1. [Proving negation: \( \exists \ a \in \mathbb{R} : \forall \ b \in \mathbb{R} : a \neq 2b/(b+1) \).]

Let \( a = 2 \in \mathbb{R} \).

[Proving \( \forall \ b \in \mathbb{R} : 2 \neq 2b/(b+1) \)]

Let \( b \in \mathbb{R} \).

Assume for contradiction that \( 2 = 2b/(b+1) \).

\( 2(b+1) = 2b \).

\( 2 = 0 \), which is a contradiction.

2. Let \( n \in \mathbb{N} \) such that \( n \geq 19 \) and \( \exists a, b \in \mathbb{N} : n = 5a + 6b \).

Let \( a_0, b_0 \in \mathbb{N} \) such that \( n = 5a_0 + 6b_0 \).

Case \( a_0 = 0 \).

\( n = 5a_0 + 6b_0 = 5 \cdot 0 + 6b_0 = 6b_0 \).

\( 6b_0 = n \geq 19 > 18 = 6 \cdot 3 \).

\( b_0 \geq 3 \).

\( b_0 \geq 4 \). # Since \( b_0 \geq 3 \) and \( b_0 \in \mathbb{N} \).

Let \( a_1 = 5 \in \mathbb{N} \), and \( b_1 = (b_0 - 4) \in \mathbb{N} \). # Since \( b_0 \in \mathbb{N} \) and \( b_0 \geq 4 \).

\( n + 1 = n - 24 + 25 = 6b_0 - 6 \cdot 4 + 5 \cdot 5 = 6(b_0 - 4) + 5 \cdot 5 = 5a_1 + 6b_1 \).

Case \( a_0 \neq 0 \).

\( a_0 \geq 1 \). # Since \( a_0 \in \mathbb{N} \).

Let \( b_1 = b_0 + 1 \in \mathbb{N} \), and \( a_1 = a_0 - 1 \in \mathbb{N} \). # Since \( a_0 \geq 1 \) and \( a_0 \in \mathbb{N} \).

\( n + 1 = n + 6 - 5 = 5a_0 + 6b_0 + 6 \cdot 1 + 5 \cdot (-1) = 5(a_0 - 1) + 6(b_0 + 1) = 5a_1 + 6b_1 \).

3. a) Let \( l, m \in \mathbb{Z} \) such that \( l < m \).

\( 0 < m - l \).

\( m - l \in \mathbb{Z} \).

\( m - l \geq 1 \). # By question's 1st assumption.

\( m - 1 \geq l \).

b) Let \( l \in \mathbb{Z} \), and \( r \in \mathbb{R} \) such that \( 0 \leq r < 1 \).

\( l = l + 0 \leq l + r \).

[Proving \( \forall z \in \mathbb{Z} : z \leq l + r \rightarrow z \leq l \)]

Let \( z \in \mathbb{Z} \) such that \( z \leq l + r \).

\( z \leq l + r \leq l + 1 \).

\( z \leq (l + 1) - 1 \). # From part (a) for \( z \in \mathbb{Z} \) and \( l + 1 \in \mathbb{Z} \).

\( z \leq l \).

\( l = l + r \.) # By question's 5th assumption, since \( l + r \in \mathbb{R} \), \( l \in \mathbb{Z} \), and

\( # l \leq l + r \wedge \forall z \in \mathbb{Z} : z \leq l + r \rightarrow z \leq l \).

\( l \leq l + r \).

[Proving \( \forall z \in \mathbb{Z} : z \leq l + r \rightarrow z \leq l + 1 \)]

Let \( z \in \mathbb{Z} \) such that \( z \geq l + r \).

\( z \geq l + r \geq l + \theta = l \).

\( z \geq l + 1 \). # From part (a) for \( l \in \mathbb{Z} \) and \( z \in \mathbb{Z} \).

\( l \leq l + r \). # By question's 6th assumption, since \( l + r \in \mathbb{R} \), \( l + 1 \in \mathbb{Z} \), and

\( # l \leq l + r \wedge \forall z \in \mathbb{Z} : z \geq l + r \rightarrow z \geq l + 1 \).

\( l \leq l + r \).

[Proving \( \forall z \in \mathbb{Z} : z \geq l + r \rightarrow z \geq l + 1 \)]

Let \( q_0 \in \mathbb{Z} \), and \( k_0 \in \mathbb{N} \) such that \( (l - 1) = q_0 \cdot n + k_0 \) and \( k_0 < n \). # By question's 2nd assumption.

\( \theta \leq k_0 < n - 1 \). # Since \( k_0 < n \) and \( k_0 \in \mathbb{Z} \).

\( l/n = (l-1)/n + 1/n = q_0 + k_0/n + 1/n = q_0 + (k_0 + 1)/n \).

\( \theta < (k_0 + 1)/n \leq (n-1 + 1)/n = 1 \).

\( \lfloor l/n \rfloor = \lfloor q_0 + (k_0 + 1)/n \rfloor 

= q_0 + 1 \). # Since \( q_0 \in \mathbb{Z} \) and \( \theta < (k_0 + 1)/n \leq 1 \) and part (c).

\( l/n = (l-1)/n + 1/n = q_0 + k_0/n + 1 = (q_0 + 1) + k_0/n \).

\( \theta \leq k_0 < n \).

\( \lfloor l/n \rfloor = \lfloor q_0 + 1 \} + k_0/n \rfloor 

= q_0 + 1 \) # Since \( q_0 + 1 \in \mathbb{Z} \) and \( \theta \leq k_0 < n \) and part (b).

\( = \lfloor l/n \rfloor \).

Part (d) was also naturally doable by cases (whether \( k_0 = 0 \) or not), but after doing so I looked at whether the cases could be unified and found the above "l - 1" idea.