Scalable Techniques for Constrained Sampling and Counting

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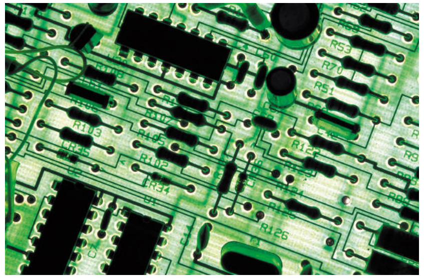
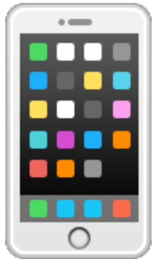
Joint work with Supratik Chakraborty (IITB), Daniel J. Fremont(UCB), Alexander Ivrii (IBM), Sharad Malik (Princeton), Sanjit A. Seshia (UCB), Moshe Y. Vardi (Rice)

How do we guarantee that systems work *correctly* ?





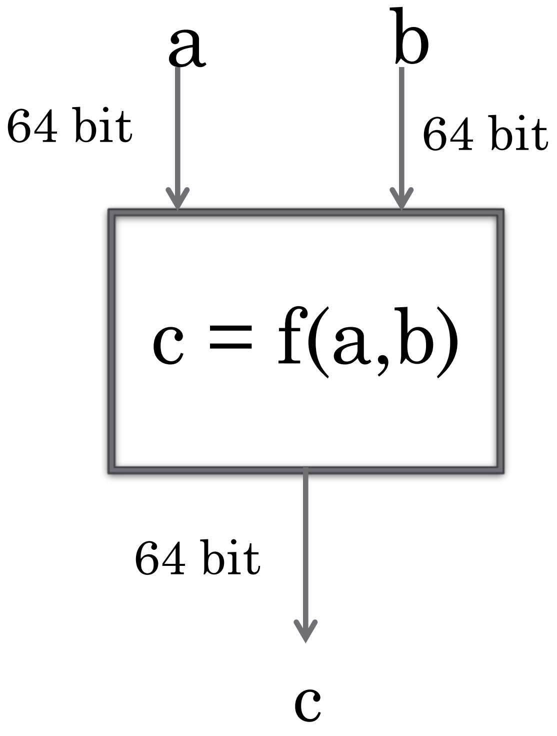
Functional Verification

- Formal verification
  - Challenges: formal requirements, scalability
  - ~10-15% of verification effort
- Dynamic verification: ***dominant approach***

Dynamic Verification

- Design is simulated with test vectors
  - Test vectors represent different verification scenarios
- Results from simulation compared to intended results
- **Challenge**: Exceedingly large test space!

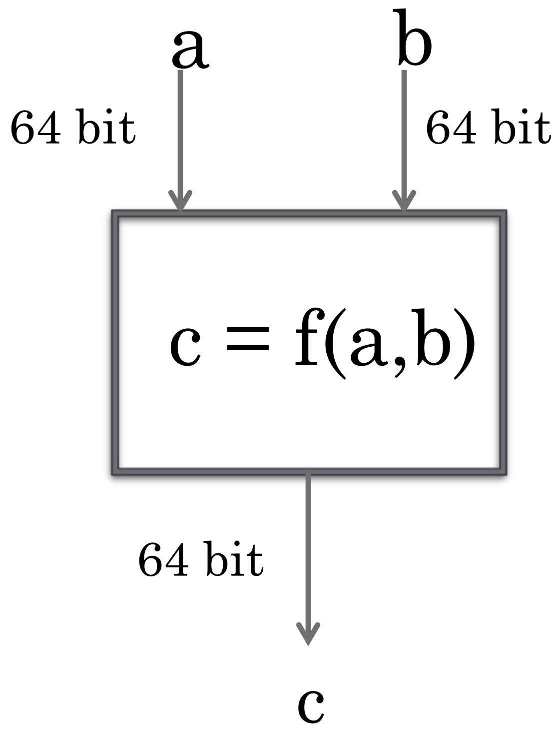
Motivating Example



How do we test the circuit works ?

- Try for all values of a and b
    - 2^{128} possibilities
    - Sun will go nova before done!
    - Not scalable

Constrained-Random Simulation

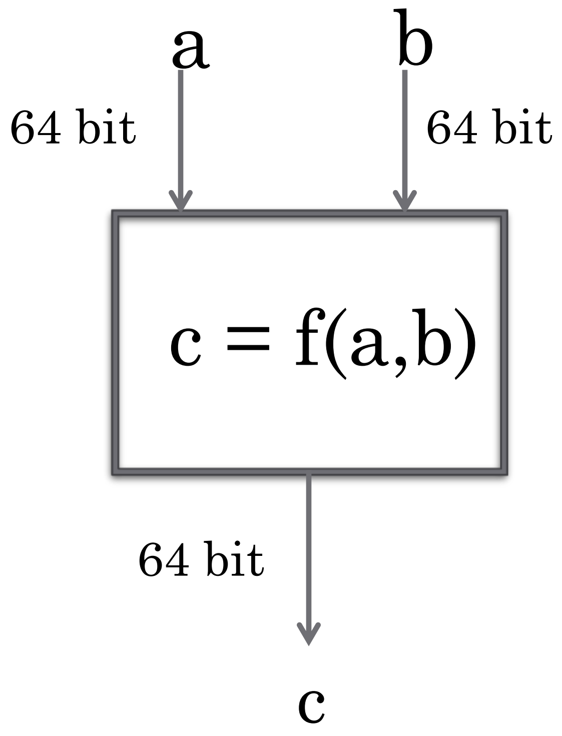


Sources for Constraints

- Designers:
  1. $a +_{64} 11 *_{32} b = 12$
  2. $a <_{64} (b >> 4)$
- Past Experience:
  1. $40 <_{64} 34 + a <_{64} 5050$
  2. $120 <_{64} b <_{64} 230$
- Users:
  1. $232 *_{32} a + b != 1100$
  2. $1020 <_{64} (b /_{64} 2) +_{64} a <_{64} 2200$

- Test vectors: solutions of constraints

Constrained-Random Simulation



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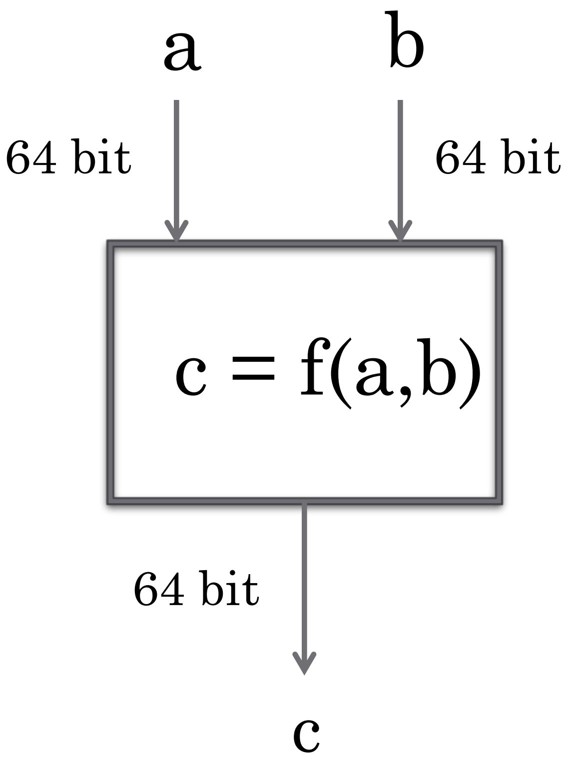
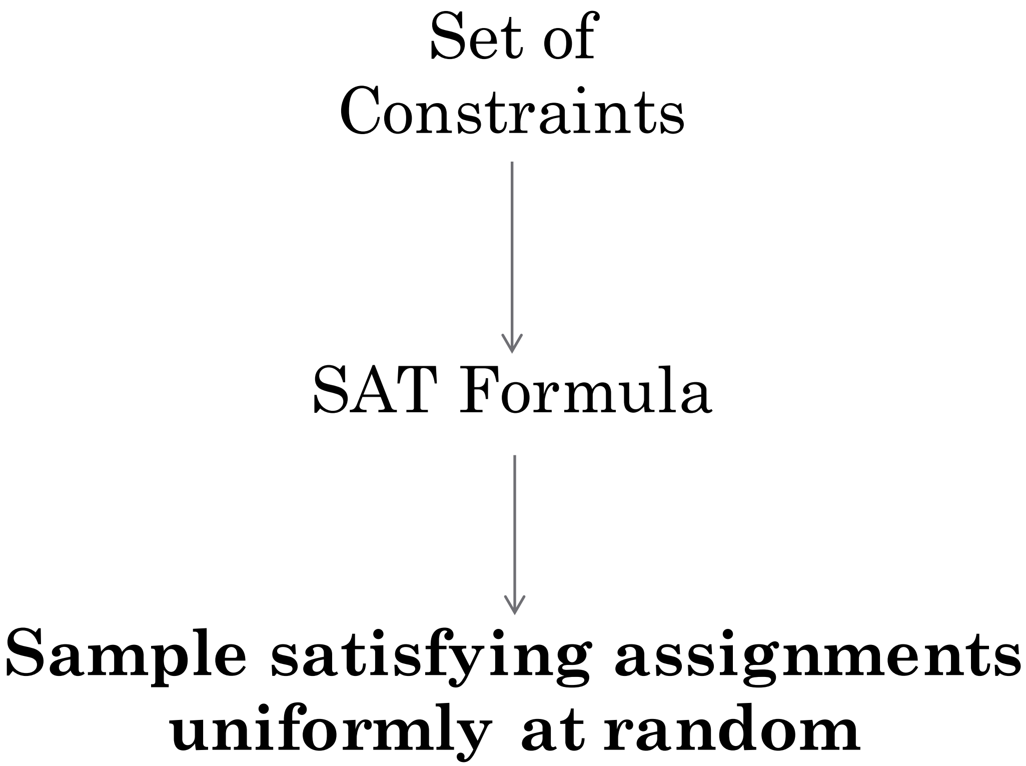
Problem: How can we <u>*uniformly*</u> sample the values of a and b satisfying the above constraints?

Problem Formulation



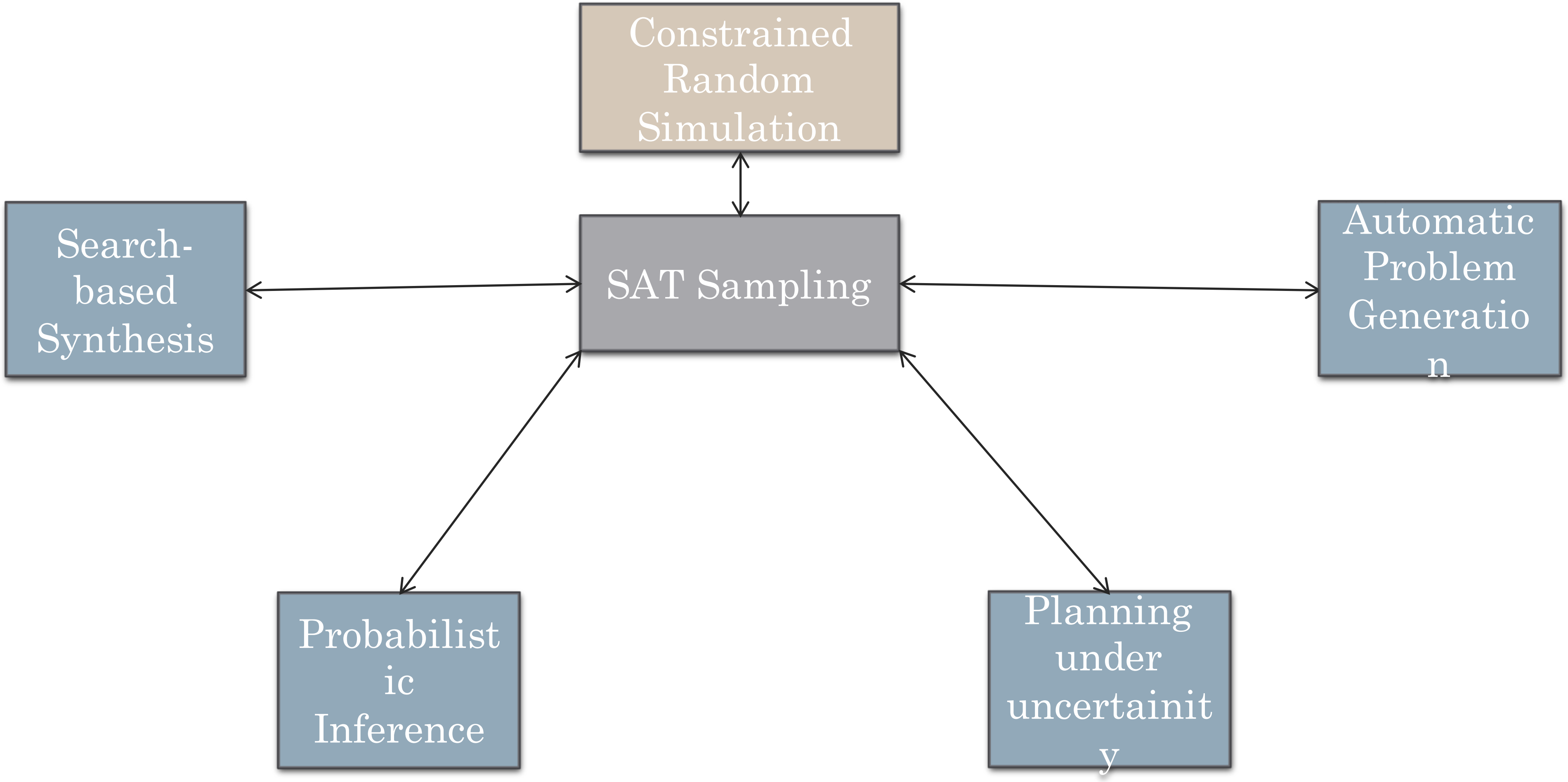


Scalable Uniform Generation of SAT Witnesses

Constraint satisfaction for random stimuli generation

Yehuda Naveh
IBM Haifa Research Lab

Diverse Applications



Search-Based Synthesis

- **Goal**: synthesize from under-constrained specifications (“sketch”)
- Large space of programs that satisfy correctness conditions
- Task: Find “optimal” program (wrt running time, memory, …)
- Method: ***Uniformly sample*** from the space of programs

Constrained Counting

- Given a SAT formula F
- R_F: Set of all solutions of F
- Problem (#SAT): Estimate the number of solutions of F (#F) i.e., what is the cardinality of R_F?
- E.g., F = (a v b)
- $R_F = \{(0,1), (1,0), (1,1)\}$
- The number of solutions (#F) = 3

#P: The class of counting problems for decision problems in NP!

Practical Applications

Wide range of applications!

- Probabilistic reasoning/Bayesian inference
- Dynamic Verification
- Planning with uncertainty
- Multi-agent/ adversarial reasoning

[Roth 96, Sang 04, Bacchus 04, Domshlak 07]

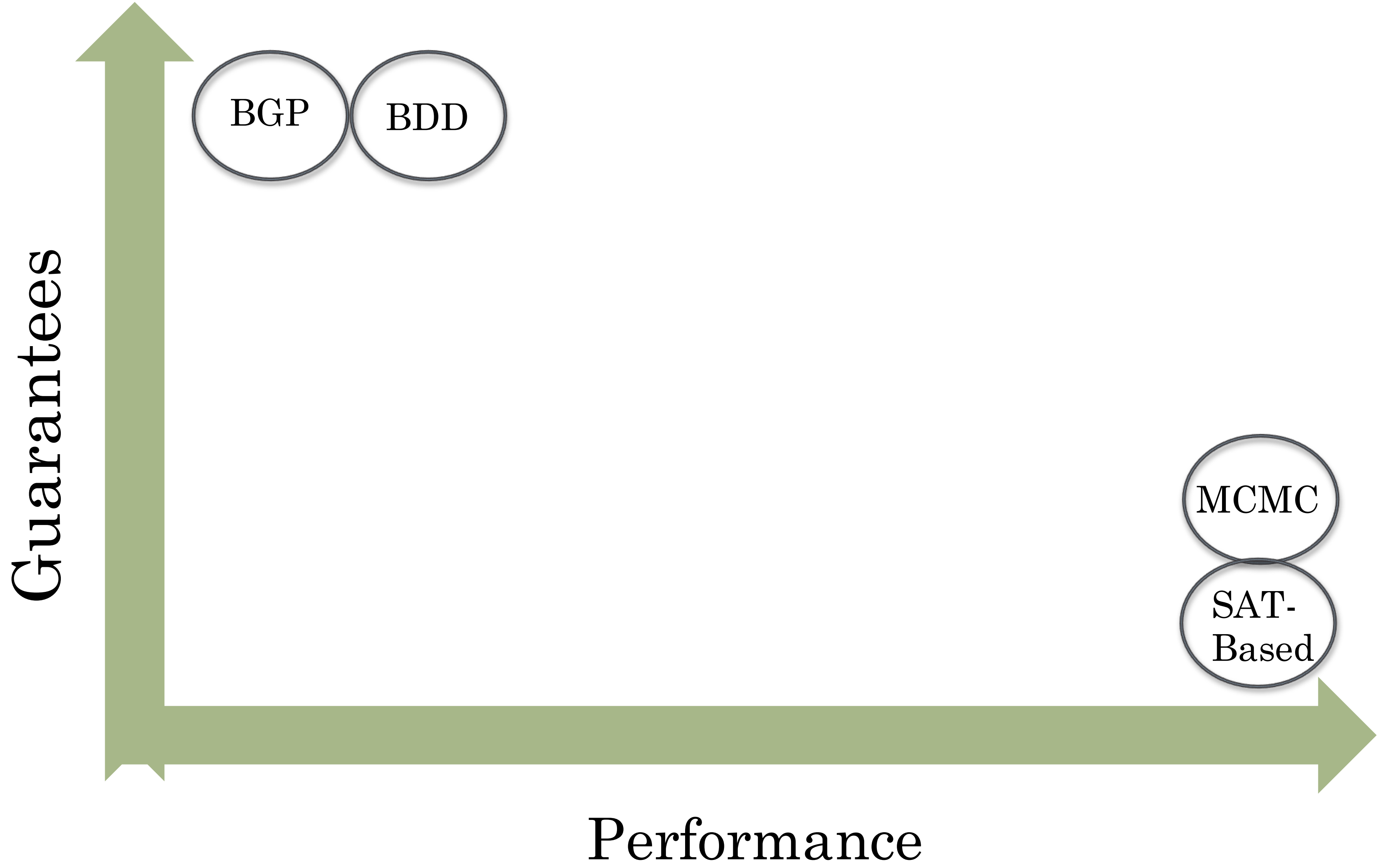
Agenda

Design Scalable Techniques for
Uniform Generation and
Model Counting
with Strong Theoretical Guarantees

Agenda

Design Scalable Techniques for
Almost-Uniform Generation and
Approximate-Model Counting
with Strong Theoretical Guarantees

Prior Work

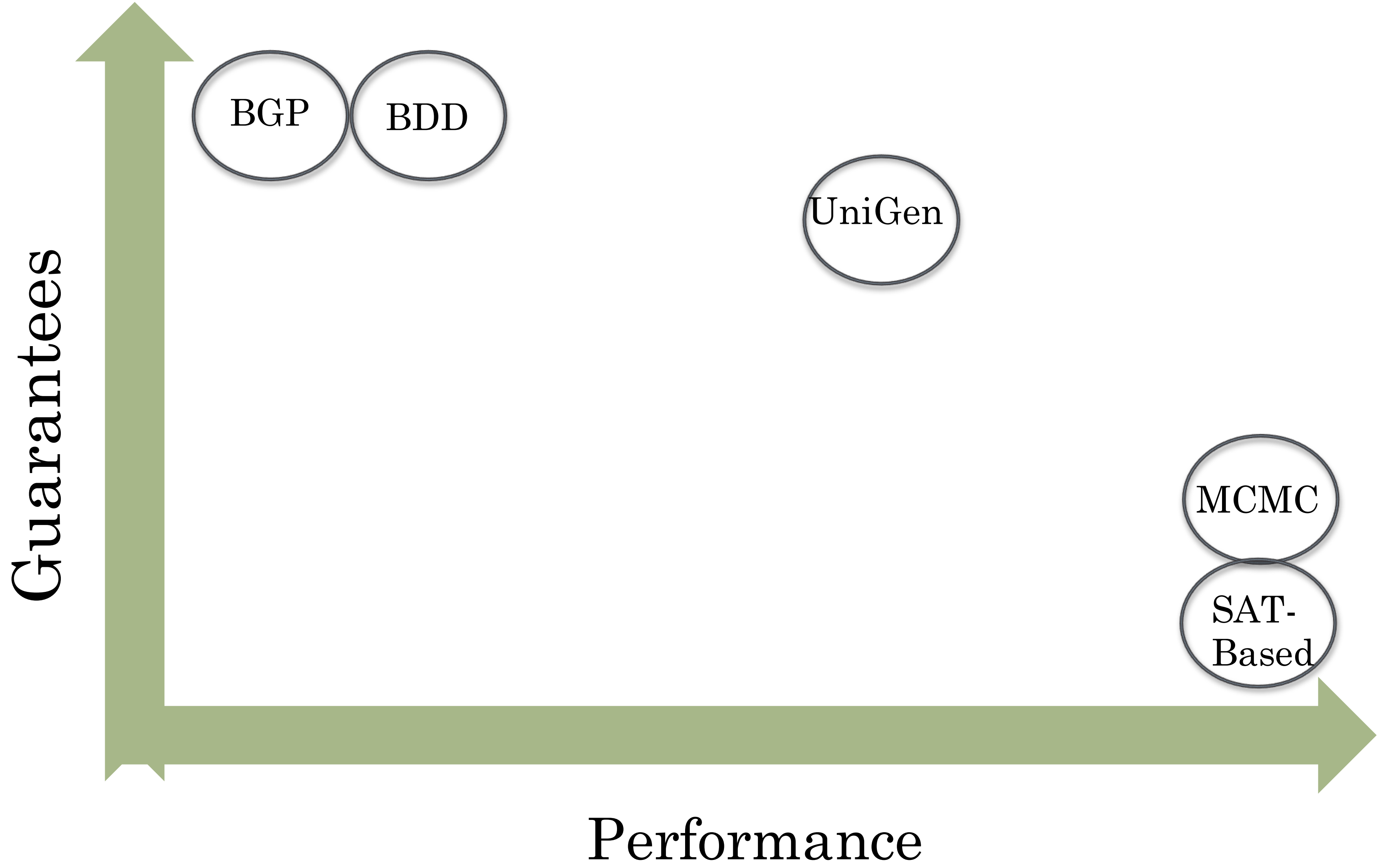


Desires

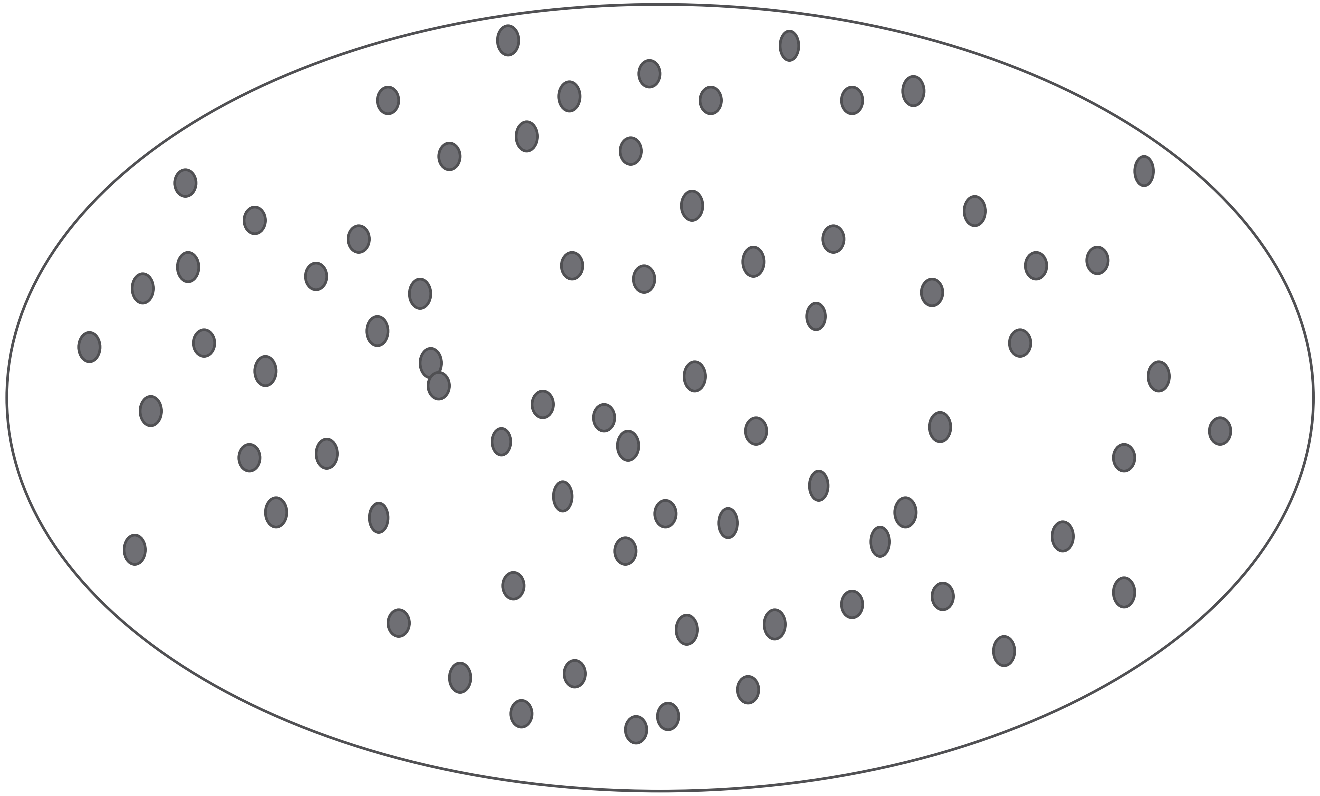
Generator	Relative runtime
State-of-the-art: XORSample’	50000
Ideal Uniform Generator*	10
SAT Solver	1

Experiments over 200+ benchmarks
*: According to EDA experts

Our Contribution



Partitioning into equal “small” cells



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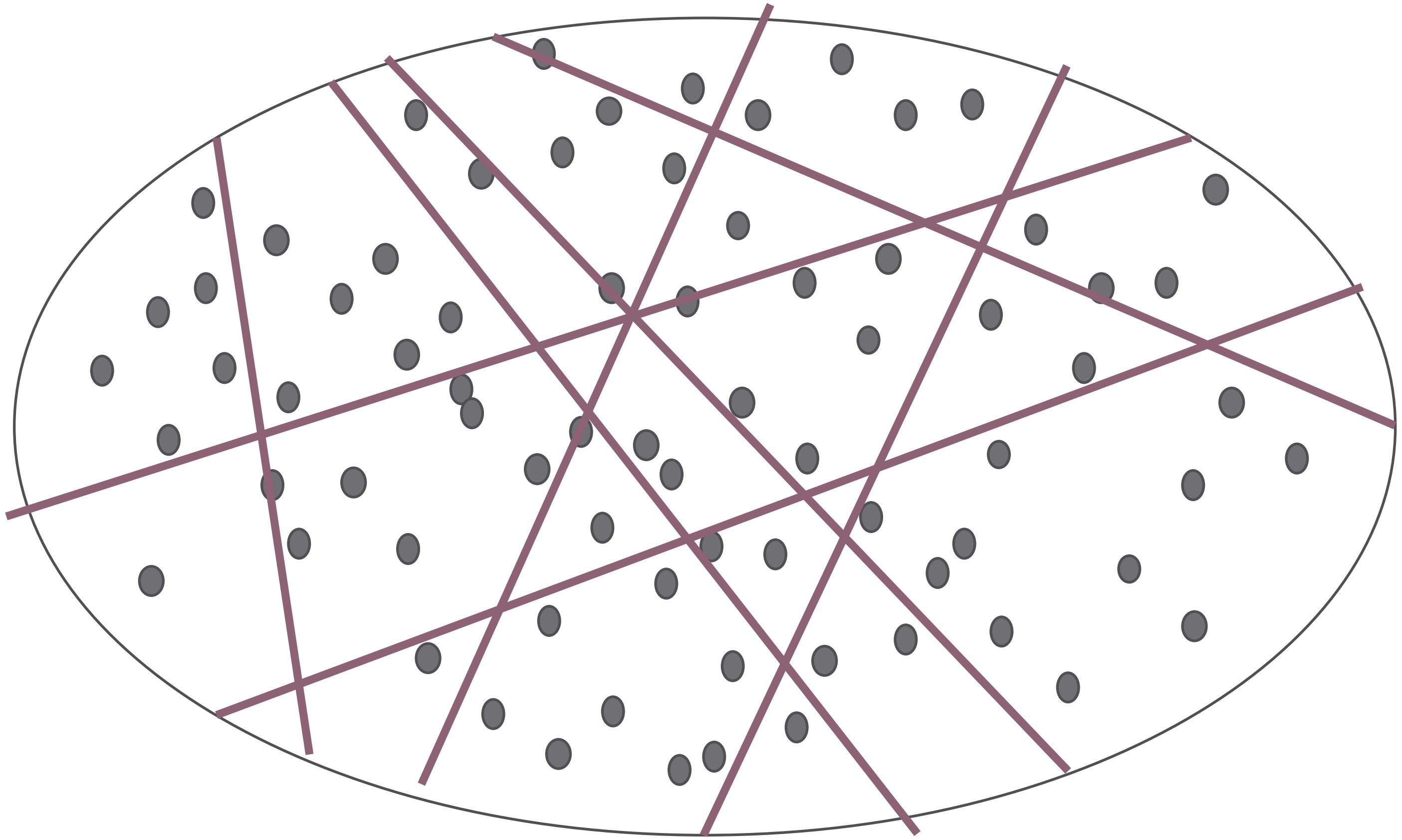
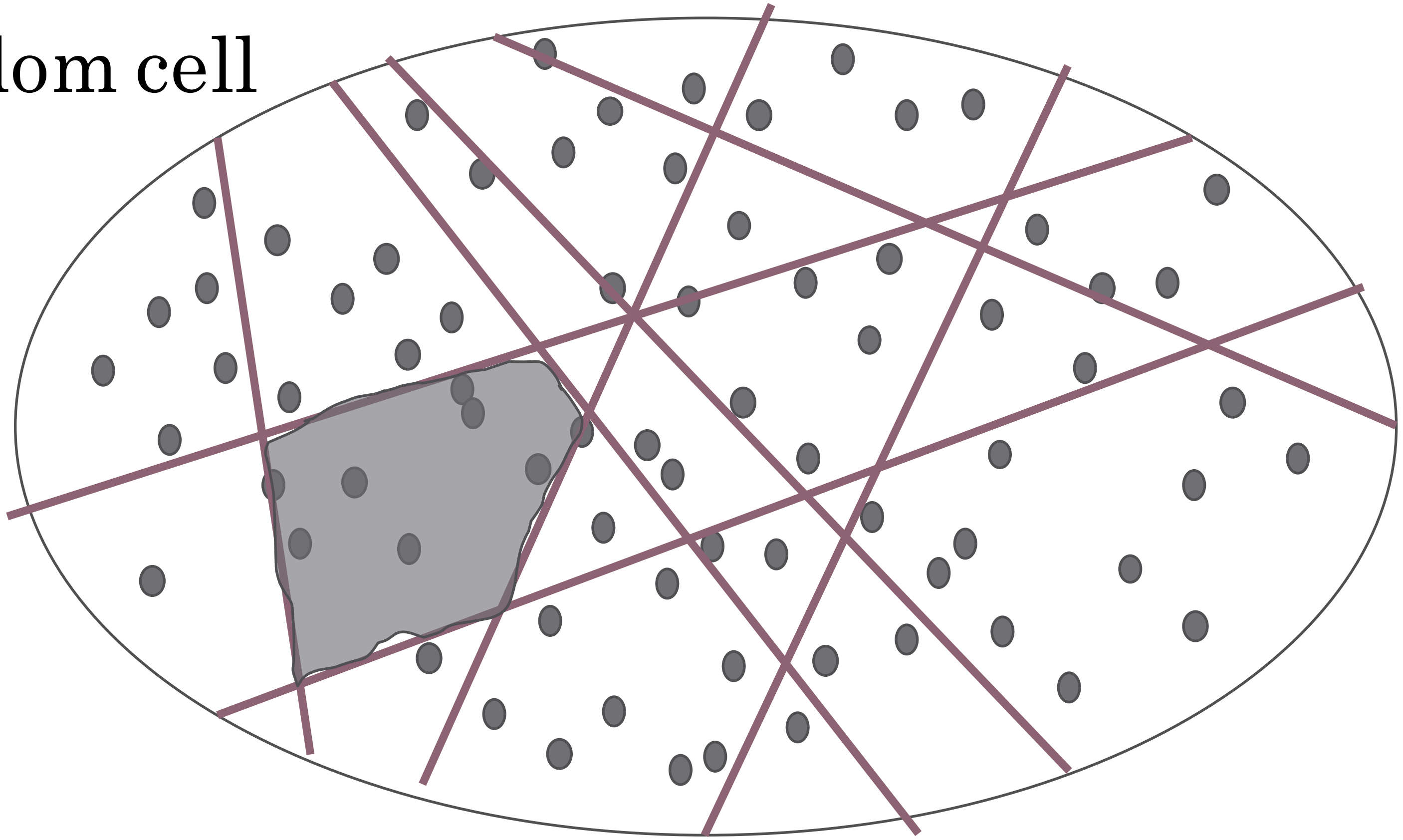


Partitioning into equal "small" cells

Pick a random cell



Pick a random solution from this cell

How to Partition?

How to partition into roughly equal small cells of solutions without knowing the distribution of solutions?

Universal Hashing
[Carter-Wegman 1979] (IBM Research)

Universal Hashing

- Hash functions: mapping $\{0,1\}^n$ to $\{0,1\}^m$
  - (2^n elements to 2^m cells)
- Random inputs => All cells are *roughly* equal (in <u>expectation</u>)
- Universal family of hash functions:
  - Choose hash function randomly from family
  - For ***arbitrary*** distribution on inputs => All cells are ***roughly equal*** (in <u>expectation</u>)

Universal Hashing and Independence

- Hash functions from mapping $\{0,1\}^n$ to $\{0,1\}^m$
  - (2^n elements to 2^m cells)
- Universal hash functions:
  - Choose hash function randomly
  - For arbitrary distribution on inputs => All cells are *roughly* equal in expectation
  - But:
    - While each input is hashed **uniformly**
    - Different inputs ***might not*** be hashed **independently**

Strong Universality

- H(n,m,r): Family of r-universal hash functions mapping $\{0,1\}^n$ to $\{0,1\}^m$ (2^n elements to 2^m cells)
  - r: degree of independence of hashed inputs

- Higher r => Stronger guarantee on ***range of size*** of cells

- r-wise universality => Polynomials of degree r-1

- Higher universality => Higher complexity

Partitioning

- How large should the cells be?

- How many cells?

Size of cell

- Too large => Hard to enumerate
- Too small => Variance can be very high

$$\text{pivot} = 5(1 + 1/\varepsilon)^2$$

How many cells?

- Our desire: $2^{\mathrm{m}} = \frac{|R_F|}{pivot}$
  - But determining $|R_F|$ is expensive (#P complete)

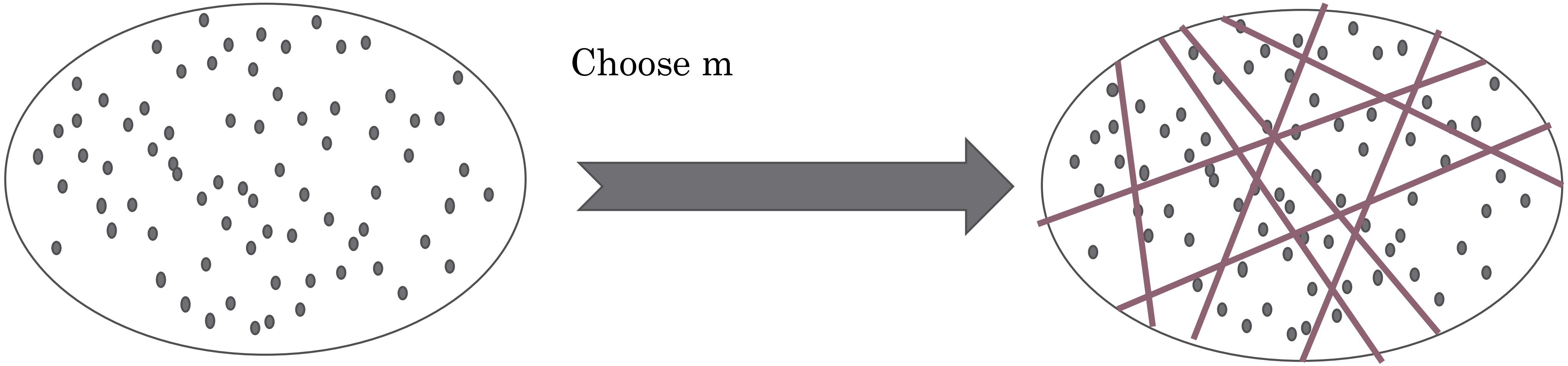
- How about approximation?
  - $ApproxMC(F, \varepsilon, \delta)$ returns C:

$$\Pr[\frac{|R_F|}{1+\varepsilon} \leq C \leq (1+\varepsilon)|R_F|] \geq 1-\delta$$

  - $q = \log C - \log pivot$

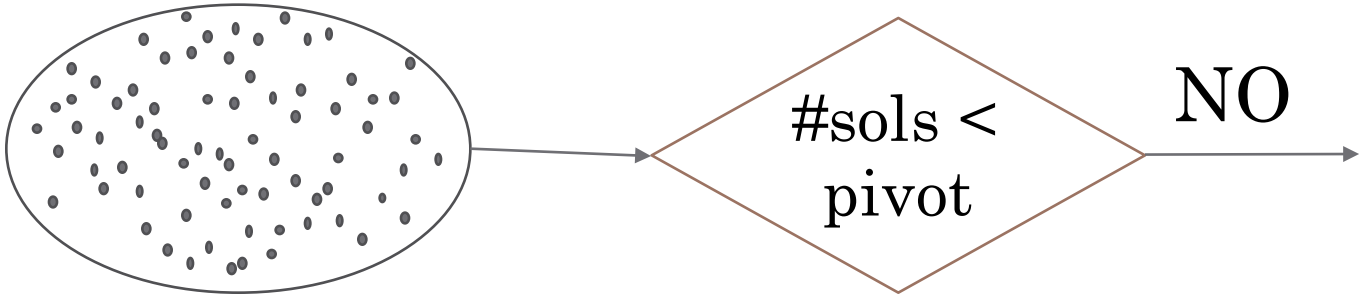
  - Concentrate on 2^m, where m = q-1, q, q+1

ApproxMC(F,ε, δ)



- For right choice of m, large number of cells are “small”
  - “almost all” the cells are “roughly” equal
- Check if a randomly picked cell is “small”
- If yes, then estimate = # of solutions in cell * 2^m

ApproxMC(F,ε, δ)



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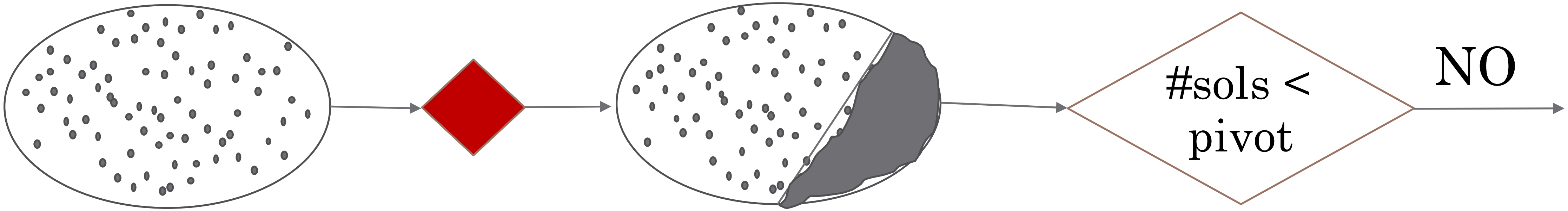

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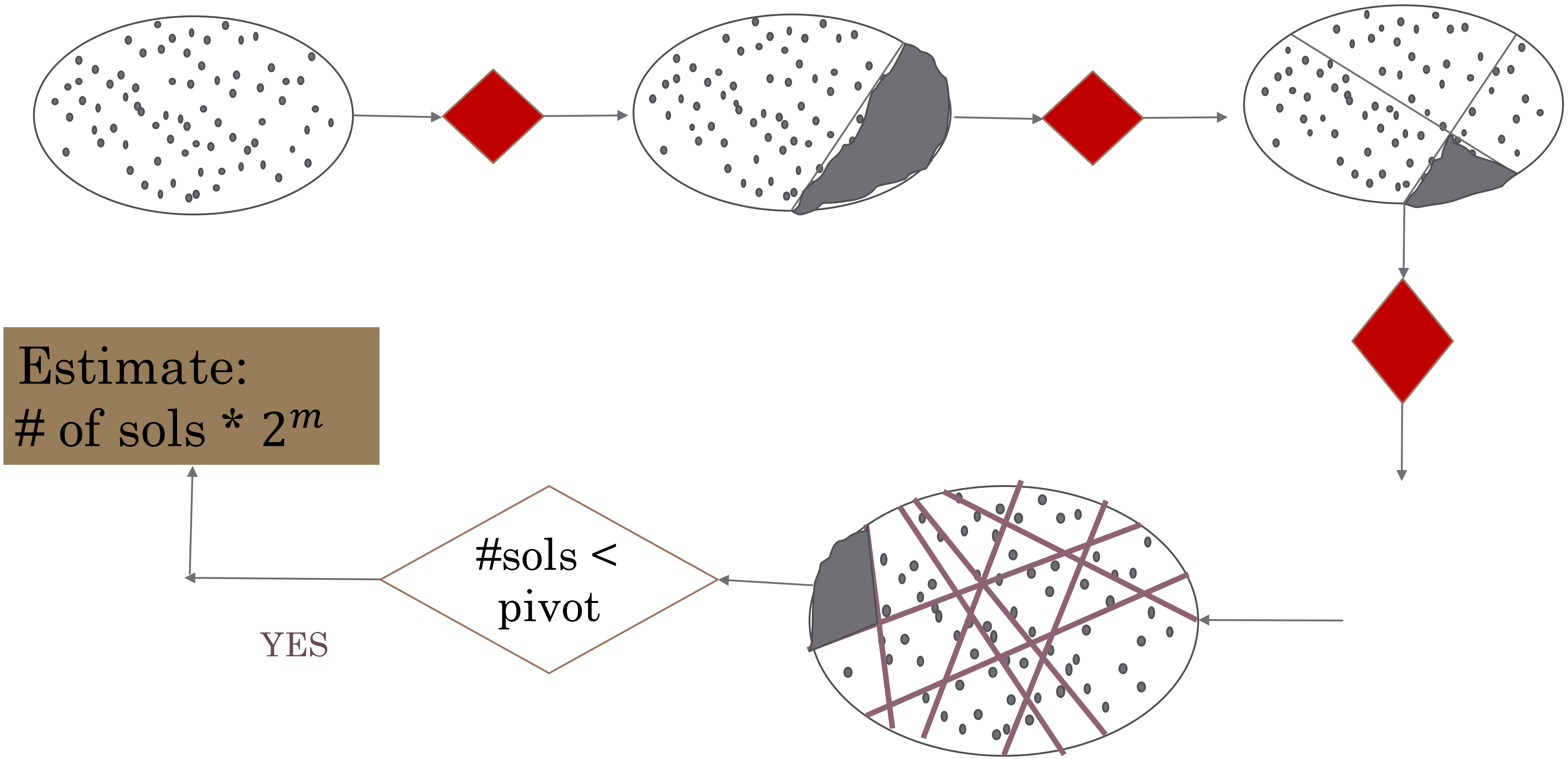
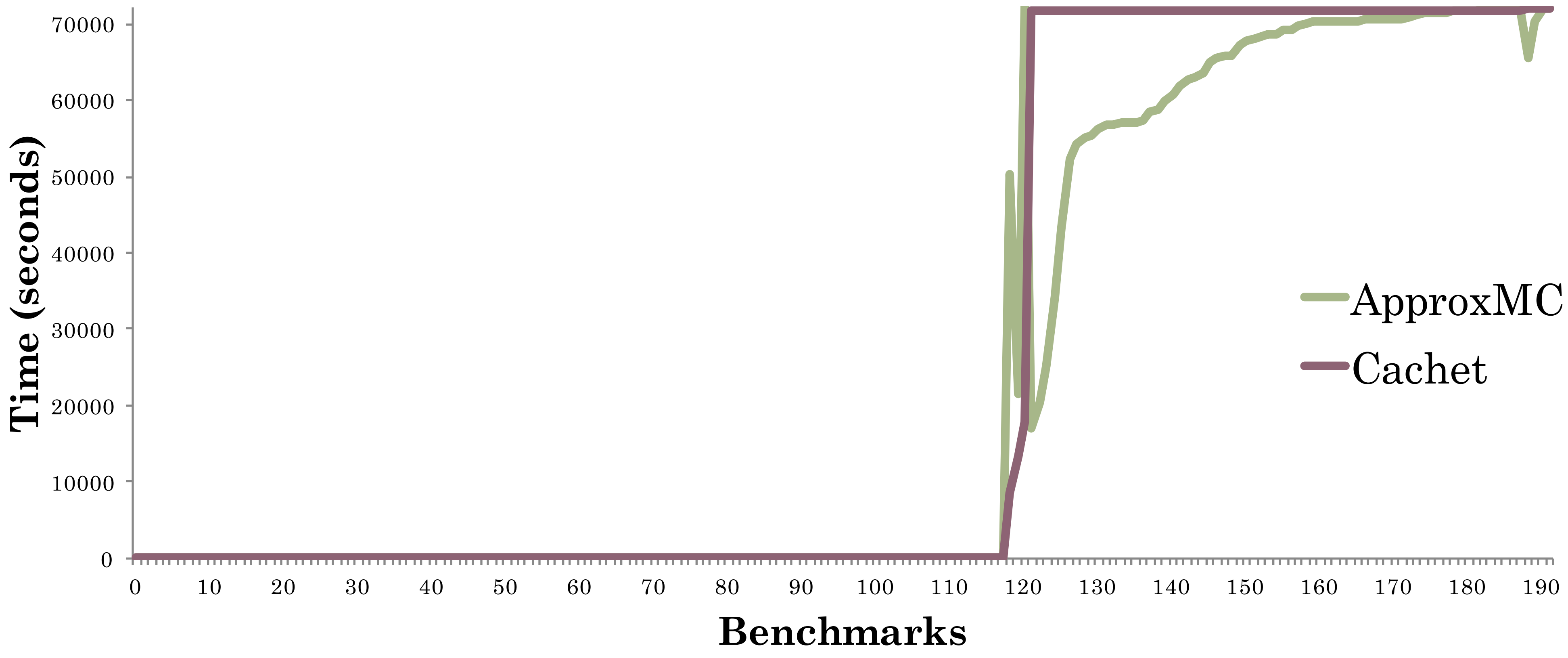


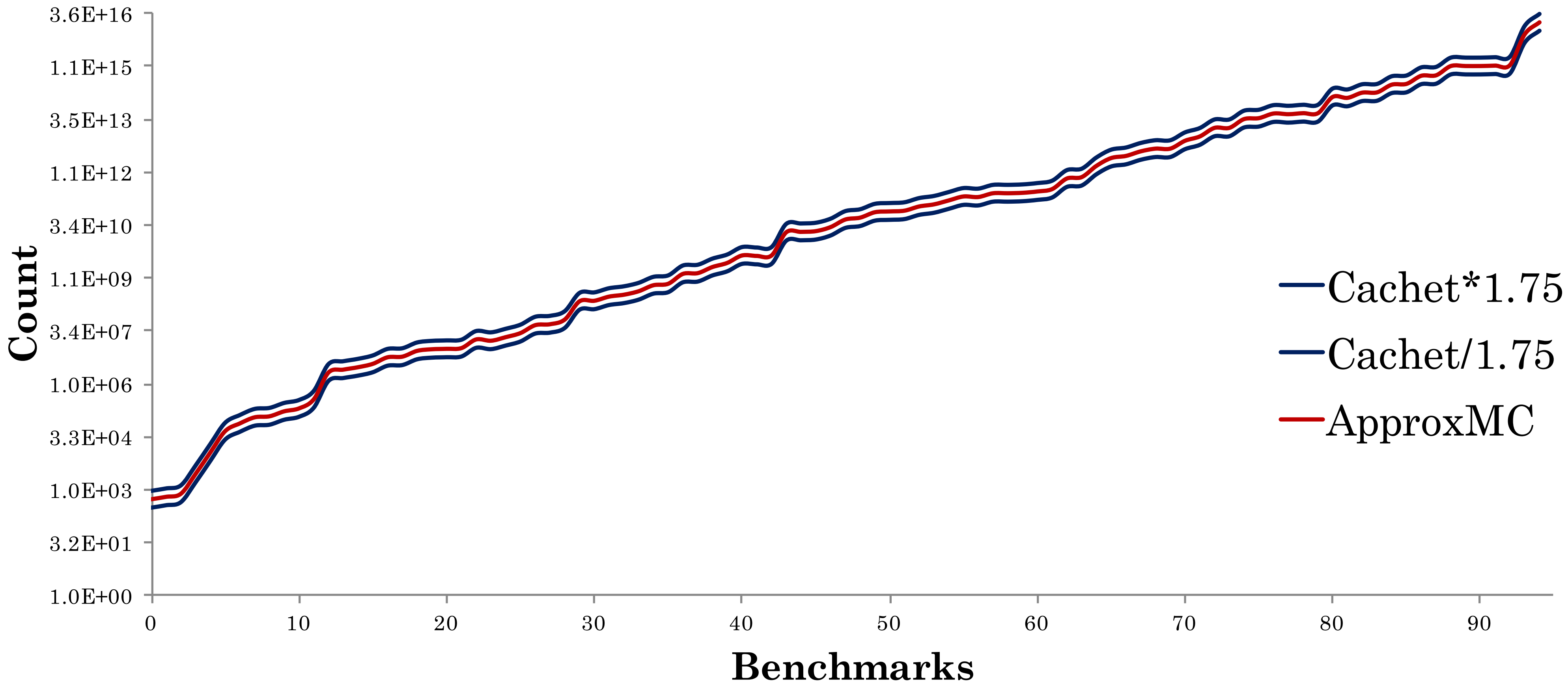
Runtime Performance of ApproxMC

Can Solve a Large Class of Problems



Large class of problems that lie beyond the exact counters but can be computed by ApproxMC

Mean Error: Only 4% (allowed: 75%)



Mean error: 4% – much smaller than the theoretical guarantee of 75%

Guarantees and Runtime performance of UniGen

Strong Theoretical Guarantees

- Almost-Uniformity

For every solution y of R_F

$1/(6.84+\varepsilon) \times 1/|R_F| <= Pr[y \text{ is output}] <= (6.84+\varepsilon)/|R_F|$

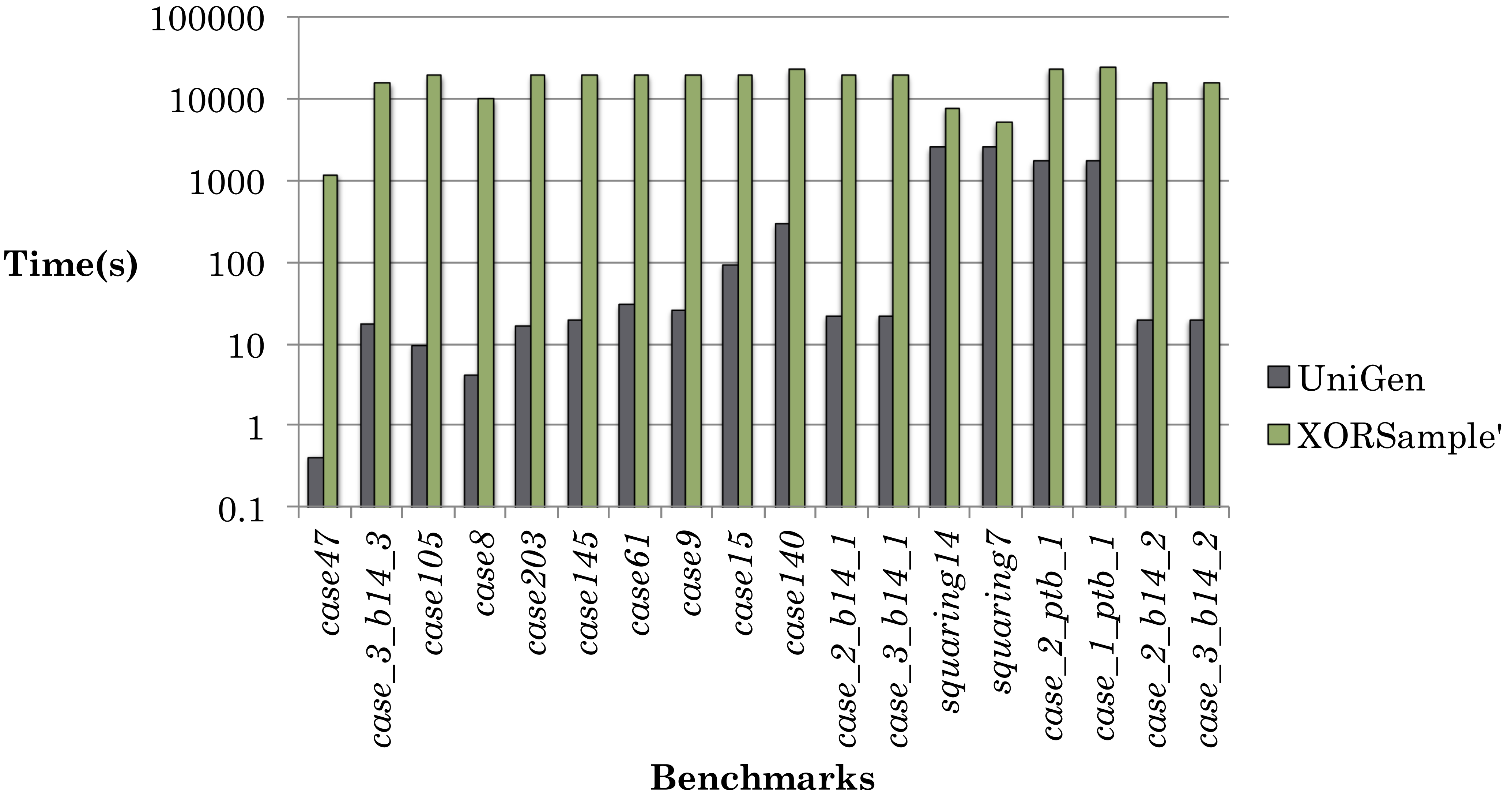
- Success Probability

UniGen succeeds with probability at least 0.52

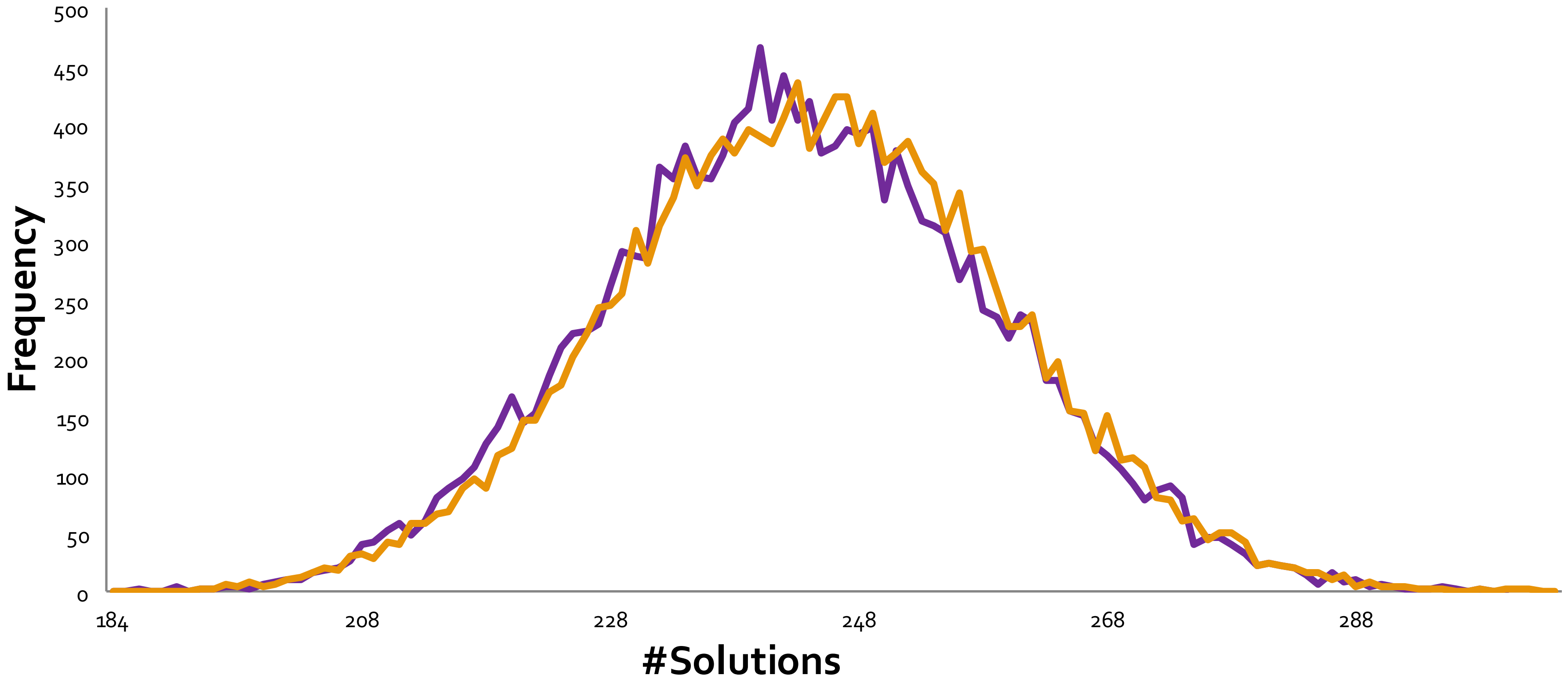
  - In practice, succ. Probability ~ 0.99

- Polynomial number of calls to SAT Solver

1-2 Orders of Magnitude Faster

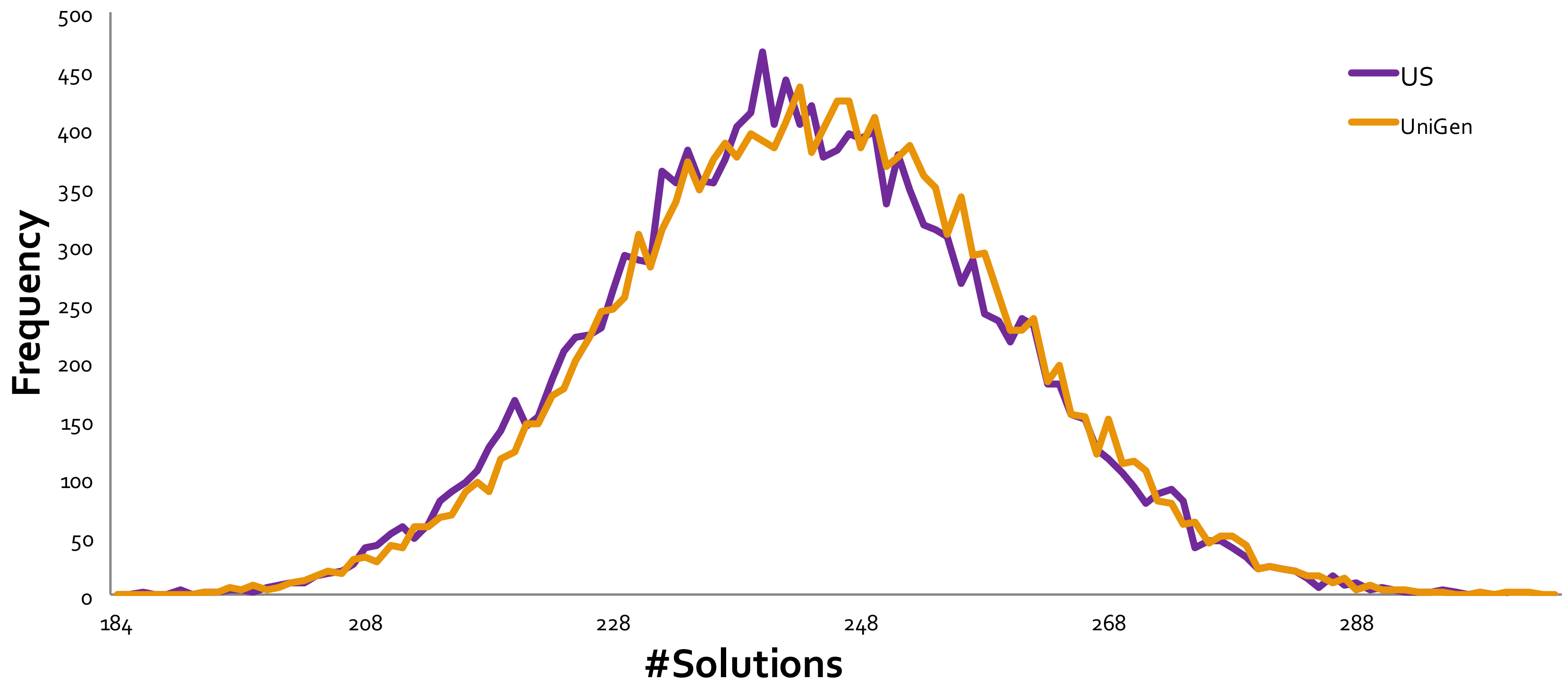


Results: Uniformity



- Benchmark: case110.cnf; #var: 287; #clauses: 1263
- Total Runs: $4x10^6$; Total Solutions : 16384

Results: Uniformity



- Benchmark: case110.cnf; #var: 287; #clauses: 1263
- Total Runs: $4x10^6$; Total Solutions : 16384

So far

- The first scalable approximate model counter
- The first scalable uniform generator
- Outperforms state-of-the-art generators/counters

Are we done?

Where are we?

Generator	Relative runtime
State-of-the-art: XORSample'	50000
UniGen	~5000
Ideal Uniform Generator*	10
SAT Solver	1

Experiments over 200+ benchmarks
*: According to EDA experts

XOR-Based Hashing

- Partition 2^n space into 2^m cells
- Variables: $X_1, X_2, X_3, \ldots, X_n$
- Pick every variable with prob. ½ ,XOR them and add 0/1 with prob. ½
- $X_1 + X_3 + X_6 + \ldots X_{n-1} + 0$
- To construct h: $\{0,1\}^n \rightarrow \{0,1\}^m$, choose m random XORs
- $\alpha \in \{0,1\}^m \rightarrow$ Set every XOR equation to 0 or 1 randomly
- The cell: $F \wedge$ XOR (CNF+XOR)

XOR-Based Hashing

- **CryptoMiniSAT**: Efficient for CNF+XOR
- Avg Length : n/2
- Smaller XORs → better performance

How to shorten XOR clauses?

Independent Support

- Set I of variables such that assignments to these uniquely determine assignments to rest of variables (for satisfying assignments)
- If σ_1 and σ_2 agree on I then $\sigma_1 = \sigma_2$
- c ⟷ (a V b) ; Independent Support I: {a, b}
- ***Key Idea***: Hash only on the independent variables
- Average size of XOR: n/2 to |I|/2

Key Idea

Minimal Independent Support (MIS) → Minimal Unsatisfiable Subset (MUS)

Minimal Unsatisfiable Subset

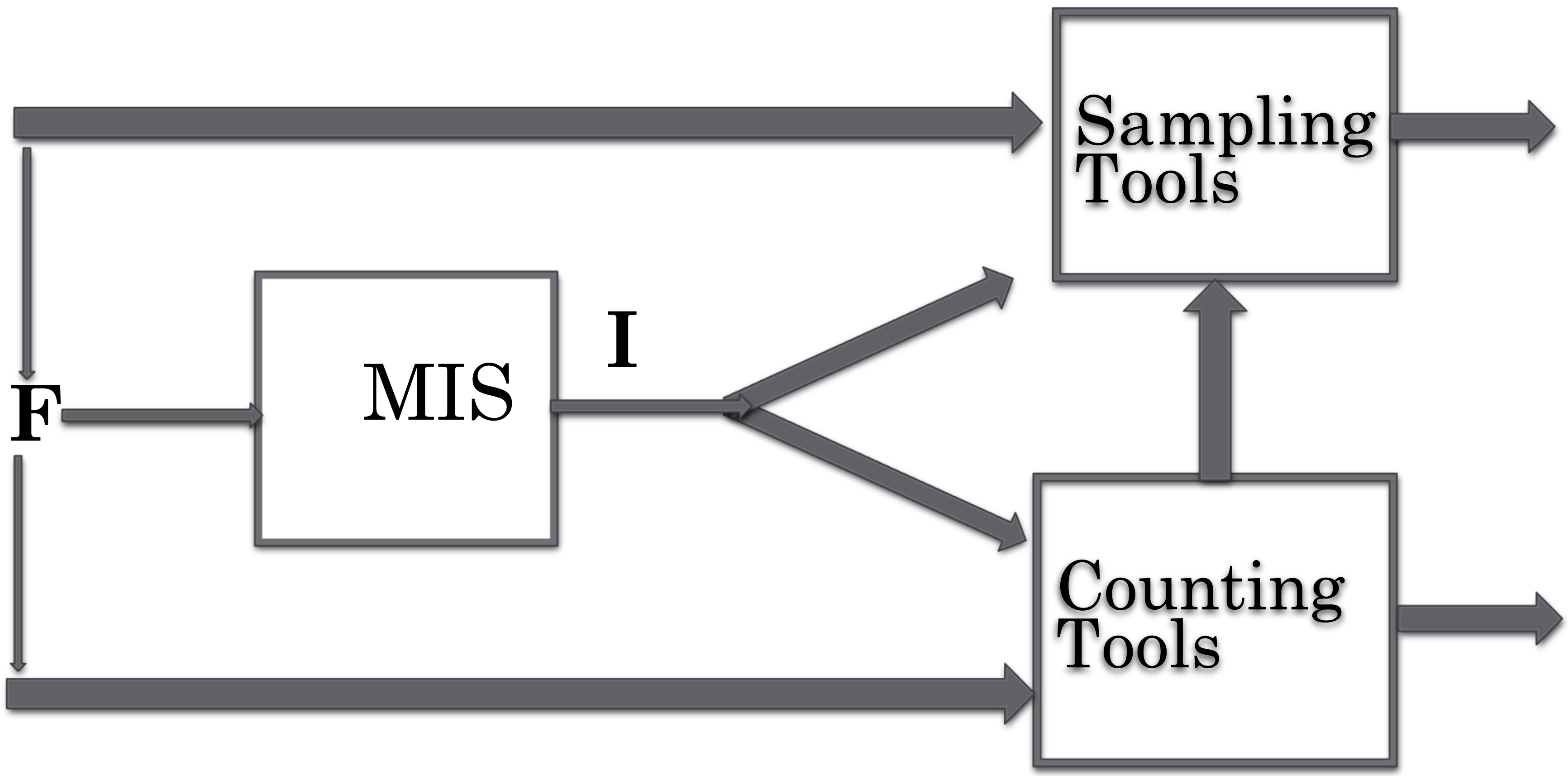
- Given $\Psi = H_1 \wedge H_2 \cdots H_m$
  - Find subset $\{H_{i1}, H_{i2}, \cdots H_{ik}\}$ of $\{H_1, H_2, \cdots H_m\}$ such that $H_{i1} \wedge H_{i2} \cdots H_{ik} \wedge \Omega$ is UNSAT

    Unsatisfiable subset

  - Find **minimal** subset $\{H_{i1}, H_{i2}, \cdots H_{ik}\}$ of $\{H_1, H_2, \cdots H_m\}$ such that $H_{i1} \wedge H_{i2} \cdots H_{ik}$ is UNSAT

    Minimal Unsatisfiable subset

Impact on Sampling and Counting Techniques

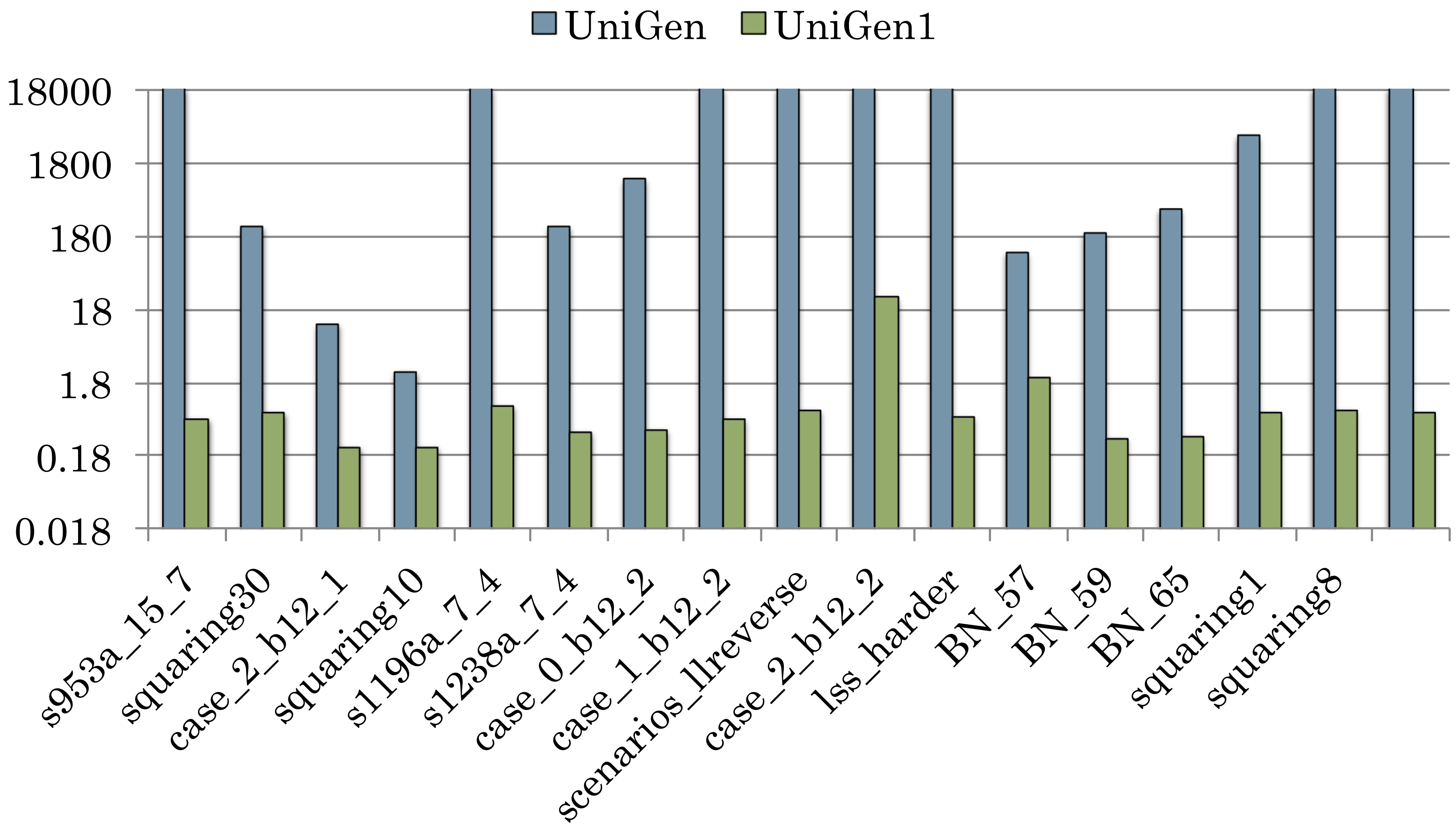


What about complexity

- Computation of MUS: FP^{NP}
- Why solve a FP^{NP} for almost-uniform generation/approximate counter (PTIME PTM with NP Oracle)

Settling the debate through practice!

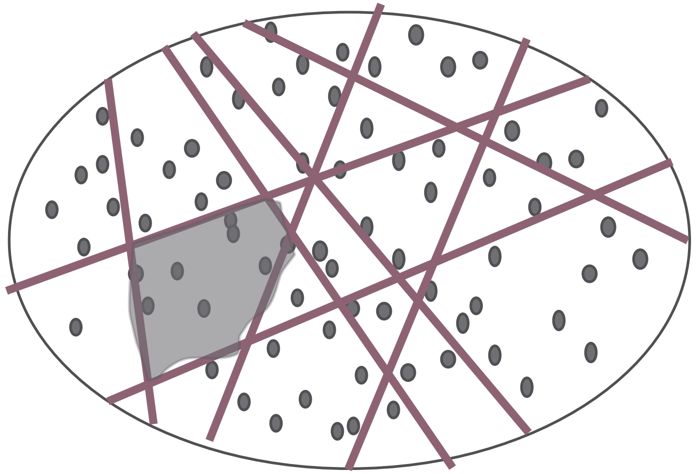
Performance Impact on Uniform Sampling



Where are we?

Generator	Relative runtime
State-of-the-art: XORSample’	50000
UniGen	5000
UniGen1	470
Ideal Uniform Generator*	10
SAT Solver	1

Back to basics



# of solutions in “small” cell $\in [loThresh, hiThresh]$

We pick one solution

“Wastage” of loThresh solutions

Pick *loThresh* samples!

Balancing Independence

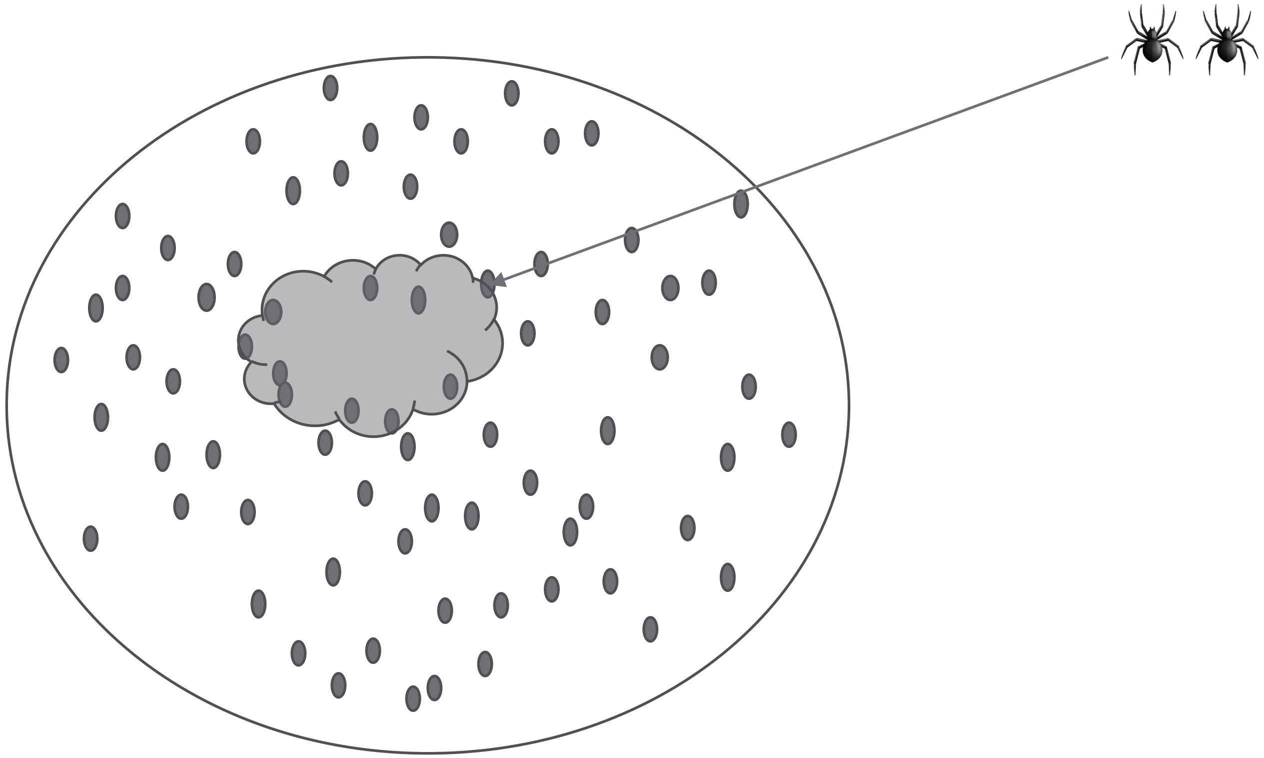
For $h \in H(n, m, 3)$

- Choosing up to 3 samples => Full independence among samples

- Choosing loThresh (>> 3) samples => Loss of independence

Why care about Independence

Convergence requires multiplication of probabilities



If every sample is independent => Faster convergence

The principle of principled compromise!

- Choosing up to 3 samples => Full independence among samples

- Choosing loThresh (>> 3) samples => Loss of independence
  - “Almost-Independence” among samples
  - Still provides strong theoretical guarantees of coverage

Strong Guarantees

- $L = \#\ of\ samples < |R_F|$

$$\frac{L}{(1+\varepsilon)|R_F|} \leq \Pr[\text{y is output}] \leq 1.02(1+\varepsilon)\frac{L}{|R_F|}$$

- ~~Polynomial~~ Constant number of SAT calls per sample
  - After one call to ApproxMC

Bug-finding effectiveness

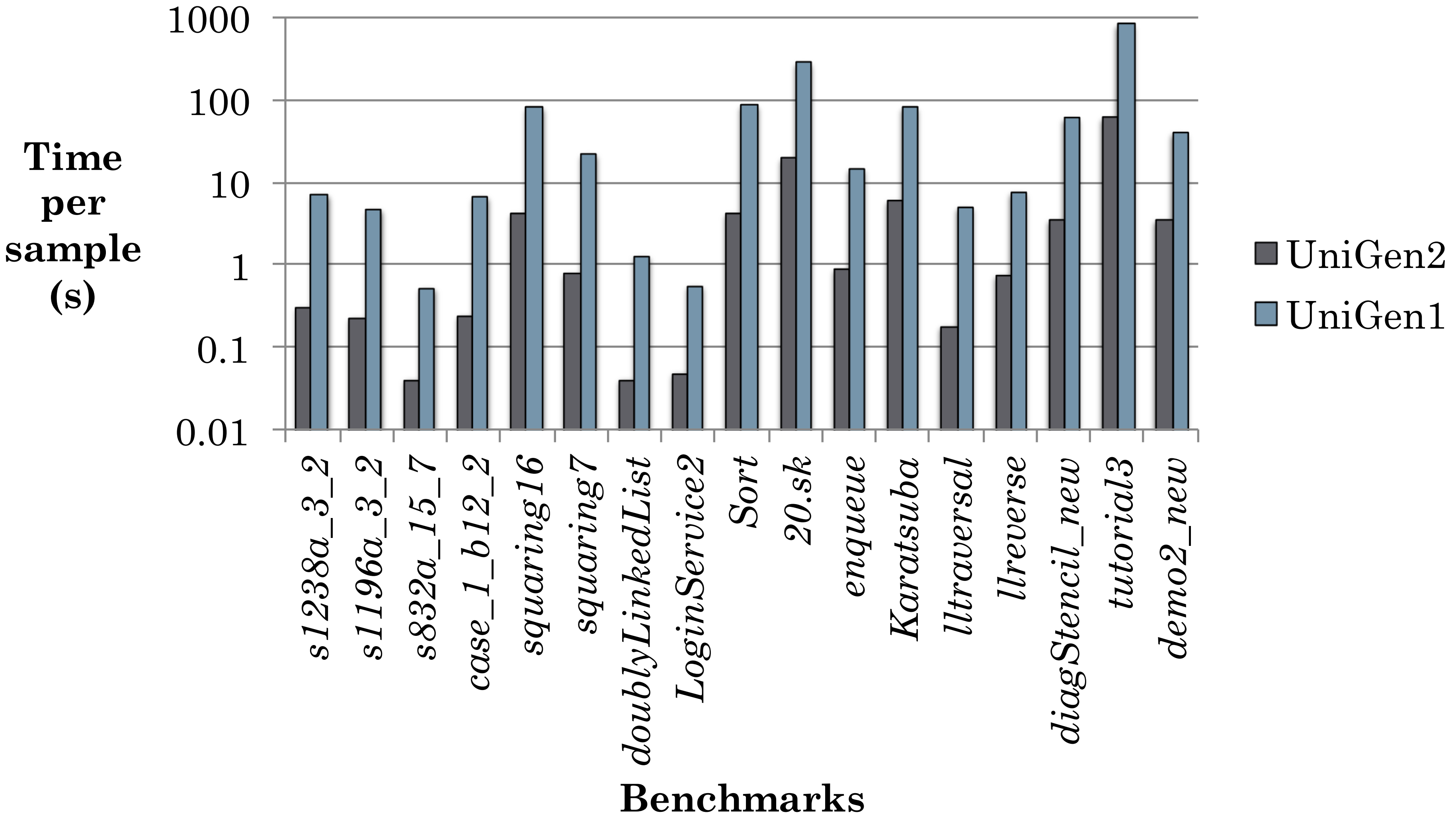
bug frequency $f = 1/10^4$

find bug with probability $\geq 1/2$

	UniGen	UniGen2
Expected number of SAT calls	4.35×10^7	3.38×10^6

An order of magnitude difference!

~20 times faster than UniGen1



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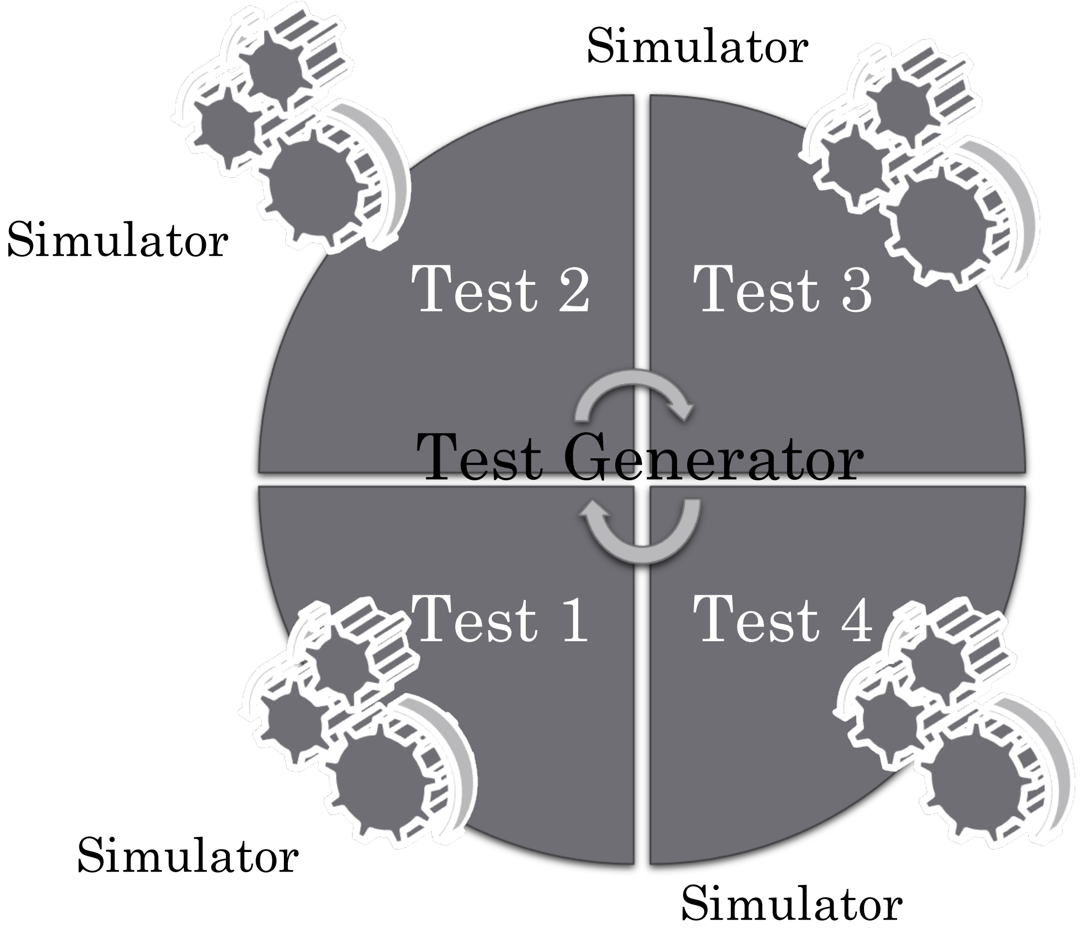
The Final Push….

- UniGen requires one time computation of ApproxMC
- Generation of samples in fully distributed fashion

(Previous algorithms lacked the above property)

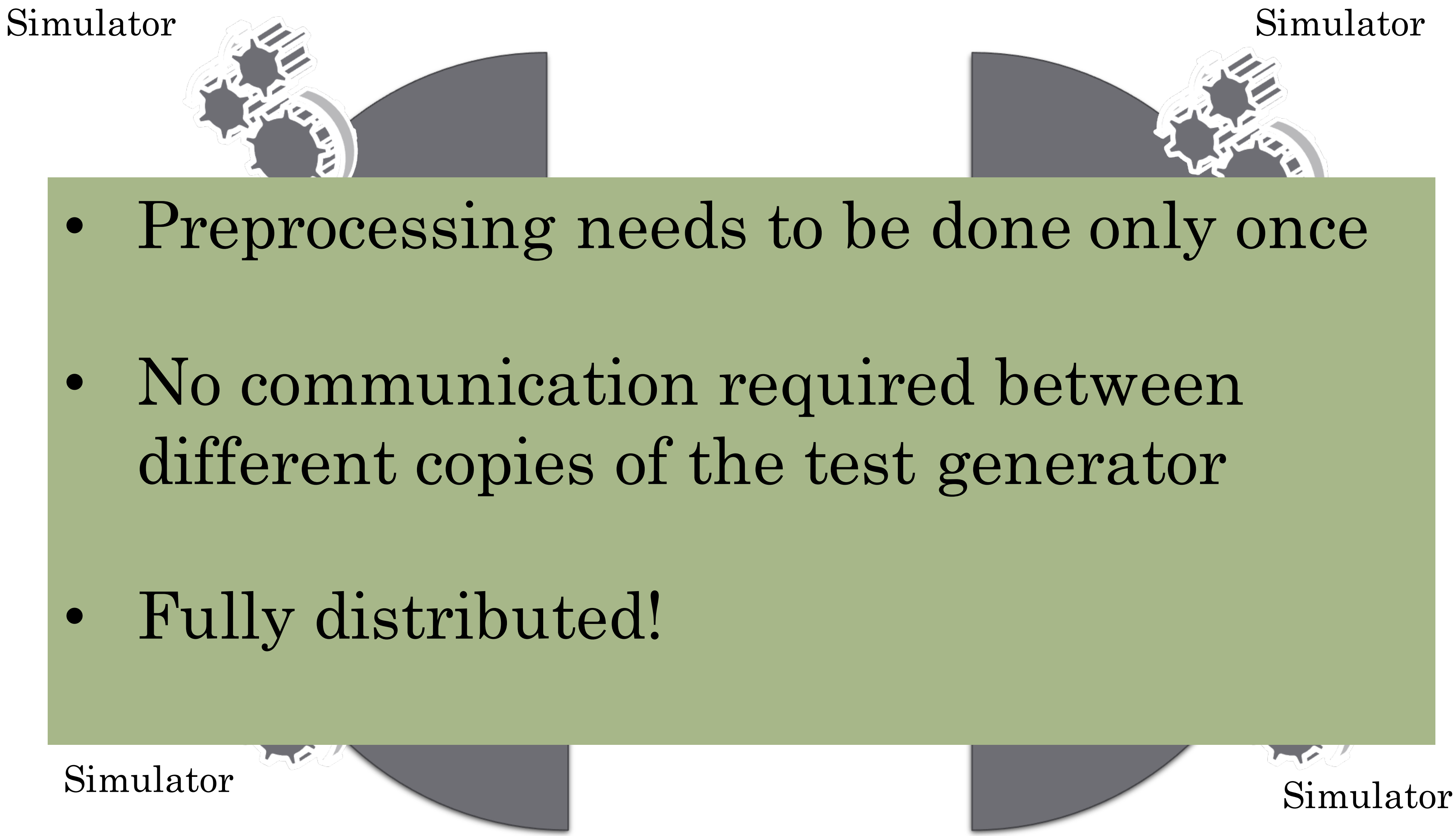
- New paradigms!

Current Paradigm of Simulation-based Verification



- Can not be parallelized since test generators maintain "global state"
- Loses theoretical guarantees (if any) of uniformity

New Paradigm of Simulation-based Verification

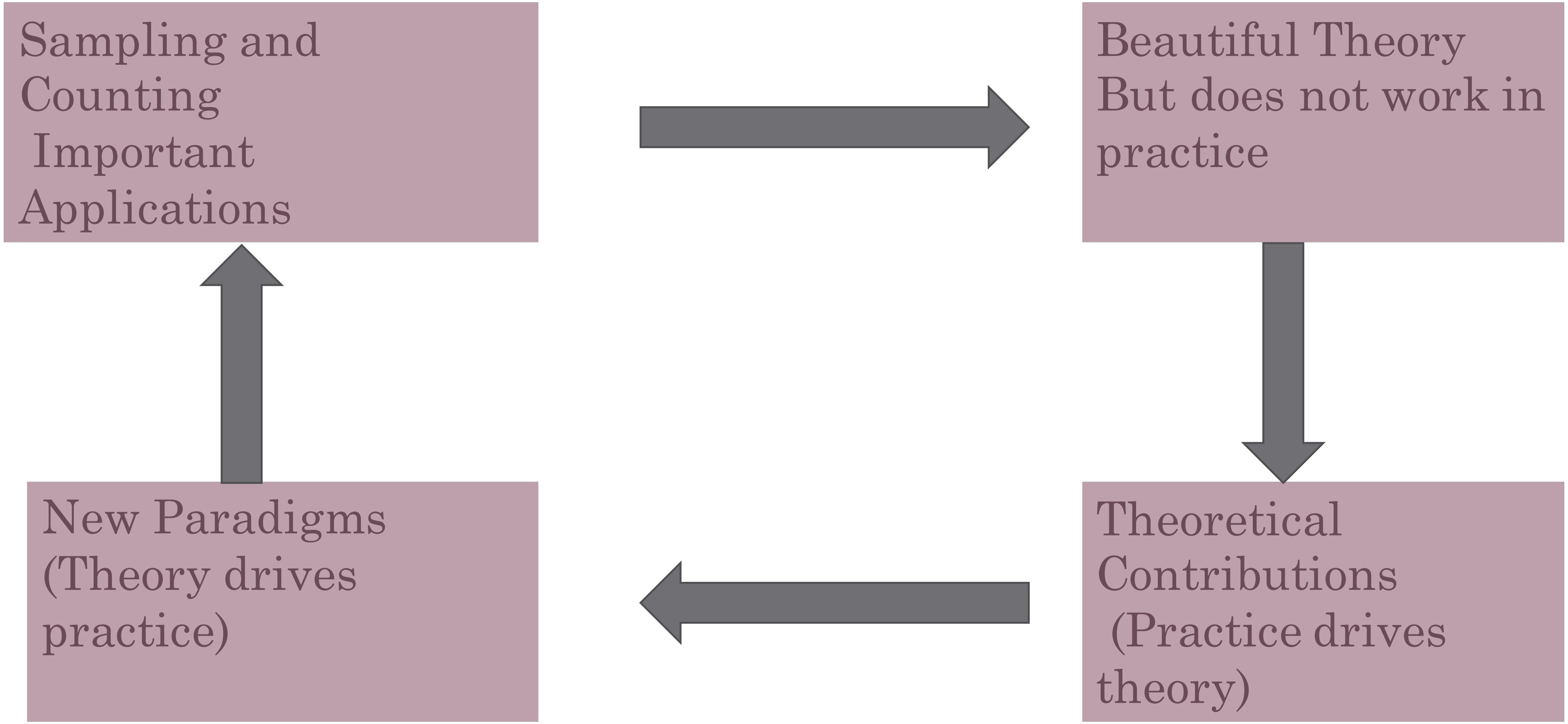


- Preprocessing needs to be done only once
- No communication required between different copies of the test generator
- Fully distributed!

Closing in…

Generator	Relative runtime
State-of-the-art: XORSample’	50000
UniGen	5000
UniGen1	470
UniGen2	20
Multi-core UniGen2	10 (two cores)
Ideal Uniform Generator*	10
SAT Solver	1

So what happened….



Future Directions

Extension to More Expressive domains

- Efficient hashing schemes
  - Extending bit-wise XOR to richer constraint domains provides guarantees but no advantage of SMT progress

- Solvers to handle F + Hash efficiently
  - CryptoMiniSAT has fueled progress for SAT domain
  - Similar solvers for other domains?

Handling Distributions

- Given: CNF formula F and Weight function W over assignments
- Weighted Counting: sum the weight of solutions
- Weighted Sampling: Sample according to weight of solution
- Wide range of applications in Machine Learning
- Extending universal hashing works only in theory so far

Thanks!

Questions?

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