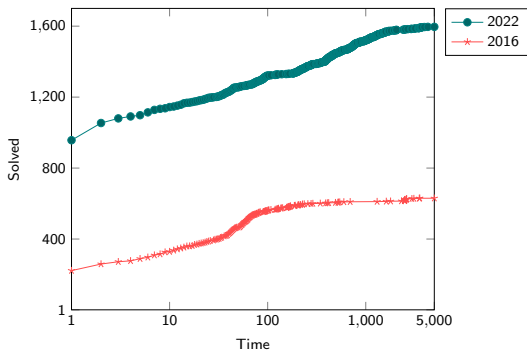
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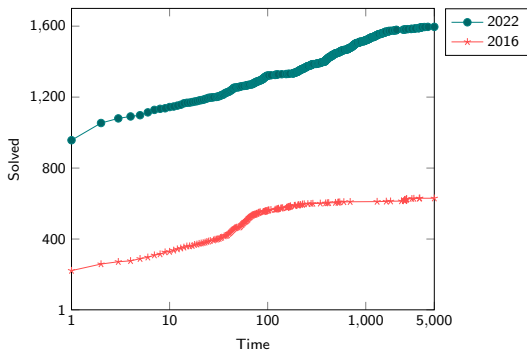


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2016 630 instances, each in ≤ 5000 seconds

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Time taken (seconds) for an instance

2016: 3552.16 **2019:** 32.83 **2020:** 19.59 **2022:** 0.15

A speedup of $20,000\times$ over 2016

Still provides (ϵ, δ) -guarantees

In Pursuit of Scalability

Theoretical Advances	Sparse hashing SAT-20 LICS-20	Duality CP-19	DNF CP-18,IJCAI-19 PODS-21,22	Phase Transition IJCAI-16,17,19 CP-20
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Performance of state of the art SAT solvers depends on the formulas

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New Architecture for CNF-XOR Formulas

Modern SAT Solvers: Conflict-Driven Clause Learning (CDCL) paradigm

- Guess an assignment to subset of variables, if conflict, remember the reason
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CDCL and XORs: Random XORs are hard for CDCL in theory and practice

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level	dec		prop
0	x_1		
1	x_3	\rightarrow	x_5
2	x_4	\rightarrow	$x_2, \neg x_5$

CDCL

x_1	x_2	x_3	
1	0	1	0
1	1	1	1
1	1	0	0

Gauss-Jordan Elimination

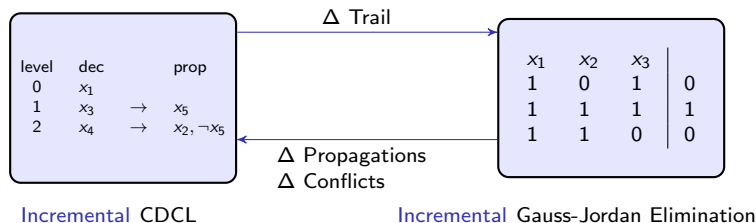
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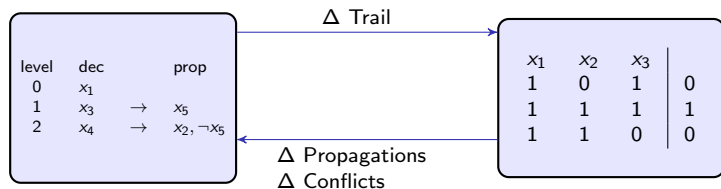
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Incremental CDCL

Incremental Gauss-Jordan Elimination

Engineering an efficient CDCL-GJE solver

[SM19; SGM20]

- Data-structures for efficient propagation and conflict analysis
- Supervised machine learning-guided heuristics

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- Not all variables are required to specify solution space of F
 - $F := X_3 \iff (X_1 \vee X_2)$
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- Approach II: n **easy** calls to SAT solver via Padoa's theorem [SM22]

Approach II + ApproxMC is up to $100\times$ faster than Approach I + ApproxMC
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- If we pick with prob $p < \frac{1}{2}$, then no guarantees of 2-wise independence
- Z_m : Number of solutions in a randomly chosen cell
- 2-wise independence $\implies \frac{\text{Var}[Z_m]}{\text{E}[Z_m]} \leq 1 \implies$ Concentration bounds

Beyond 2-wise Independence

[MA20]

Open problem (2013-19): Sparse ($p < \frac{1}{2}$) XORs that work in theory and practice

Beyond 2-wise Independence

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Theorem (Log-Sparse XORs suffice)

If we pick m -th XOR with $p_m = \frac{\log m}{m}$, we have $\frac{\text{Var}[Z_m]}{\mathbb{E}[Z_m]} \leq 1.1$

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Key Idea: In the context of Z_m , It suffices to assume $|\text{Sol}(F)| < 2^{m+u}$ for small u .

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Reliability of Critical Infrastructure Networks

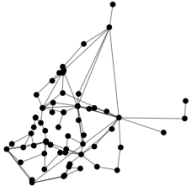
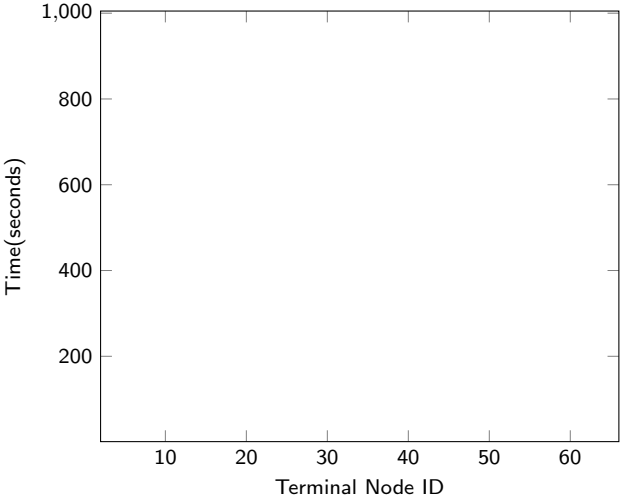


Figure: Plantersville, SC



Timeout = 1000 seconds

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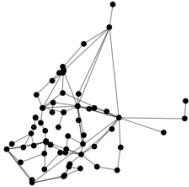
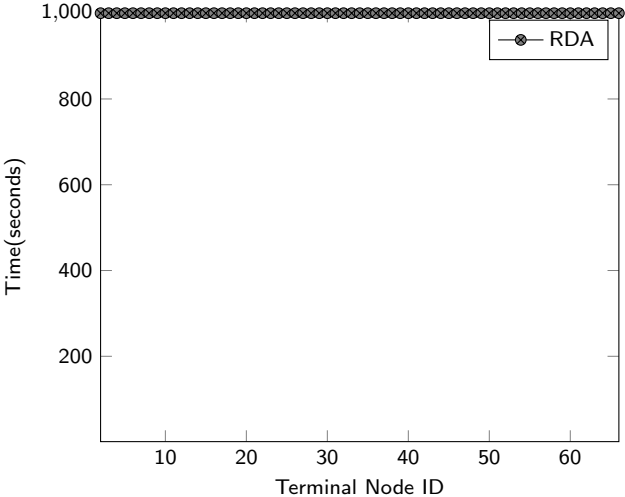


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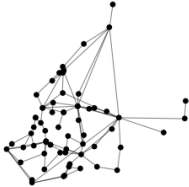
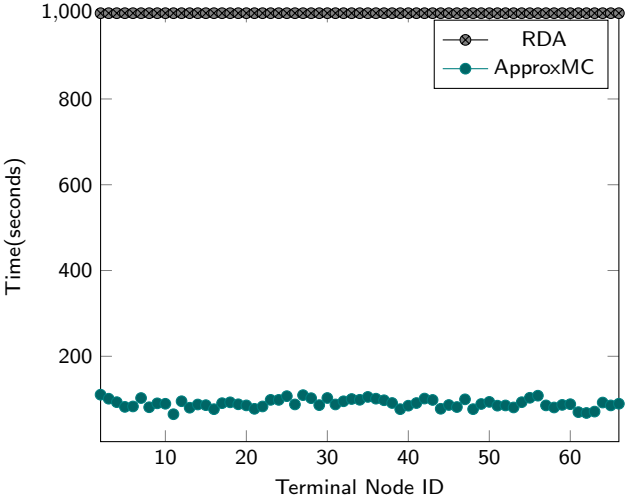


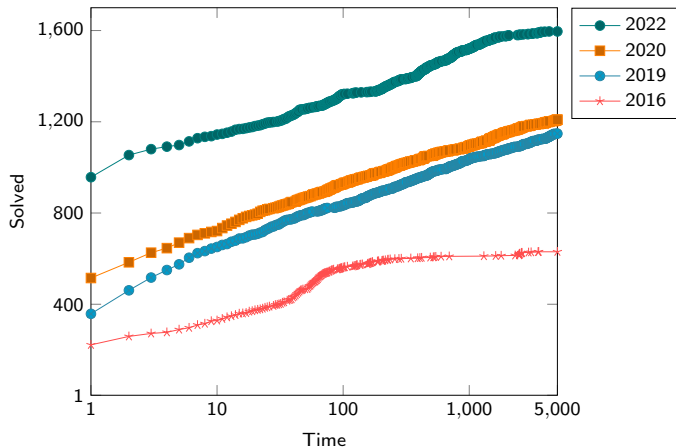
Figure: Plantersville, SC



Timeout = 1000 seconds

Impact: The first theoretically sound estimates of resilience in power transmission networks of ten medium sized cities in US

ApproxMC: Progress over the years



1896 benchmarks from diverse applications

Time taken (seconds) for an instance

2016: 3552.16

2019: 32.83

2020: 19.59

2022: 0.15

A speedup of 20,000 \times over 2016

Another Iteration of Virtuous Cycle

B. Cook, 2022: Virtuous cycle: ...application areas drives more investment in foundational tools, while improvements in the foundational tools drive further applications. Around and around.

SharpTNI: Counting and Sampling Parsimonious Transmission Networks under a Weak Bottleneck

Palash Sashittal¹ and Mohammed El-Kebir^{2*}

Check before You Change: Preventing Correlated Failures in Service Updates

Ennan Zhai¹, Ang Chen¹, Ruzica Piskac², Mahesh Balakrishnan^{1,3},
Bingchuan Tian¹, Bo Song⁴, Haoliang Zhang^{4*}

**Automating the Development of
Chosen Ciphertext Attacks**

Gabrielle Beck, Maximilian Zinkus, and Matthew Green,
Johns Hopkins University

Static Evaluation of Noninterference using
Approximate Model Counting

Ziqiao Zhou

Zhiyun Qian

Michael K. Reiter

Yinqian Zhang

A Study of the Learnability of Relational Properties

Model Counting Meets Machine Learning (MCMML)

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**Quantifying Software Reliability
via Model-Counting**

Suzuel Teuber^{ORCID} and Alexander Weigl^{ORCID}

**IN SEARCH FOR A SAT-FRIENDLY BINARIZED NEU-
RAL NETWORK ARCHITECTURE**

Nina Narodytska

Hongze Zhang*

Quantifying the Efficacy of Logic Locking Methods

Joseph Sweeney, Deepali Garg, Lawrence Pileggi

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Model Counting (MC-2020)

Competition

* Track(s): Model Counting, Weighted Model Counting, and Projected Model Count
(description, format, winners, report, slides, software, competition instances, submitter)

Workshop

* Workshop 2020
(program, abstracts, slides, recordings)

Model Counting (MC-2021)

Competition

* Track(s): Model Counting, Weighted Model Counting, and Projected Model Count
(description, StarKee Community, format, winners, report (PDF), slides, software, vote)

Workshop on Counting and Sampling

* Workshop 2021
(program, abstracts, slides, recordings)

Model Counting (MC-2022)

Competition

Part of StarKee Open-Source Contest
* Track(s): Model Counting, Weighted Model Counting, Projected Model Counting, 1
(description, StarKee Community, format, winners, report (PDF), slides, software, vote)

Workshop on Counting and Sampling

* Workshop 2022 (in person event)
(program, abstracts, slides)

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Generalizability

Union of Sets ApproxMC is Fully Polynomial Randomized Approximation Scheme (FPRAS) – fundamentally different from the Monte-Carlo based FPRAS

- IJCAI-19 Sister Conferences Best Paper Award Track

[MSV19]

Generalizability

Union of Sets ApproxMC is Fully Polynomial Randomized Approximation Scheme (FPRAS) – fundamentally different from the Monte-Carlo based FPRAS

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[MSV19]

Streaming Counting over a stream: Distinct Elements
Example: How many unique customers visit website?
Fundamental problem in Databases

- CACM Research Highlights
- ACM SIGMOD 2022 Research Highlight
- “Best of PODS 2021” by ACM TODS

[PVBM21]

Generalizability

Union of Sets ApproxMC is Fully Polynomial Randomized Approximation Scheme (FPRAS) – fundamentally different from the Monte-Carlo based FPRAS

- IJCAI-19 Sister Conferences Best Paper Award Track [MSV19]

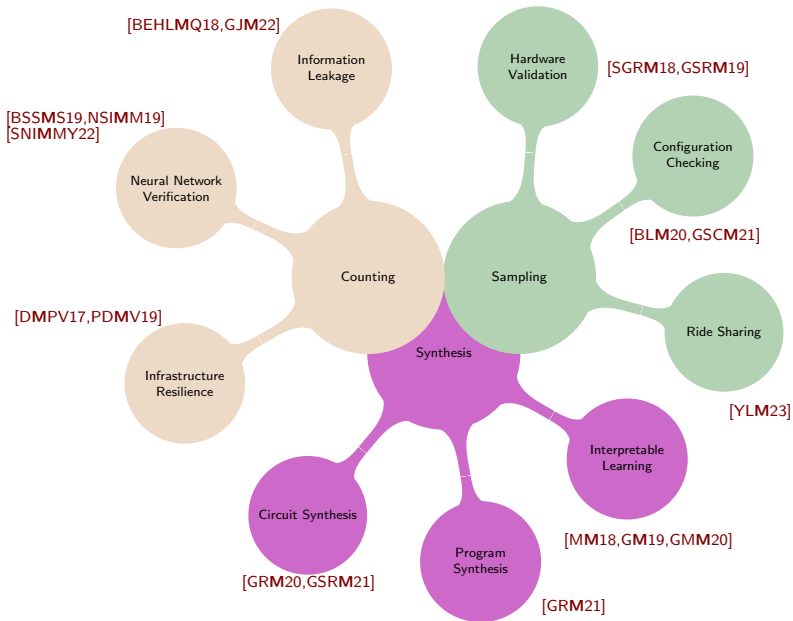
Streaming Counting over a stream: Distinct Elements
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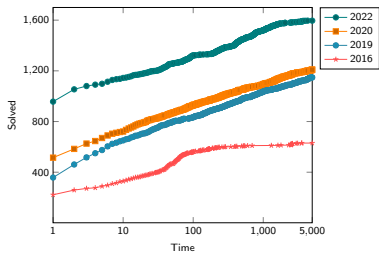
Unsatisfiable Subsets Count minimal subsets of clauses that are unsatisfiable.
Diagnosis metric for systems

- “Best Papers of CAV-20” by FMSD [BM20]

Counting, Sampling, and Synthesis

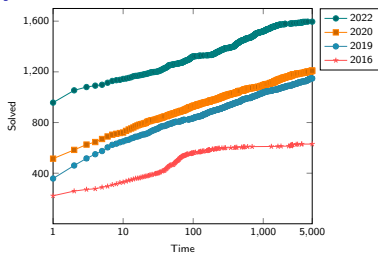


In Pursuit of Scalability

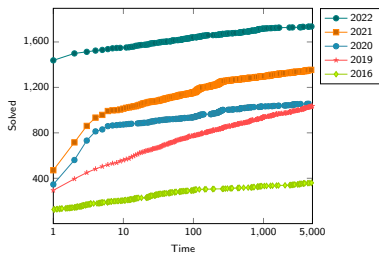


Counting over the years

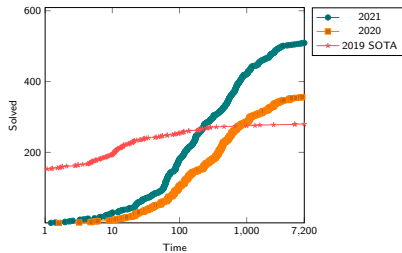
In Pursuit of Scalability



Counting over the years



Sampling over the years



Synthesis over the years

Where do we go from here?

Where do we go from here?

The Quest for Scalability is Endless

Today's Counters/Samplers/Synthesis Engines \approx SAT Solvers in early 2000s

Industrial Practice: 100 \times Speedup

The Pursuit of Scalability

Mission 2028: 100× Speedup for Counting, Sampling, and Synthesis

Challenge Problems (for Counting)

Civil Engineering Rigorous resilience estimation for power grid of Los Angeles

Quantitative Evaluation Binarized neural network with 1M neurons

Software Engineering Information Flow analysis of programs with 10K lines of code

Technical Directions (for Counting)

Theoretical Advances Native reasoning over expressive theories (*Beyond SMT*)

Algorithmic Engineering Machine Learning-guided heuristic design

Software Development Hardware-accelerator aware tools

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Certification: Approximate count is “correct” or the distribution generated is correct

- Applications to verification of probabilistic programming
- Building on recent advances in distribution testing
- **Preliminary Work:** AAI-19, NeurIPS-20, NeurIPS-21, CP-22, NeurIPS-22

It Takes a Village

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Roland H. C. Yap(NUS, SG)

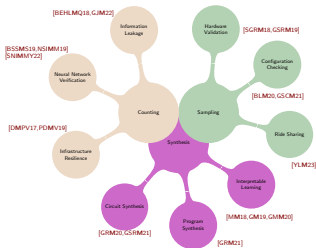
Funding Agencies: National Research Foundation, Ministry of Education, Defense Service Organization

Industrial Support: Grab Taxiholdings, Amazon, Microsoft Research

Counting, Sampling, and Synthesis

```

PC2 (char[] SP, char[] UI) {
  match = true;
  for (int i=0; i<UI.length(); i++) {
    if (SP[i] != UI[i]) match=false;
    else match = match;
  }
  if match return Yes;
  else return No;
}
    
```

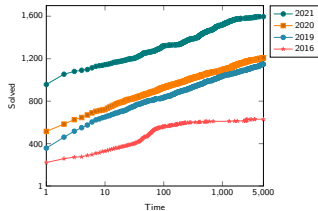


Theoretical Advances

Sparse hashing SAT-20 LICS-20	Duality CP-19	DNF CP-18, IJCAI-19 PODS-21, 22	Phase Transition IJCAI-16, 17, 19 CP-20
Indep Supp Constraints-16 ICCAD-22	Chain Formulas IJCAI-15 NeurIPS-20	Symmetry TACAS-20, AAAI-21	Prob. Caching IJCAI-19 AAAI-23
CNF-XOR AAAI-19, CAV-20	Pseudo-Boolean CP-21	MaxSAT-XOR KR-21	Hardware Accelerator SAT-21

Algorithmic Engineering

Software Development



These slides are available at tinyurl.com/meel-talk

Detailed Future Directions

Applications: Infrastructure Resilience, Information Leakage, Prob. Databases, Configuration Testing, Partition Function, BNN Verification

Theoretical Advances

Formula-based Sparse-XORs DNF, Minimal Solutions, Chain formula

Revisiting FPRAS Permanent, Automata, Linear Extensions

Parameterized Complexity Addition of XORs

Streaming Delphic Sets

Synthesis A theory of learning from relations

Entropy Reduction in the number of queries

Algorithmic Engineering

Incremental Incremental Counting Queries

Bit-vectors Partitioning; Independent Support

Heuristic ML-guided heuristic synthesis

Distributed Streaming techniques

SMT Synthesis SMT Formula Learning

Beyond Qualitative Synthesis Optimal Functions, Approximate Synthesis

Software Development

Tighter Integration Multiple Queries

Hybrid Constraints Callbacks

XOR Handling PB-XOR, BNN-XOR, MaxSAT-XOR, ASP-XOR

Accelerators GPU

Knowledge Compilation SMT, Portfolio

Certification

Distribution Probabilistic Programming Equivalence

Counting Certificate for Approximation