CSC165, Summer 2005, Assignment 1
Due: Thursday June 9th, 10 am

Danny Heap

INSTRUCTIONS

Please work on all questions. Turn in the outline and structure of a solution, even if you cannot provide every step, and we will try to assign some part marks. However, if there is any question you cannot see how to even begin, leave it blank you will receive 20% of the marks for that question.

Be sure to give full credit to any sources you consult (other than course notes, TAs, and the instructor) in preparing this problem set. If you try to pass off somebody else’s work as your own for credit, you are committing an academic offense, and that can entail serious consequences. Any ideas that you do not attribute to someone else are assumed to be the ideas of the author(s) listed below, and will be evaluated for grading.

Write your name(s) and student number(s) (maximum of two names and two student numbers) in the space below.

Name

Student #

Name

Student #
1. Prerequisites

The following fragments are from the U of T Arts & Science Calendar, specifying the prerequisites for CSC165:

CSC107H1/CSC108H1/(CSC148H1/CSC150H1 taken concurrently); U Advanced Functions and Introductory Calculus, and one of U Geometry and Discrete Mathematics or U Mathematics of Data Management OR OAC Calculus and one of Algebra & Geometry or Finite Mathematics.

Explanation of Symbols The comma (,) the semi-colon (;) the ampersand (&) and the plus sign (+) all mean “AND”. The slash (/) means “OR”.

Devise symbolic notation for each of the predicates connecting a single course to a student. For example, \( \text{CSC107}(x) \) may represent “\( x \) has completed CSC107H1.” Using the operators \( \land \) and \( \lor \), combine your predicates into three different predicates, each of which is consistent with the given English description of the prerequisites. By different I mean the predicates specify different sets of students, and by consistent I mean that there is a reasonable interpretation of the English description that has the same meaning as your predicate. Justify your answers.

2. The Venn of statements

Let \( S \) represent the set of stories, and

- Let \( A(s) \) represent “\( s \) is apocryphal.”
- Let \( B(s) \) represent “\( s \) is blasphemous.”
- Let \( C(s) \) represent “\( s \) is convoluted.”

Consider this Venn diagram, where shaded regions are exactly those containing one or more stories.

Which of the following are true, which are false? Justify your answers using the Venn diagram.

(a) \( \forall s \in S, A(s) \Rightarrow B(s) \).
(b) Every blasphemous story is apocryphal.
(c) \( \exists s \in S, C(s) \Rightarrow A(s) \).
(d) Some story is both apocryphal and convoluted only if it is blasphemous.
(e) \( \exists s \in S, B(s) \Rightarrow C(s) \).
(f) Any story that is both apocryphal and blasphemous must be convoluted.
3. **Various Venns**

For some domain $D$, consider the statement:

$$\forall x \in D, E(x) \Rightarrow (F(x) \Rightarrow G(x)).$$

Draw three unshaded Venn diagrams with domain $D$ and intersecting sets $E$, $F$, and $G$ that create 8 regions analogous to the diagram in question 2. Now shade each diagram to indicate which regions are non-empty, so that no diagram contradicts the above statement. In addition, each pair of diagrams must be distinct, having at least one region that’s shaded in one and unshaded in another. Justify your diagrams.

4. **Virtue Rewarded**

The following piece of propaganda undertakes to show the correlation between the number of hours per week (including lectures and tutorials) spent on CSC165 and final mark.

<table>
<thead>
<tr>
<th>student</th>
<th>mark</th>
<th>hours/week</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>65</td>
<td>4</td>
</tr>
<tr>
<td>$B$</td>
<td>70</td>
<td>5</td>
</tr>
<tr>
<td>$C$</td>
<td>75</td>
<td>6</td>
</tr>
<tr>
<td>$D$</td>
<td>80</td>
<td>7</td>
</tr>
<tr>
<td>$E$</td>
<td>85</td>
<td>8</td>
</tr>
<tr>
<td>$F$</td>
<td>90</td>
<td>3</td>
</tr>
</tbody>
</table>

If our domain $D$ is $\{A, B, C, D, E, F\}$, then which of the following conclusions are justified or not justified? Explain your answers.

(a) Any student who spends at least 7 hours per week on the course earns a mark of at least 80.
(b) All students who earn a mark of at least 80 study at least 7 hours per week.
(c) All students who study at least 9 hours per week earn a mark of at least 90.
(d) All students who study less than 3 hours per week earn a mark of at least 95.
(e) Not all students who study less than 2 hours per week earn a mark of less than 65.