1. Let $f$ be a boolean function on $X \times Y$. prove that if all of the rows of $M_f$ are distinct, then $D(f) \geq \log \log |X|$. Prove that $D(f) \leq \text{rank}(f) + 1$.

2. $MED(x, y)$ is defined to be the median of the multiset $x \cup y$. (Here we are viewing $x$ and $y$ as $n$-bit binary strings each representing subsets of $[n]$.) Using binary search, one can show that $D(MED) = O(\log^2 n)$. Give an $O(\log n)$-bit protocol for MED.

3. $GT(x, y)$, the greater-than function, is 1 if and only if $x > y$ (viewing $x$ and $y$ as numbers expressed in binary, each as $n$-bit numbers). What is the communication complexity of the greater-than function?

   (a) Prove a lower bound of $n$ on the deterministic communication complexity of $GT$.

   (b) Given an upper bound of $O(\log^2 n)$ on the randomized complexity.

4. For $x, y \in \{0, 1\}^n$, let $d(x, y)$ denote the Hamming distance between $x$ and $y$, that is, the number of indices $i$ such that $x_i \neq y_i$. Let $R$ be a relation consisting of all triples $(x, y, m)$ such that $|m - d(x, y)| \leq n/3$. That is, computing $R$ is the problem of approximating the Hamming distance between $x$ and $y$. Prove that $D(R) = \Omega(n)$. (Observe that computing the Hamming distance exactly is as hard as computing the equality function.)