Functional Programming—
Illustrated in Scheme

References:
- Dybvig, (available online and in the library)
- Sebesta 6th ed., chapter 15,

Modified and updated © S. McIlraith 2004.
Additional slides use material taken from © G. Baumgartner 2001.

The Spirit of Lisp-like Languages

We shall first define a class of symbolic expressions in terms of ordered pairs and lists.
Then we shall define five elementary functions and predicates, and build from them by composition, conditional expressions and recursive definitions an extensive class of functions
of which we shall give a number of examples. We shall then show how these functions can themselves be expressed as symbolic expressions, and we shall give a universal function apply that allows us to compute from the expressions for a given function its value for given arguments. Finally, we shall define some functions with functions as arguments and give some useful examples.


Jumping right in

A Scheme procedure

(define increment
  (lambda (n)
    (+ n 1))
)

or

(define (increment n)
  (+ n 1))

A call to the procedure

(increment 21)

Pure Functional Languages

Fundamental concept: application of (mathematical) functions to values

1. Referential transparency: The value of a function application is independent of the context in which it occurs
   - value of \( f(a,b,c) \) depends only on the values of \( f, a, b \) and \( c \)
   - It does not depend on the global state of computation
     \( \Rightarrow \) all vars in function must be parameters
**Pure Functional Languages (cont.)**

2. The concept of assignment is **not** part of functional programming
   - no explicit assignment statements
   - variables bound to values only through
     the association of actual parameters to
     formal parameters in function calls
   - function calls have no side effects
   - thus no need to consider global state

3. Control flow is governed by function calls
   and conditional expressions
   ⇒ no iteration
   ⇒ recursion is widely used

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**A Functional Program**

A program includes:

1. A set of function definitions
2. An expression to be evaluated

E.g. in Scheme:

```scheme
1 ]=> (define (abs-val x)
    (if (>= x 0)
        x
        (- x)))

;Value: abs-val

1 ]=> (abs-val (- 3 5))

;Value: 2
```

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**Pure Functional Languages (cont.)**

4. All storage management is implicit
   - needs garbage collection

5. Functions are First Class Values
   - Can be returned as the value of an expression
   - Can be passed as an argument
   - Can be put in a data structure as a value
   - Unnamed functions exist as values

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**Jumping Back In**

**The MIT Scheme Interface**

werewolf 1\% scheme
Scheme Microcode Version ...

1 ]=> (+ 8 3 5 16 9)
;Value: 41

1 ]=> (define increment (lambda (n) (+ n 1)))
;Value: increment

1 ]=> (increment 21)
;Value: 22

1 ]=> (load "incr")
;Loading "incr.scm" -- done
;Value: increment-list

1 ]=> (increment-list (1 32 7))
;The object 1 is not applicable.
;To continue, call RESTART with an option number:
; (RESTART 2) => Specify a procedure to use in its place.
; (RESTART 1) => Return to read-eval-print level 1.

2 error> (restart 1)
;Abort!

1 ]=> (increment-list '1 32 7))
;Value: 1: (2 33 8)
\[ \text{(trace increment-list)} \]
\[ ; \text{Unspecified return value} \]
\[ 1 \] \( \Rightarrow \) \[ \text{(increment-list '(1 32 7))} \]

\[ \text{[Entering #[compound-procedure 2 increment-list]} \]
\[ \text{Args: (1 32 7)} \]
\[ \text{[Entering #[compound-procedure 2 increment-list]} \]
\[ \text{Args: (32 7)} \]
\[ \text{[Entering #[compound-procedure 2 increment-list]} \]
\[ \text{Args: (?)} \]
\[ \text{[Entering #[compound-procedure 2 increment-list]} \]
\[ \text{Args: ()} \]
\[ 0 \]
\[ \text{[}] \]
\[ \text{[8]} \]
\[ \text{[33 8]} \]
\[ \text{[2 33 8]} \]
\[ \text{[Value 3: (2 33 8)} \]
\[ \text{]} \]
\[ 1 \] \( \Rightarrow \) \[ \text{(exit)} \]

Kill Scheme (y or n)? Yes
Happy Happy Joy Joy.
werewolf 2%

\[ \text{Formal Roots: } \lambda\text{-Calculus} \]

- Defined by Alonzo Church, a logician, in 1930s as a computational theory of recursive functions
- \( \lambda \)-calculus is equivalent in computational power to Turing machines
- Recall: what’s a Turing machine? Turing machines are abstract machines that emphasize computation as a series of state transitions driven by symbols on an input tape (which leads naturally to an imperative style of programming based on assignment)
- How is \( \lambda \)-calculus different?
  - \( \lambda \)-calculus emphasizes typed expressions and functions (which naturally leads to a functional style of programming).
  - No state transitions.

\[ \text{Formal Syntax in BNF} \]
\[ \langle \lambda\text{-term} \rangle :: \langle \text{variable} \rangle \]
\[ | \lambda \langle \text{variable} \rangle \ . \langle \lambda\text{-term} \rangle \]
\[ | (\langle \lambda\text{-term} \rangle \langle \lambda\text{-term} \rangle) \]

\[ \langle \text{variable} \rangle :: x \ | \ y \ | \ z \ | \ ... \]

Or more compactly
\[ \text{E ::=} \ y \ | \ \lambda \text{.E} \ | \ (E1 \ E2) \]
\[ \text{V ::=} \ x \ | \ y \ | \ z \ | \ ... \]

Where \( \text{V} \) is an arbitrary variable and \( \text{E} \) is an arbitrary \( \lambda \)-expression. We call \( \text{\#} \) the head of the \( \lambda \)-expressions and \( \text{E} \) the body.

\[ \text{\[ \lambda\text{-Calculus (cont.)} \]} \]

\[ \lambda\text{-calculus is a formal system for defining recursive functions and their properties.} \]

- Expressions are called \( \lambda \)-expressions.
- Every \( \lambda \)-expression denotes a function.

- A \( \lambda \)-expression consists of 3 kinds of terms:
  - Variables: \( x, y, z \) etc
  - Abstractions: \( \lambda V.E \)
    - where \( V \) is some variable and \( E \) is another \( \lambda \)-term.
  - Applications: \( (E1 \ E2) \) where \( E1 \) and \( E2 \) are \( \lambda \)-terms. Applications are sometimes called combinations.
### λ-Calculus: Functional Forms

A higher-order function (functional form):
- Takes functions as parameters
- Yields a function as a result

E.g.: Given
  
  \[ f(x) = x + 2 \, \quad g(x) = 3 \times x \]

then,
  
  \[ h(x) = f(g(x)) \text{ and} \]
  \[ h(x) = (3 \times x) + 2 \]

\( h(x) \) is called a higher-order function.

#### Types of Functional Forms:

**Construction form:** E.g.,

\[ g(x) = x \times x, \quad h(x) = 2 \times x, \quad i(x) = x \div 2 \]

\[ [g, h, i](4) = (16, 8, 2) \]

**Apply-to-all form:** E.g.,

\[ h(x) = x \times x \]

\[ y(h, (2, 3, 4)) = (4, 9, 16) \]

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### λ-Calculus

Is it really Turing Complete?

Can we represent the class of Turing computable functions?

Yes, we can represent:
- Boolean and conditional functions
- Numerical and arithmetic functions
- Data structures: ordered pairs, lists, etc.
- Recursion

But, doing so in λ-calculus is tedious;

- Need syntactic sugar to simplify task,
- λ-calculus more suitable as an abstract model of a programming language rather than a practical programming language.

*Both Turing machines and λ-calculus are idealized, mathematical models of computation.*

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### Scheme: A Functional Programming Language

1958: Lisp
1975: Scheme (revised over the years)
1980: Common Lisp (“CL”)
1980s: Lisp Machines (e.g., Symbolics, TI Explorer, etc.)

Lisp, Scheme and CL contrasted on following pages.

Some features of Scheme:
- denotational semantics based on the λ-calculus.
  I.e., the meaning of programming constructs in the language is defined in terms of mathematical functions.
- lexical scoping
  I.e., all free variables in a λ-expression are assigned values at the time that the λs defined (i.e., evaluated and returned).
- arbitrary ctrl structures w/ continuations.
- functions as first-class values
- automatic garbage collection.

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### LISP

- Functional language developed by John McCarthy in 1958.
- Semantics based on λ-Calculus
- All functions operate on lists or atomic symbols: (called “S-expressions”)
- Only five basic functions: list functions cons, car, cdr, equal, atom and one conditional construct: cond
- Uses dynamic scoping
- Useful for list-processing applications
- Programs and data have the same syntactic form: S-expressions
- Used in Artificial Intelligence
**SCHEME**

- Developed in 1975 by G. Sussman and G. Steele
- A version of LISP
- Consistent syntax, small language
- Closer to initial semantics of LISP
- Provides basic list processing tools
- Allows functions to be first class objects
- Provides support for *lazy evaluation*
- lexical scoping of variables

**COMMON LISP (CL)**

- Implementations of LISP did not completely adhere to semantics
- Semantics redefined to match implementations
- COMMON LISP has become the standard
- Committee-designed language (1980s) to unify LISP variants
- Many defined functions
- Simple syntax, large language

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**Commonalities between LISP and SCHEME**

- Expressions are written in prefix, parenthesized form
  - (function arg₁ arg₂ ...argₙ)
  - (+ 4 5)
  - (+ (* 3 4 5) (- 5 3))

- In order to evaluate an expression:
  1. evaluate function to a function value
  2. evaluate each argᵢ in order to obtain its value
  3. apply the function value to these values

**S-expressions**

Common structure for both procedures and data. In Scheme, functions are called procedures.

When an expression is evaluated it creates a value or list of values that can be embedded into other expressions. Therefore programs can be written to manipulate other programs.

```
<expression> --> <variable>
| <literal>
| <procedure call>
| <lambda expression>
| <conditional>
| <assignment>
| <derived expression>
| <macro use>
| <macro block>
#t (true)
() (false)
(a b c)
(a b c) d
((a b c) (d e (f)))
(1 (b) 2)
(+ '1 2)
```

Lists have nested structure.
### Built-In Procedures

- `eq?`: identity on atoms
- `null?`: is list empty?
- `car`: selects first element of list
- `cdr`: selects rest of list
- `(cons element list)`: constructs lists by adding element to front of list
- `quote` or `'`: produces constants

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**More about lists**

**Proper lists:**
- `()`, `(a b (c d) e)`
- `(cons 'a '(b)) → (a b)`

**Dotted pairs (improper lists):**
- `(cons 'a 'b) → (a . b)`
- `(car '(a . b)) → a`
- `(cdr '(a . b)) → b`
- `(cons a '(b . c)) → (a b . c)`
- `(a b c) → (a . (b . (c . ())))`
Things you should know about cons, pairs and lists

The **pair or cons cell** is the most fundamental of Scheme's structured object types.

A **list** is a sequence of **pairs**; each pair's cdr is the next pair in the sequence.

The cdr of the last pair in a **proper list** is the empty list. Otherwise the sequence of pairs forms an **improper list**. I.e., an empty list is a proper list, and any pair whose cdr is a proper list is a proper list.

An improper list is printed in **dotted-pair notation** with a period (dot) preceding the final element of the list. A pair whose cdr is not a list is often called a **dotted pair**.

**cons vs. list**: The procedure **cons** actually builds pairs, and there is no reason that the cdr of a pair must be a list, as illustrated on the previous page.

The procedure **list** is similar to **cons**, except that it takes an arbitrary number of arguments and always builds a proper list.

E.g., (list 'a 'b 'c) → (a b c)

Other (Predicate) Procedures

Predicate procedures return #t or () (i.e., false).

- + - * / numeric operators, e.g.,
  (+ 5 3) = 8, (- 5 3) = 2
  (* 5 3) = 15, (/ 5 3) = 1.666666
- = < > <= >= number comparison ops
- Run-time type checking procedures:
  - All return Boolean values: #t and ()
    - (number? 5) is #t
    - (zero? 0) is #t
    - (symbol? 'sam) is #t
    - (list? '(a b)) is #t
    - (null? '()) is #t

Other Predicate Procedures

- (number? 'sam) evaluates to ()
- (null? '(a)) evaluates to ()
- (zero? (- 3 3)) evaluates to #t
- (zero? '(- 3 3)) → type error
- (list? (+ 3 4)) evaluates to ()
- (list? '(+ 3 4)) evaluates to #t

READ-EVAL-PRINT Loop

**READ**: Read input from user:
- a procedure application

**EVAL**: Evaluate input:
- (f arg₁ arg₂ ...argₙ)
  1. evaluate f to obtain a procedure
  2. evaluate each argᵢ to obtain a value
  3. apply procedure to argument values

**PRINT**: Print resulting value:
- the result of the procedure application
READ-EVAL-PRINT Loop Example

1 ]=> (cons 'a (cons 'b '(c d)))
;Value 1: (a b c d)

1. Read the procedure application
   (cons 'a (cons 'b '(c d)))

2. Evaluate cons to obtain a procedure

3. Evaluate 'a to obtain a itself

4. Evaluate (cons 'b '(c d)):
   (a) Evaluate cons to obtain a procedure
   (b) Evaluate 'b to obtain b itself
   (c) Evaluate '(c d) to obtain (c d) itself
   (d) Apply the cons procedure to b and (c d) to obtain (b c d)

5. Apply the cons procedure to a and (b c d) to obtain (a b c d)

6. Print the result of the application:
   (a b c d)

Quotes Inhibit Evaluation

;;Same as before:
1 ]=> (cons 'a (cons 'b '(c d)))
;Value 2: (a b c d)

;;Now quote the second argument:
1 ]=> (cons 'a (cons 'b '(c d)))
;Value 3: (a cons (quote b) (quote (c d)))

;;Instead, un-quote the first argument:
1 ]=> (cons a (cons 'b '(c d)))
;Unbound variable: a
;To continue, call RESTART...
2 error> ^C^C
1 ]=>

Quotes vs. Eval

;;Some things evaluate to themselves:
1 ]=> (list 1 42 #f ())
;Value 4: (1 2 #t () ())

;;They can also be quoted:
1 ]=> (list '1 '42 '#t '#f '())
;Value 5: (1 2 #t () ())

Eval Activates Evaluation

1 ]=> '(+ 1 2)
;Value 6: (+ 1 2)

;;Eval can be used to evaluate an expression
1 ]=> (eval '(+ 1 2))
;Value 7: 3

READ-EVAL-PRINT Loop

Can also be used to define procedures.

READ: Read input from user:
   a symbol definition

EVAL: Evaluate input:
   store function definition

PRINT: Print resulting value:
   the symbol defined

Example:
1 ]=> (define (square x) (* x x))
;Value: square
Procedure Definition

Two syntaxes for definition:

1. (define (<fcn-name> <fcn-params>)
   <expression>)
   (define (square x)
     (* x x))

2. (define <fcn-name> <fcn-value>)
   (define mean
     (lambda (x y) (/ (+ x y) 2)))

Lambda procedure syntax enables the creation of anonymous procedures. More on this later!

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Conditional Execution: if

(if <condition> <result1> <result2>)

1. Evaluate <condition>
2. If the result is a “true value” (i.e., anything but () or #f), then evaluate and return <result1>
3. Otherwise, evaluate and return <result2>

(define (abs-val x)
  (if (>= x 0) x (- x))

(define (rest-if-first e lst)
  (if (eq? e (car lst)) (cdr lst) '()))

---

Conditional Execution: cond

(cond (<condition1> <result1>)
  (<condition2> <result2>)
  ...
  (<conditionN> <resultN>)
  (else <else-result>);optional else
  )

1. Evaluate conditions in order until obtaining one that returns a true value
2. Evaluate and return the corresponding result
3. If none of the conditions returns a true value, evaluate and return <else-result>

(define (abs-val x)
  (cond ((>= x 0) x)
        (else (- x))
    )

(define (rest-if-first e lst)
  (cond ((null? lst) '())
        ((eq? e (car lst)) (cdr lst))
        (else '())
    )

)
Conditional vs. Boolean Expressions

Write a procedure that takes a parameter x and returns #t if x is an atom, and false otherwise. Using cond:

(define (atom? x)
  (cond ((symbol? x) '#t)
        ((number? x) '#t)
        ((char? x) '#t)
        ((string? x) '#t)
        ((null? x) '#t)
        (else 'false)))

Better atom? procedure

Any list is a pair (dotted pair with CAR and CDR), except the empty list (which is both list and atom).

(define (atom? x)
  (if (pair? x) () '#t))

(define (atom? x)
  (cond ((pair? x) (false))
        (else '#t)))

Recursion:
Five Steps to a Recursive Function

1. **Strategy:** How to reduce the problem?
2. **Header:**
   - What info needed as input and output?
   - Write the function header. Use a noun phrase for the function name.
3. **Spec:** Write a method specification in terms of the parameters and return value. Include preconditions.
4. **Base Cases:**
   - When is the answer so simple that we know it without recursing?
   - What is the answer in these base case(s)?
   - Write code for the base case(s).
5. **Recursive Cases:**
   - Describe the answer in the other case(s) in terms of the answer on smaller inputs.
   - Simplify if possible.
   - Write code for the recursive case(s).
Recursive Scheme Procedures: Sum-N

Parameter: integer \( n \geq 0 \).

Result: sum of integers from 0 to \( n \).

\[
\text{(define (sum-n n)}
\]

\[
\begin{align*}
& \quad \text{(cond (} \\
& \quad \quad \text{)} \\
& \quad \quad \text{(else } \\
& \quad \quad \quad \text{)} \\
& \quad \quad \text{)} \\
& \quad \text{)}
\end{align*}
\]

This is called "cdr-recursion."

Note: There is a built-in length procedure.

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Recursive Scheme Procedures: Length

\[
\text{(define (length x)}
\]

\[
\begin{align*}
\text{)}
\end{align*}
\]

Recursive Scheme Procedures: Abs-List

- \( \text{(abs-list '}(1 -2 -3 4 0)) \Rightarrow (1 2 3 4 0) \)
- \( \text{(abs-list '}()) \Rightarrow () \)

\[
\text{(define (abs-list lst)}
\]

\[
\begin{align*}
\text{)}
\end{align*}
\]

Length (cont.)

\[
1 \Rightarrow (\text{trace length)}
\]

;No value

\[
1 \Rightarrow (\text{length '}(a \ b \ c))
\]

[Entering \#[compound-procedure 5 length]
 Args: (a b c)]

[Entering \#[compound-procedure 5 length]
 Args: (b c)]

[Entering \#[compound-procedure 5 length]
 Args: (c)]

[Entering \#[compound-procedure 5 length]
 Args: ()]

[0
 <= \#[compound-procedure 5 length]
 Args: ()]]

[1
 <= \#[compound-procedure 5 length]
 Args: (c)]]

[2
 <= \#[compound-procedure 5 length]
 Args: (b c)]]

[3
 <= \#[compound-procedure 5 length]
 Args: (a b c)]

;Value: 3
Recursive Scheme Procedures: Append

(append '(1 2) '(3 4 5)) ⇒ (1 2 3 4 5)
(append '(1 2) '(3 (4) 5)) ⇒ (1 2 3 (4 5))
(append '() '(1 4 5)) ⇒ (1 4 5)
(append '(1 4 5) '()) ⇒ (1 4 5)
(append '() '()) ⇒ ()

(define (append x y))

)

Note: There is a built-in append procedure.