You will be responsible for Lecture slides 13 to 17 (i.e. chapter 13 and 14 of the book). Below are some sample questions for test 3.

1. Two astronomers in different parts of the world make measurements $M_1$ and $M_2$ of the number of stars $N$ in some small region of the sky, using their telescopes. Normally, there is a small probability $e$ or error of up to one star in each direction. Each telescope can also be badly out of focus (with probability $f$, much smaller than $e$). Let $F_1$ and $F_2$ be boolean variables with $F_i = \text{true}$ being that the $i$-th telescope is out of focus. If the telescope is out of focus then the scientist will undercount by 3 or more starts (or, if $N$ is 3 or less, fail to detect any stars at all). Consider the three networks:

(i) \[ \begin{array}{c}
\text{F1} \\
\downarrow \\
\text{M1} \\
\downarrow \\
\text{N} \\
\end{array} \]

(ii) \[ \begin{array}{c}
\text{F1} \\
\downarrow \\
\text{N} \\
\downarrow \\
\text{M1} \\
\downarrow \\
\text{M2} \\
\downarrow \\
\text{N} \\
\end{array} \]

(iii) \[ \begin{array}{c}
\text{M1} \\
\downarrow \\
\text{M2} \\
\downarrow \\
\text{N} \\
\downarrow \\
\text{F1} \\
\downarrow \\
\text{F2} \\
\end{array} \]

(a) Which of these Bayesian Networks are correct representations of the preceding information?
(b) Which is the best network? Explain.
(c) Write out the CPT for $Pr(M_1|N,F_1)$ for the case where $M_1 \in \{0,1,2,3,4\}$ and $N \in \{1,2,3\}$. Express the entries in terms of $e$ and $f$.
(d) Use your CPT for $Pr(M_1|N,F_1)$ to compute the CPT for $Pr(M_1|N)$ (again expressed in terms of $e$ and $f$).
(e) Suppose $M_1 = 1$ and $M_2 = 3$. What are the possible numbers of stars.

2. Consider the Bayes Network given below.
(a) What is the product decomposition specified by this network?

(b) Say that variable $X_7$ has 3 possible values, $X_6$ has 2 possible values, and $X_4$ has 4 possible values. How many values will be contained in the conditional probability table for $X_6$?

(c) Are $X_1$ and $X_5$ conditionally independent given $X_2$, given $X_7$, given $X_6$, given $X_4$?

(d) What are the relevant variables given the query $X_3$ and the evidence items $X_6$, given evidence $X_5$, given evidence $X_4$?

3. Consider the bayes net

In this network all variables are binary (including the alarm sound) having values either true or false. We use upper case to indicate a variable and lower case to indicate the values of a variable. When you are asked to give a probability involving some variables, you must the value of this probability for all values of the variables. Hint, many of the questions can be answered directly without any numeric calculations.

Let the conditional probability tables for the network be:

<table>
<thead>
<tr>
<th></th>
<th>e</th>
<th>-e</th>
<th>b</th>
<th>-b</th>
<th>s</th>
<th>-s</th>
<th>w</th>
<th>-w</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>1/10</td>
<td>9/10</td>
<td>1/10</td>
<td>9/10</td>
<td></td>
<td></td>
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<tr>
<td>B</td>
<td></td>
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<td>1/10</td>
<td>9/10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>e \land b</td>
<td>9/10</td>
<td>1/10</td>
<td>e \land -b</td>
<td>2/10</td>
<td>8/10</td>
<td>-e \land b</td>
<td>8/10</td>
</tr>
<tr>
<td></td>
<td>-e \land -b</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
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<tr>
<td>W</td>
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<td>s</td>
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<td></td>
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<td></td>
<td></td>
<td>-s</td>
<td>2/10</td>
<td>8/10</td>
<td></td>
</tr>
</tbody>
</table>

(a) Given that Mrs. Gibbons phones you (g) what is the probability that the alarm went off (s)?
(b) Say that there was a burglary (b) and but no earthquake (-e), what is the expression specifying the posterior probability of Dr. Watson phoning you (w) given the evidence. (You do not need to calculate a numeric answer, just give the probability expression).

(c) What is $\Pr(G|S)$? (i.e., the four probability values $\Pr(g|s)$, $\Pr(-g|s)$, $\Pr(g|s =s)$, $\Pr(-g|s =s)$).

(d) What is $\Pr(G|S \land W)$? (i.e., the 8 probability values $\Pr(g|s \land w)$, $\Pr(g|s \land -w)$, $\Pr(-g|s \land w)$, $\Pr(-g|s \land -w)$).

(e) What do these values tell us about the relationship between $G$, $W$ and $S$?

(f) What is $\Pr(G|W)$? (i.e., the four probability values $\Pr(g|w)$, $\Pr(-g|w)$, $\Pr(g|-w)$, and $\Pr(-g|-w)$).

(g) What do these values tell us about the relationship between $G$ and $W$, and why does this relationship differ when we know $S$?