This assignment is due in your tutorial on 21 March 2001.

1. [2 marks]

Write a MatLab function \texttt{runge} that, given a vector argument \( x \), returns a vector \( y \) of the same size and shape as \( x \) satisfying

\[
y_i = \frac{1}{1 + 25 x_i^2}
\]

Your function should not contain any loops.

Hint: we’ve used Runge’s function before in an assignment.

2. [8 marks]

Copy the script \texttt{ShowPWL2a} from our course webpage

http://www.cs.toronto.edu/~krj/courses/282/

onto your computer. Also copy the functions that it uses from Van Loan’s webpage


onto your computer. To check that you’ve got all the functions you need, try running \texttt{ShowPWL2a}.

Modify \texttt{ShowPWL2a} so that it compares \texttt{pwLStatic} and \texttt{pwLAdapt} on the interval \([-1, 1]\) using your function \texttt{runge}, rather than \texttt{humps} on the interval \([0, 3]\). Also modify the \texttt{plot} calls so that each graph shows

(a) Runge’s function,

(b) the \textit{knots} for the piece linear approximation, and

(c) the piece linear approximation itself.

Note that you have to make quite a few more changes to \texttt{ShowPWL2a} than just changing \texttt{‘humps’} to \texttt{‘runge’} and the arguments of the \texttt{plot} function.

You don’t have to hand in anything for the next three paragraphs, but you should observe the points made there.

The approximation for the variably-spaced grid looks quite poor for \( \text{delta} = 1 \) because we’re allowing the approximation to have an error which is the same size as the function itself. Note that, as \( \text{delta} \) gets smaller, the approximation becomes quite good.

In Assignment 5, you used a higher-order polynomial to interpolate Runge’s function. Recall that, for an evenly spaced grid, the polynomial interpolant produced a very poor approximation to Runge’s function. In fact, as the degree of the polynomial increased, the approximation got worse.

Contrast this to the behaviour of the piecewise linear approximations in this assignment. You should observe that the approximation improves as more mesh points are used for both the evenly-spaced and the variably-spaced grids.