Definition 1

The *Cartesian product* (or *cross product*) of \( A \) and \( B \), denoted by \( A \times B \), is the set

\[
A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}
\]

1. the elements \((a, b)\) of \( A \times B \) are *ordered pairs*
2. for pairs \((a, b), (c, d)\) we have

\[
(a, b) = (c, d) \iff a = c \text{ and } b = d
\]

Definition 2

The *n-fold product* of sets \( A_1, A_2, \ldots, A_n \) is the set of *n-tuples*

\[
A_1 \times A_2 \times \ldots \times A_n = \{(a_1, a_2, \ldots, a_n) \mid a_i \in A_i \text{ for all } 1 \leq i \leq n\}
\]
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The *$n$-fold product* of sets $A_1, A_2, \ldots, A_n$ is the set of *$n$-tuples*

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Definition 1

\[ A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\} \]

\[ A = \{2, 3, 4\} \]
\[ B = \{4, 5\} \]

a) \[ A \times B = \{(2, 4), (2, 5), (3, 4), (3, 5), (4, 4), (4, 5)\} \]
b) \[ B \times A = \{(4, 2), (4, 3), (4, 4), (5, 2), (5, 3), (5, 4)\} \]
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\[ A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\} \]

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\( B = \{4, 5\} \)

a) \( B^2 = B \times B = \{(4, 4), (4, 5), (5, 4), (5, 5)\} \)
b) \( B^3 = B \times B \times B = \{(4, 4, 4), (4, 4, 5), (4, 5, 4), (4, 5, 5), (5, 4, 4), (5, 4, 5), (5, 5, 4), (5, 5, 5)\} \)
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\{\triangle, \square\} \times \{x, y\} \times \{\heartsuit, \spadesuit, \clubsuit\}

Tree Diagram

\begin{itemize}
\item \((\triangle, x)\rightarrow (\triangle, x, \heartsuit)\)
\item \((\triangle, x)\rightarrow (\triangle, x, \spadesuit)\)
\item \((\triangle, x)\rightarrow (\triangle, x, \clubsuit)\)
\item \((\triangle, y)\rightarrow (\triangle, y, \heartsuit)\)
\item \((\triangle, y)\rightarrow (\triangle, y, \spadesuit)\)
\item \((\triangle, y)\rightarrow (\triangle, y, \clubsuit)\)
\item \((\square, x)\rightarrow (\square, x, \heartsuit)\)
\item \((\square, x)\rightarrow (\square, x, \spadesuit)\)
\item \((\square, x)\rightarrow (\square, x, \clubsuit)\)
\item \((\square, y)\rightarrow (\square, y, \heartsuit)\)
\item \((\square, y)\rightarrow (\square, y, \spadesuit)\)
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\end{itemize}
**Definition 3**

A *(binary) relation* from $A$ to $B$ is a subset of $A \times B$. A *(binary) relation* on $A$ is a subset of $A \times A$.

$A = \{2, 3, 4\}$ and $B = \{4, 5\}$

a) $R_1 = \{(2, 4), (3, 5)\}$

b) $R_2 = \{(2, 4), (3, 4), (4, 4)\}$

c) $R_3 = \{(2, 4), (2, 5), (4, 4), (4, 5)\}$

d) $R_4 = \emptyset$
Definition 3

A (binary) relation from $A$ to $B$ is a subset of $A \times B$.
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c) $R_3 = \{(2, 4), (2, 5), (4, 4), (4, 5)\}$

d) $R_4 = \emptyset$
Relation $\mathcal{R} = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid 1 \leq x \leq y \leq 4\} = 
= \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)\}$
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Notation

$(x, y) \in \mathcal{R} \iff x \mathcal{R} y$

(think of $\mathcal{R}$ as $\leq$)
\{\text{Justin, Joey, Kevin, Nick}\} \times \{\text{Britney, Christina, Jessica, Kelly, Sarah}\}
Who dated whom? \{ (Ju, Br), (Ju, Je), (Jo, Ke), (Jo, Sa), (Ke, Br), (Ke, Ch), (Ni, Ch), (Ni, Ke), (Ni, Sa) \}
Who is dating whom? \( \{(Ju, Je), (Ke, Br), (Ni, Ch), (Ni, Ke)\} \)
Theorem 4

For any set $A$, we have $A \times \emptyset = \emptyset$ (and $\emptyset \times A = \emptyset$)

Proof. If $(a, b) \in A \times \emptyset$, then $a \in A$ and $b \in \emptyset$, impossible. \qed

Theorem 5

For any sets $A, B, C$

a) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

b) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

c) $(A \cap B) \times C = (A \times C) \cap (B \times C)$

d) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

Proof. a) $(a, b) \in A \times (B \cap C) \iff a \in A$ and $b \in B \cap C \iff a \in A$ and $b \in B$ and $b \in C \iff (a, b) \in A \times B$ and $(a, b) \in A \times C \iff (a, b) \in (A \times B) \cap (A \times C)$ \qed
**Theorem 4**

For any set $A$, we have $A \times \emptyset = \emptyset$ (and $\emptyset \times A = \emptyset$)

**Proof.** If $(a, b) \in A \times \emptyset$, then $a \in A$ and $b \in \emptyset$, impossible. □

**Theorem 5**

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d) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

**Proof.** a) $(a, b) \in A \times (B \cap C) \iff a \in A$ and $b \in B \cap C \iff a \in A$ and $b \in B$ and $b \in C \iff (a, b) \in A \times B$ and $(a, b) \in A \times C \iff (a, b) \in (A \times B) \cap (A \times C)$ □
Observation 6

For any two sets $A, B$, the number of elements in $A \times B$ is

$$|A \times B| = |A| \cdot |B|$$

Hence there are exactly $|\mathcal{P}(A \times B)| = 2^{|A \times B|} = 2^{|A| \cdot |B|}$ different relations from $A$ to $B$. 
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Exercises:

5.1.7 - If \( A = \{1, 2, 3, 4, 5\} \) and \( B = \{w, x, y, z\} \), how many elements are there in \( \mathcal{P}(A \times B) \).

Answer: \( 2^{20} = 1,048,576 \)

5.1.3 - For \( A = \{1, 2, 3\} \) and \( B = \{2, 4, 5\} \)

a) \( |A \times B| = ? \)

Answer: \( 9 \)

b) \# of relations from \( A \) to \( B \) ?

Answer: \( 2^9 = 512 \)

c) \# of relations on \( A \) ?

Answer: \( 2^9 = 512 \)

d) \# of relations from \( A \) to \( B \)
that contain \( (1, 2) \) and \( (1, 5) \) ?

Answer: \( 2^7 = 128 \)

e) \# of relations from \( A \) to \( B \)
that contain exactly five ordered pairs ?

Answer: \( \binom{9}{5} = 126 \)

f) \# of relations on \( A \) that
contain at least seven elements ?

Answer: \( \binom{9}{7} + \binom{9}{8} + \binom{9}{9} = 121 \)
Exercises:

5.1.7 - If $A = \{1, 2, 3, 4, 5\}$ and $B = \{w, x, y, z\}$, how many elements are there in $\mathcal{P}(A \times B)$. Answer: $2^{20} = 1,048,576$

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d) # of relations from $A$ to $B$
that contain $(1, 2)$ and $(1, 5)$ ? Answer: $2^7 = 128$

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that contain exactly five ordered pairs ? Answer: $\binom{9}{5} = 126$

f) # of relations on $A$ that contain at least seven elements ? Answer: $\binom{9}{7} + \binom{9}{8} + \binom{9}{9} = 121$
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5.1.7 - If \( A = \{1, 2, 3, 4, 5\} \) and \( B = \{w, x, y, z\} \), how many elements are there in \( \mathcal{P}(A \times B) \).  
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b) \# of relations from \( A \) to \( B \)?  
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Answer: \( 2^9 = 512 \)

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Answer: \( \binom{9}{5} = 126 \)

f) \# of relations on \( A \) that contain at least seven elements?  
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Exercises:

5.1.7 - If \( A = \{1, 2, 3, 4, 5\} \) and \( B = \{w, x, y, z\} \), how many elements are there in \( P(A \times B) \).

Answer: \( 2^{20} = 1,048,576 \)

5.1.3 - For \( A = \{1, 2, 3\} \) and \( B = \{2, 4, 5\} \)

a) \( |A \times B| =? \)  
   Answer: 9

b) \# of relations from A to B ?  
   Answer: \( 2^9 = 512 \)

c) \# of relations on A ?  
   Answer: \( 2^9 = 512 \)

d) \# of relations from A to B  
   that contain (1, 2) and (1, 5) ?  
   Answer: \( 2^7 = 128 \)

e) \# of relations from A to B  
   that contain exactly five ordered pairs ?  
   Answer: \( \binom{9}{5} = 126 \)

f) \# of relations on A that  
   contain at least seven elements ?  
   Answer: \( \binom{9}{7} + \binom{9}{8} + \binom{9}{9} = 121 \)
Exercises:

5.1.7 - If $A = \{1, 2, 3, 4, 5\}$ and $B = \{w, x, y, z\}$, how many elements are there in $\mathcal{P}(A \times B)$. 
Answer: $2^{20} = 1,048,576$

5.1.3 - For $A = \{1, 2, 3\}$ and $B = \{2, 4, 5\}$

a) $|A \times B| = ?$
Answer: $9$

b) # of relations from $A$ to $B$ ?
Answer: $2^9 = 512$

c) # of relations on $A$ ?
Answer: $2^9 = 512$

d) # of relations from $A$ to $B$
that contain $(1, 2)$ and $(1, 5)$ ?
Answer: $2^7 = 128$

e) # of relations from $A$ to $B$
that contain exactly five ordered pairs ?
Answer: $\binom{9}{5} = 126$

f) # of relations on $A$ that
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Answer: $\binom{9}{7} + \binom{9}{8} + \binom{9}{9} = 121$
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that contain exactly five ordered pairs ?

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Answer: \( \binom{9}{7} + \binom{9}{8} + \binom{9}{9} = 121 \)
Exercises:

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5.1.3 - For $A = \{1, 2, 3\}$ and $B = \{2, 4, 5\}$

a) $|A \times B| = ?$ Answer: 9

b) # of relations from $A$ to $B$? Answer: $2^9 = 512$

c) # of relations on $A$? Answer: $2^9 = 512$

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that contain (1, 2) and (1, 5)? Answer: $2^7 = 128$

e) # of relations from $A$ to $B$
that contain exactly five ordered pairs? Answer: $\binom{9}{5} = 126$

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Exercises:

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5.1.3 - For $A = \{1, 2, 3\}$ and $B = \{2, 4, 5\}$

a) $|A \times B| =$? Answer: 9

b) # of relations from $A$ to $B$ ? Answer: $2^9 = 512$

c) # of relations on $A$ ? Answer: $2^9 = 512$

d) # of relations from $A$ to $B$ that contain (1, 2) and (1, 5) ? Answer: $2^7 = 128$

e) # of relations from $A$ to $B$ that contain exactly five ordered pairs ? Answer: $\binom{9}{5} = 126$

f) # of relations on $A$ that contain at least seven elements ? Answer: $\binom{9}{7} + \binom{9}{6} + \binom{9}{9} = 121$
Exercises:

5.1.7 - If $A = \{1, 2, 3, 4, 5\}$ and $B = \{w, x, y, z\}$, how many elements are there in $\mathcal{P}(A \times B)$.  
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d) # of relations from $A$ to $B$ that contain $(1, 2)$ and $(1, 5)$ ?  
Answer: $2^7 = 128$

e) # of relations from $A$ to $B$ that contain exactly five ordered pairs ?  
Answer: $\binom{9}{5} = 126$

f) # of relations on $A$ that contain at least seven elements ?  
Answer: $\binom{9}{7} + \binom{9}{8} + \binom{9}{9} = 121$