CSC165 Quiz 7, Thursday July 14th

Name: Student number:

Suppose \( j, k \in \mathbb{N}, 25 \leq j \leq 200, 3200 \leq k \leq 102400 \). Suppose \( j \) has binary representation \( (b_n \cdots b_0)_2 \), and \( k \) has binary representation \( (b'_m \cdots b'_0)_2 \). For the following questions, reading symbols from a calculator doesn’t constitute justification.

1. What are the possible values of \( n \) and \( m \)? Justify your answer.

**Sample Solution:** Since \( 25 \leq j \leq 200 \), \( j \)'s binary representation has at least as many digits as the binary representation of \( 25 \) (or \( (11001)_2 \), since \( 25 = 16 + 8 + 1 \)), which is 5 digits, and at most the number of binary digits as \( 25 \times 8 \), which shifts the binary representation of \( 25 \) 3 times to the left, so 8 digits. Thus \( n \) can have values from 4 to 7, inclusive (1 less than the number of digits). Since \( 32000 \leq k \leq 102400 \), it has a minimum of the number of digits in \( 200 \times 16 \), which is 200 shifted left 4 times, or 12 digits. It has a maximum of the number of digits in \( 102400 \), or \( 3200 \times 32 \), which is 3200 shifted left 5 times, or 17 digits. Thus \( 11 \leq m \leq 16 \) (the number of digits minus 1).

2. How many bits could there be in \( 9j/4 \)? Justify your answer.

**Sample Solution:** \( 9j \) is calculated by adding \( 8j \) to \( j \), which means shifting \( j \) to the left 3 times and then adding the result to \( j \). The sum \( j + 8j \) is no more than \( 2 \times 8j \), so the result will be at most 1 digit more than \( 8j \). Dividing by 4 shifts to the right 2 places, losing 2 digits. We calculated above that \( j \) has from 5 to 8 digits, so \( 9j \) has from 8 to 12 digits, and \( 9j/4 \) has from 6 to 10 digits.