Assignment #1

Details
- BNF, CFG, regular expression, regular grammar and warm-up exercises for Scheme
- weight 6%

Due Date
This assignment is due on Thursday Sept. 24, 2009, 6:10pm sharp.

Silent Policy
A silent policy will take effect 24 hours before this assignment is due. This means that no question about this assignment will be answered, whether it is asked on the newsgroup, by email, or in person.

Handing in this Assignment
You should hand in this assignment on paper in the tutorial room to your TA.

Please include this cover sheet

Last Name:___________________  First Name:____________________
Student #:___________________  CDF Login:____________________
Email:_______________________  Date & Time:___________________
Grace Days used to date:_______
Grace days used for this assignment: ________

I understand that collaboration is not allowed. All answers are my own, written in isolation, without help from others. This submission is in accordance with the University of Toronto Code of Behavior on Academic Matters (http://www.artsandscience.utoronto.ca/ofr/calendar/rules.htm#behaviour)

Signature: _____________________
Assignment # 1

[1 mark] Fill out the cover sheet correctly.

[28 marks, 4 marks each]
1. For each of the following languages, (1) provide a context-free grammar in BNF that generates all strings in the language and no other strings or say it cannot be done, and (2) provide a regular expression that accepts all strings in the language or say it cannot be done. If you claim that a context-free grammar/regular expression cannot be provided, you do not have to explain why.

a) The set of all strings of 0's and 1's not containing 111 as a substring

b) The set of all strings of 0's and 1's, where every pair of adjacent 0's appear before any pair of adjacent 1's

c) The set of all strings of 0's and 1's with equal number of 0's and 1's, such that no prefix has two more 0's than 1's, nor two more 1's than 0's

d) The set of all strings of 0's and 1's whose number of 0's is divisible by five.

e) The set of all strings over alphabet \{a,b,c\} containing at least one a and at least one c.

f) The set of all strings over alphabet \{a,b,c\} of the form \(a^n b^m c^{3n}\) \((m>0, n\geq 0)\).

g) The set of all strings over alphabet \{a,b,c\} of the form \(a^n b^m c^k\) \((m\neq n \text{ or } m\neq k)\).

[8 marks]
2. Consider the following CFG \(G\) defined by productions:

\[
<S> ::= a <S> b <S> \mid b <S> a <S> \mid \varepsilon
\]

a) What are the terminals in this grammar? What is the non-terminal in this grammar? What is the start symbol in this grammar? [1 mark]

b) Give three strings generated by this grammar. [1 mark]

c) Give three strings over \{a,b\} that are not generated by this grammar. [1 mark]

d) Show a right-most derivation for string baabbaaba [2 mark]

e) Show a parse tree for baabbaaba [2 mark]

f) What the language \(L(G)\), use one English sentence to describe it. [1 mark]
Consider the following grammar G:

**terminals:** \{ a, b, c, d \}

**non-terminals:** \{ <S>, <A>, <B>, <C>, <D> \}

**start symbol:** <S>

**productions:**

\[
\begin{align*}
<S> & ::= <A><B> | <C> \\
<A> & ::= a<A>b | ab \\
<B> & ::= c<B>d | cd \\
<C> & ::= a<C>d | a<D>d \\
<D> & ::= b<D>c | bc
\end{align*}
\]

Is the grammar ambiguous? If you think the grammar is ambiguous, prove it. If you think it is not, explain why (no formal proof required).

[15 marks]

We now define a special logic (say, K Logic) inductively as follows:

a) In this logic, the basic components are primitive concepts and primitive roles.

b) Each primitive concept \( C \) can be any English word starting with an upper case character.

c) Each primitive role \( R \) can be any English word starting with a lower case character.

d) Any primitive concept is a K Logic formula.

e) If \( A \) and \( B \) are K Logic formulas, then \( \neg A, A \& B, A | B, \) and \( A \rightarrow B \) are K Logic formulas.

f) If \( A \) is a K Logic formula and \( R \) is a primitive role, then \( \exists R.A \) and \( \forall R.A \) are K Logic formulas.

For example, suppose that we have formulas \( P, Q, \) and \( T \) and primitive role \( r \) and \( q \). Then

\[
(\neg \exists r. P) \& (\forall q. \neg Q) \rightarrow R
\]

is a K Logic formula.

Your task is to give an unambiguous context-free grammar using BNF that generates the language of K Logic Formulas.

In addition, the following properties should hold:

a) The precedence order is as follows (from highest to lowest priority):

1. \( \neg \)
2. \( \& \)
3. \( | \)
4. \( \rightarrow \)
5. \( \exists , \forall \)

and parentheses override the precedence order.
b) & and | are left-associative, \( \rightarrow \) is right-associative.

[10 marks]
5. The is a question to help you review some knowledge about recursive functions, so that you will get a little bit prepared for learning Scheme.

[2 marks]
(a) Find out the relationship among the following sequence of numbers, and give a recursive definition for \( f(n) \) (\( n \geq 0 \)):

\[
\begin{align*}
f(0) &= 1, \ f(1) = 1, \ f(2) = 2, \ f(3) = 1, \ f(4) = 3, \ f(5) = 2, \ f(6) = 5, \ f(7) = 6, \\
f(8) &= 11, \ f(9) = 30, \ldots
\end{align*}
\]

[b] [8 marks]
(b) Complete the following class so that \( m_1 \) and \( m_2 \) each return \( f(n) \).

```java
class compSeq{
    public static int m1(int n){
        // Implement by using loop statements without calling any other methods.
    }

    public static int m2(int n){
        // Recursion implement without using loop statements
    }
}
```