1. Let $G$ denote the set of video games. Let Tetris and MarioKart be specific games in $G$. Consider the following predicates, along with their meaning: $C(x)$: “$x$ is a console game”; $W(x)$: “$x$ is a web game”; $P(x)$: “I have played / will play game $x$”; $V(x,y)$: “$x$ is more violent than $y$”; $B(x,y)$: “$x$ is better than $y$”.

Using only the domain, constants, and predicates above (in addition to appropriate connectives and quantifiers), translate each sentence below, i.e., give a natural English sentence that corresponds to each symbolic sentence, and give a clear symbolic sentence that corresponds to each English sentence. State clearly any assumptions you might need to make.

(a) If I have played every console game, then I have played every web game.
(b) There exists $x \in G$, there exists $y \in G$, $W(x) \land B(y, x) \land \neg P(y)$
(c) Some console game is more violent than every game that I have played.
(d) For all $x \in G$, $W(x) \land V(MarioKart, x) \Rightarrow \neg P(x)$
(e) No console game that I have played is better than any web game.
(f) For all $x \in G$, $C(x) \land P(x) \Rightarrow V(MarioKart, x)$
(g) I have played Tetris and MarioKart, and neither game is more violent than any web game.
(h) For all $x \in G$, for all $y \in G$, (there exists $z \in G$, there exists $w \in G$, $W(z) \land B(x, z) \land W(w) \land P(w) \land B(y, w)$) $\Rightarrow V(x, y)$
(i) I will play Tetris, unless it is more violent than every console game I have played.
(j) There exists $x \in G$, $C(x) \land P(x) \land V(x, Tetris) \land \forall y \in G, B(y, Tetris) \Rightarrow V(x, y)$

2. Consider the following sentence about positive integers $x$ and $y$:

If $xy$ is even, then $x$ is even or $y$ is even.

For each sentence below, state whether it is the negation of, the converse of, the contrapositive of, unrelated to, or equivalent to the sentence above. Justify each of your answers briefly (e.g., by writing both sentences in symbolic notation).

(a) If $x$ is even or $y$ is even, then $xy$ is even.
(b) If $xy$ is even and $x$ is odd, then $y$ is even.
(c) If $x$ is odd and $y$ is odd, then $xy$ is even.
(d) $xy$ is even and $x$ is odd and $y$ is odd.
(e) If $x$ is odd or $y$ is odd, then $xy$ is odd.
3. Consider the following statements about sequences of natural numbers \(a_0, a_1, a_2, \ldots\) (recall that in this course, the natural numbers start at 0, i.e., \(\mathbb{N} = \{0, 1, 2, 3, \ldots\}\)):

\[(S_1) \exists i \in \mathbb{N}, i > 1 \land \forall j \in \mathbb{N}, j > i \Rightarrow a_j > a_i\]
\[(S_2) \forall i \in \mathbb{N}, \exists j \in \mathbb{N}, j \neq i \land a_j = a_i \land \forall k \in \mathbb{N}, k \neq i \land k \neq j \Rightarrow a_k \neq a_i\]

And the following sequences:

\[(A_1)\] 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 6, 6, \ldots
\[(A_2)\] 9, 8, 7, 6, 5, 4, 3, 2, 1, 0, 0, 0, \ldots

For each sequence and each statement, state whether the statement is true or false for the sequence and justify your answer (you are not expected to write a formal proof to support your claim: just give an informal, but rigorous, explanation in clear, unambiguous English).

4. An “interpretation” for a logical statement consists of a domain \(D\) (any non-empty set of elements) and a meaning for each predicate symbol. For example, \(D = \{1, 2\}\) and \(P(x)\): “\(x > 0\)” is an interpretation for the statement \(\forall x \in D, P(x)\) (in this case, one that happens to make the statement True). For each statement below, provide one interpretation under which the statement is true and another interpretation under which the statement is false—if either case is not possible, explain why clearly and concisely.

(a) \(\forall x \in D, P(x) \land Q(x) \Rightarrow \exists y \in D, R(x, y)\)
(b) \(\forall x \in D, P(x) \Rightarrow Q(x)\) \iff \(\exists x \in D, P(x) \Rightarrow \exists y \in D, Q(x)\)
(c) \(\forall x \in D, \exists y \in D, P(x, y) \land \forall z \in D, P(z, y) \Rightarrow z = x\)

5. For each equivalence below, either provide a derivation from one side of the equivalence to the other (justify each step of your derivation with a brief explanation—for example, by naming one of the equivalences in the “Summary of Manipulation Rules” on page 30 of the course notes), or show that the equivalence does not hold (warning: you cannot use a derivation to show non-equivalence—instead, think carefully about what an equivalence means, and how you can disprove it).

(a) \((P \Rightarrow R) \land (Q \Rightarrow R)\) \iff \((P \lor Q) \Rightarrow R\)
(b) \(P \Rightarrow (Q \Rightarrow R)\) \iff \((P \Rightarrow Q) \Rightarrow (P \Rightarrow R)\)
(c) \(\exists x \in D, P(x) \land \forall y \in D, y \neq x \Rightarrow \lnot P(y)\) \iff \(\exists x \in D, \forall y \in D, P(y) \iff y = x\)