Due: By 12:00 noon on Thursday, December 7.

You must complete and sign an assignment cover page, and attach it (with a staple) to the front of your assignment. Assignments should be handed into the drop box in BA 2220.

1. Consider the following program, where A is an array of n integers:
   
m = A[0] + 1;
   for (i = 0; i < A.length; i++) {
     if (A[i] < m) {
       m = A[i];
       s = A[i];
       for (j = i+1; j < A.length; j++)
         s = s + A[j];
     }
   }

(a) Describe an array of length n that is a worst-case input for this algorithm (in terms of number of lines executed). Explain why your answer is a worst-case input.

(b) Let $T(n)$ be the worst-case number of lines executed by this program over all arrays of length n. Prove that $T(n) \in \Theta(n^2)$.

2. Consider the normalized floating point system $F$ with $\beta = 2$, $t = 6$, $e_{\text{max}} = 7$, $e_{\text{min}} = -8$ that uses round-to-nearest.

(a) How many real numbers are representable exactly in $F$? Justify your answer.

(b) Give the decimal (base 10) representation of the largest and smallest positive real numbers representable in $F$.

(c) Give an example of a decimal number that will cause overflow in $F$. Explain.

(d) Give an example of a decimal number that will cause underflow in $F$. Explain.

(e) Suppose a real number $x$ is represented in $F$ by $x' = 1.01101 \times 2^1$. What range of decimal values could $x$ have been? Justify your answer.

3. Computing the expression $x^2 - 4$ is susceptible to error for certain values of $x$.

(a) Explain what kind of error can occur here and for what values of $x$ it occurs. Illustrate your claim with an example.

(b) Reformulate the expression to avoid this error. Explain why this error is avoided.

4. Suppose we need to compute the expression $(x - 1)^3$ for $x = 0.85$ in the normalized floating point system with $\beta = 10$, $t = 3$, $e_{\text{max}} = 2$, $e_{\text{min}} = -3$ that uses round-to-nearest.

(a) Evaluate $x^3 + -3x^2 + 3x + -1$ by performing the addition operations in this system from left to right. (Remember order of operations says to do the multiplications first!) Show your work.

(b) Evaluate $x^3 + -3x^2 + 3x + -1$ by performing the addition operations in this system from right to left. Show your work.

(c) Which computation is more stable for this value of $x$? Why?