LOGIC PROGRAMMING
AND PROLOG

Reading:
• Sebesta, chapter 16

References:
• Clocksin and Mellish, Ch. 1-4, 6, 8
• Online Resources (tutorials, SWI page, etc.)

Modified by Sheila McIlraith 2004.

Logic Programming and Prolog

Logic programming languages are not procedural or functional.

• Specify relations between objects
  - larger(3,2)
  - father(tom,jane)

• Separate logic from control:
  - Programmer declares what facts and relations are true.
  - System determines how to use facts to solve problems.
  - System instantiates variables in order to make relations true!

• Computation engine: theorem-proving and recursion (Unification, Resolution, Backward Chaining, Backtracking)
  - Higher-level than imperative languages

Jumping Right In

Suppose we state these facts:

\[
\begin{align*}
\text{male(albert).} & \quad \text{parent(albert,edward).} \\
\text{female(alice).} & \quad \text{parent(victoria,edward).} \\
\text{male(edward).} & \quad \text{parent(albert,alice).} \\
\text{female(victoria).} & \quad \text{parent(victoria,alice).}
\end{align*}
\]

We can then make queries:

?- male(albert).
Yes

?- male(victoria).
No

?- female(Person).
Person = alice;
Person = victoria;
No

?- parent(Person, edward).
Person = albert;
Person = victoria;
No

?- parent(Person, edward), female(Person).
Person = victoria;
No

We can also state rules, such as this one:

\[
\text{sibling(X, Y) :- parent(P, X), parent(P, Y).}
\]

Then the queries become more interesting:

?- sibling(albert, victoria).
No

?- sibling(edward, Sib).
Sib = edward;
Sib = alice;
Sib = edward;
Sib = alice;
No
Prolog vs Scheme

In Scheme, we program with functions ("procedures").

- A function’s arguments are different from the function’s value.
- Give a single Scheme function, we can only ask one kind of question:

  Here are the argument values; tell me what is the function’s value.

In Prolog, we program with relations.

- There is no bias; all arguments are the same.
- Given a single Prolog predicate, we can ask many kinds of question:

  Here are some of the argument values; tell me what the others have to be in order to make a true statement.

Logic Programming

- A program consists of facts and rules.
- Running a program means asking queries.
- The language tries to find one way (or more) to prove that the query is true.
- This may have the side effect of freezing variable values.
- The language determines how to do all of this, not the program.
- How does the language do it? Using unification, resolution, and backtracking.

The swi Interface on cdf

cdf% ls
family.pl

cdf% svi
Welcome to SWI-Prolog (Multi-threaded, Version 5.2.11)
Copyright (c) 1990-2003 University of Amsterdam.
SWI-Prolog comes with ABSOLUTELY NO WARRANTY. This is free sof
and you are welcome to redistribute it under certain condition
Please visit http://www.swi-prolog.org for details.

For help, use ?- help(Topic), or ?- apropos(Word).

?- ['family'].  "--------- load file family.pl
% family compiled 0.00 sec, 5,264 bytes
Yes

?- parent(Person, edward).
Person = albert;  "" to get more
Person = victoria;
No

?- parent(Person, edward).
Person = albert  ""a", CR, space to break
Yes

?- trace.  Yes
[trace]
[trace] ?- parent(Person, edward).
[trace] ?- parent(Person, edward).
Call: (7) parent(_G283, edward) ? creep <- CR to continue
Exit: (7) parent(albert, edward) ? creep

Person = albert ;
Redo: (7) parent(_G283, edward) ? creep
Exit: (7) parent(victoria, edward) ? creep

Person = victoria
Yes
[debug] ?-

(0) Call: parent(_G7,edward) ?
(0) Exit: parent(albert,edward) ?
Person = albert;
(0) Redo: parent(albert,edward) ?
(0) Exit: parent(victoria,edward) ?
Person = victoria
Yes
[trace]
?- notrace.
   Yes

?- parent(Person, edward).
   Person = albert;
   Person = victoria;
   No

?- halt.
cdf%

cdr% pl
Welcome to SWI-Prolog (Multi-threaded, Version 5.2.11) ...

For help, use ?- help(Topic). or ?- apropos(Word).

?- [family].
   [family loaded]
   Yes

?- parent(Person, edward).
   Person = albert;
   Person = victoria;
   No

--- edit family.pl and remove parent(albert,edward). ---

?- ['family'].

% family compiled 0.00 sec, 5,200 bytes
   Yes

?- parent(Person, edward).
   Person = victoria;
   No

?- halt.
cdf%

Some Prolog Syntax

Lexical Rules:

- Variables are capitalized.
- Constants begin with a lower case letter.
- Predicate names begin with a lower case letter.

Simplified Grammar:

<clause> ::= <pred> |
          <pred> ::= <pred> , <pred> .

<term> ::= <name> '(' ' <term> ( , <term> ) ')' |
          <int> | <atom> | <var>

Note: No blank between predicate name and opening bracket.

Prolog Queries

A query is a proposed fact that is to be proven.

- If the query has no variables, returns yes/no.
- If the query has variables, returns appropriate values of variables (called a substitution).
Horn Clauses (Rules)

A Horn Clause is: \( c \leftarrow h_1 \land h_2 \land h_3 \land \ldots \land h_n \)

- Antecedents: conjunction of zero or more conditions which are atomic formulae in predicate logic
- Consequent: an atomic formula in predicate logic

Meaning of a Horn clause:

- “The consequent is true if the antecedents are all true”
- \( c \) is true if \( h_1, h_2, h_3, \ldots, h_n \) are all true

Prolog Horn Clause Examples

A Horn clause with no tail:

```
male(albert).
```

I.e., a fact: albert is a male dependent on no other conditions

A Horn clause with a tail:

```
father(albert, edward):-
male(albert), parent(albert, edward).
```

I.e., a rule: albert is the father of edward if albert is male and albert is a parent of edward’s.

Horn Clause Terminology

- Horn Clause = Clause
- Consequent = Goal = Head
- Antecedents = Subgoals = Tail
- Horn Clause with No Tail = Fact
- Horn Clause with Tail = Rule

In Prolog, a Horn clause

\[ c \leftarrow h_1 \land \ldots \land h_n \]

is written

\[ c : - h_1, \ldots, h_n. \]

Syntax elements: ‘:’ ‘-’ ‘,’ ‘!’

Meaning of Prolog Rules Without Variables

A prolog rule must have this form:

\[ c : - a_1, a_2, a_3, \ldots, a_n. \]

which means in logic:

\[ a_1 \land a_2 \land a_3 \land \ldots \land a_n \rightarrow c. \]

Restrictions

- There can be zero or more antecedents, but they are conjoined; we cannot disjoin them.
- There cannot be more than 1 consequent.
Bending the Restrictions

Getting disjoined antecedents

Example: \( a_1 \lor a_2 \lor a_3 \rightarrow c. \)

Solution:

Getting more than 1 consequent, conjoined

Example: \( a_1 \land a_2 \land a_3 \rightarrow c_1 \land c_2. \)

Solution:

Getter more than 1 consequent, disjoined

Example: \( a_1 \land a_2 \land a_3 \rightarrow c_1 \lor c_2. \)

Solution:

Why Can’t We Disjoin Consequents?

Why did the designers of Prolog disallow this?

Logic Review

Horn Clauses with Variables

Variables may appear in the antecedents and consequent of a Horn clause:

- \( c(X_1, \ldots, X_n) \leftarrow h(X_1, \ldots, X_n). \)
  
  “For all values of \( X_1, \ldots, X_n \), the formula \( c(X_1, \ldots, X_n) \) is true if the formula \( h(X_1, \ldots, X_n) \) is true”

- \( c(X_1, \ldots, X_n) \leftarrow h(X_1, \ldots, X_n, Y_1, \ldots, Y_k). \)
  
  “For all values of \( X_1, \ldots, X_n \), the formula \( c(X_1, \ldots, X_n) \) is true if there exist values of \( Y_1, \ldots, Y_k \) such that the formula \( h(X_1, \ldots, X_n, Y_1, \ldots, Y_k) \) is true”
Meaning of Prolog Rules
With Variables

Example:

\[ \text{isaMother}(X) \leftarrow \text{female}(X), \text{parent}(X, Y). \]

Logic:

\[ \text{parent}(X, Y) \land \text{female}(X) \supset \text{isaMother}(X). \]

But this is meaningless without quantifiers for the variables.

The rule

A Prolog rule of this form \((n \geq 0, m \leq n, k \geq 0)\):

\[ c(X_1, \cdots X_n) : \leftarrow a(X_1, \cdots X_m, Y_1, \cdots Y_k), \]

means:

\[ \forall X_1, \cdots X_n \]
\[ \exists Y_1, \cdots Y_k \ [a(X_1, \cdots X_m, Y_1, \cdots Y_k) \supset c(X_1, \cdots X_n)] \]

Rule Ordering and Unification

1. rule ordering used in search
2. unification requires two instances of the same variable in the same rule to get the same value
3. unification does not require differently named variables to get different values: hence, sibling(Edward, Edward).
4. all rules searched if requested by ';'
Trace it by hand

Prolog Search Trees

- Each node is an ordered list of goals.
- Each edge is labelled with the variable bindings that occurred due to applying a rule. (The binding are in effect throughout the subtree.)
- Each leaf represents either success or failure.

Trace it in Prolog

[trace] ?- grandfather(X,Y).
Call: (7) grandfather(_G283, _G284) ? creep
Call: (8) male(_G283) ? creep
Exit: (8) male(tom) ? creep
Call: (8) parent(tom, _L205) ? creep
Exit: (8) parent(tom, william) ? creep
Call: (8) parent(william, _G284) ? creep
Exit: (8) parent(william, _G284) ? creep
Exit: (8) male(_G283) ? creep
Exit: (8) male(peter) ? creep
Call: (8) parent(peter, _L205) ? creep
Fail: (8) parent(peter, _L205) ? creep
Redo: (8) male(_G283) ? creep
Exit: (8) male(doug) ? creep
Call: (8) parent(doug, _L205) ? creep
Exit: (8) parent(doug, susan) ? creep
Call: (8) parent(susan, _G284) ? creep
Exit: (8) parent(susan, _G284) ? creep
Redo: (8) parent(doug, _L205) ? creep
Exit: (8) parent(doug, david) ? creep
Call: (8) parent(david, _G284) ? creep
Fail: (8) parent(david, _G284) ? creep
Redo: (8) parent(doug, _L205) ? creep
Exit: (8) parent(doug, tom) ? creep
Call: (8) parent(tom, _G284) ? creep
Exit: (8) parent(tom, william) ? creep
Exit: (7) grandfather(doug, william) ? creep
X = doug
Y = william
Yes

Example 2

Database:

1) male(albert).
2) female(alice).
3) male(edward).
4) female(victoria).
5) parent(albert,edward).
6) parent(victoria,edward).
7) parent(albert,alice).
8) parent(victoria,alice).
9) sibling(X, Y) :- parent(P, X), parent(P, Y).

Query:

?- sibling(alice,Asib).
Asib = edward ;
Asib = alice ;
Asib = edward ;
Asib = alice ;
No
?- sibling(Asib, alice).
Asib = edward ;
Asib = edward ;
Asib = alice ;
Asib = alice ;
No
Trace it by hand

Trace it in Prolog

[trace] \( \leftarrow \) sibling(alice, Asib).
Call: (7) sibling(alice, _G284) \( \leftarrow \) creep
Call: (8) parent(_,L206, alice) \( \leftarrow \) creep
Exit: (8) parent(albert, alice) \( \leftarrow \) creep
Call: (8) parent(albert, _G284) \( \leftarrow \) creep
Exit: (8) parent(albert, edward) \( \leftarrow \) creep
Exit: (7) sibling(alice, edward) \( \leftarrow \) creep

Asib = edward ;
Redo: (8) parent(albert, _G284) \( \leftarrow \) creep
Exit: (8) parent(albert, alice) \( \leftarrow \) creep
Exit: (7) sibling(alice, alice) \( \leftarrow \) creep

Asib = alice ;
Redo: (8) parent(_,L206, alice) \( \leftarrow \) creep
Exit: (8) parent(victoria, alice) \( \leftarrow \) creep
Call: (8) parent(victoria, _G284) \( \leftarrow \) creep
Exit: (8) parent(victoria, edward) \( \leftarrow \) creep
Exit: (7) sibling(alice, edward) \( \leftarrow \) creep

Asib = edward ;
Redo: (8) parent(victoria, _G284) \( \leftarrow \) creep
Exit: (8) parent(victoria, alice) \( \leftarrow \) creep
Exit: (7) sibling(alice, alice) \( \leftarrow \) creep

Asib = alice ;
No

The Anonymous Variable

If a rule has a variable that appears only once, that variable is called a “singleton variable”.

Its value doesn’t matter — it doesn’t have to match anything elsewhere in the rule.

\texttt{isMother(X) :- female(X), parent(X, Y).}

Such a variable consumes resources at run time.

Procedural Semantics of Prolog

Notice the recursion in this algorithm: “find” calls “find”. This reasoning is recursively applied until we reach rules that are facts.

This process is called \textbf{Backward Chaining}.

We can replace it with “\_”, the anonymous variable. It matches anything.

If we don’t, Prolog will warn us.
Logic Programming vs. Prolog

cousin(X, Y) :- parent(W, X), sister(W, Z),
    parent(Z, Y).

cousin(X, Y) :- parent(W, X), brother(W, Z),
    parent(Z, Y).

| ?- cousin(X, jane). % a query

Rule and Goal Ordering:
- There are two rules for cousin
- Which rule do we try first?
- Each rule for cousin has several subgoals
- Which subgoal do we try first?

Logic Programming: Nondeterministic
- Arbitrarily choose rule to expand first
- Arbitrarily choose subgoal to explore first
- Results don't depend on rule and subgoal ordering

Prolog: Deterministic
- Expand first rule first
- Explore first subgoal first
- Results may depend on rule and subgoal ordering