Due: September 29, 2004 by 11am

Please attach the completed COVER PAGE to the front of your assignment.

1. Consider the following statement:

   (S1) If a number is even, then it is not prime.

   (a) Rewrite (S1) in precise symbolic notation. Clearly define the domain and predicates that you are using.
   (b) Write the contrapositive of (S1) in:
      i. English
      ii. precise symbolic notation
   (c) Write the converse of (S1) in:
      i. English
      ii. precise symbolic notation
   (d) Write the contrapositive of the converse of (S1) in:
      i. English
      ii. precise symbolic notation

2. Let the domain \( C \) be the set of all computers from UofT. For \( x \in C \), consider the following predicates:

   - Let \( L(x) \) represent computer \( x \) is running the Linux Operating System.
   - Let \( M(x) \) represent computer \( x \) is running the Mac Operating System.
   - Let \( W(x) \) represent computer \( x \) is running the Windows Operating System.

Consider the following sentences, where \( x \in C \):

   (S2) \( M(x) \rightarrow \neg L(x) \)
   (S3) \( W(x) \rightarrow \neg M(x) \)

Consider the Venn diagram from Figure 1, where \( c_1, c_2, \ldots, c_8 \in C \).

   (a) For each sentence (S2) and (S3) and for each computer \( c_1, c_2, \ldots, c_8 \) from the Venn Diagram, state whether the sentence is true or false for that computer.
   (b) Consider the amended sentences, (S2a) and (S3a):

   (S2a) \( \forall x \in C, M(x) \rightarrow \neg L(x) \)
   (S3a) \( \forall x \in C, W(x) \rightarrow \neg M(x) \)

   Assuming that (S2a) and (S3a) are true, draw a Venn diagram (make sure that your sets overlap to divide the diagram into eight regions) and put an X in the area(s) that cannot contain any computers.

   (c) Assume that (S2a) and (S3a) are true. Let \( c \) be a computer from \( C \) and such that the following sentence is true:

   (S4) \( L(c) \rightarrow W(c) \)

      i. Assuming that \( c \) runs Windows, draw a Venn diagram with sets to represent W, L and M (make sure that your sets overlap to divide the diagram into eight regions), and shade the area(s) where \( c \) could be located.
      ii. Assuming that \( c \) runs Linux, draw a Venn diagram with sets to represent W, L and M (make sure that your sets overlap to divide the diagram into eight regions), and shade the area(s) where \( c \) could be located.
      iii. Assuming that \( c \) runs Mac, draw a Venn diagram with sets to represent W, L and M (make sure that your sets overlap to divide the diagram into eight regions), and shade the area(s) where \( c \) could be located.
3. Consider the following hockey teams, which we will identify by the first letter of the city name.

   B Brandon Wheat Kings
   R Red Deer Optimist Chiefs
   T Toronto Marlboros
   K Kenora Stars
   C Cornwall Thunder

All of the teams listed above have played against each other at most once.

Let the domain $\mathcal{D}$ be the set of all teams $\{B, R, T, K, C\}$.

For any two teams $x$ and $y$ from $\mathcal{D}$, let $P(x, y)$ represent teams $x$ and $y$ have played each other.
(Note: $P(x, y)$ is equivalent to $P(y, x)$.)

For any two teams $x$ and $y$ from $\mathcal{D}$ who have played each other,
let $W(x, y)$ represent team $x$ won the game against team $y$.
(Note: In this league, there can be no tie games.)

(a) Consider (S5): Team “Brandon Wheat Kings” has played against all the other teams.
Rewrite (S5) in precise symbolic notation.

(b) Consider (S6): Team “Toronto Marlboros” has won all the games that it played.
Rewrite (S6) in precise symbolic notation.

(c) Consider (S7): Team “Toronto Marlboros” did not play any games.
Rewrite (S7) in precise symbolic notation.

(d) Consider (S8): Team “Kenora Stars” did not lose all of the games that it played.
Rewrite (S8) in precise symbolic notation.

(e) Consider (S9): Team “Red Deer Optimist Chiefs” has lost all of the games that it played.
Rewrite (S9) in precise symbolic notation.
(f) All the results of the games are summarized in Table 1. For any teams \( x \) and \( y \), the winner of the game between \( x \) and \( y \) is at the intersection of the row \( x \) and the column \( y \). If the intersection of the row \( x \) and the column \( y \) is empty, then the teams did not play each other.

For each of the statements (S5), ..., (S9), using the information provided in Table 1, state whether the statement is true or false. If false, give a counterexample. If true, justify.

4. Two of the hockey teams from Question 3, the “Red Deer Optimistic Chiefs” and the “Kenora Stars”, came last in the league. They noticed that the results of their games is affected by the color of the jerseys that they or their opponents wear during the game. In order to win next year’s championship, they have organized a testing tournament in which they have tested all of the possible combinations of jerseys having colors red, blue and yellow. They have recorded all the results of the games and the color of the jerseys in the Table 2.

<table>
<thead>
<tr>
<th>Game</th>
<th>Red Deer Optimistic Chiefs</th>
<th>Kenora Stars</th>
<th>Winner</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>red</td>
<td>blue</td>
<td>Red Deer Optimistic Chiefs</td>
</tr>
<tr>
<td>2</td>
<td>red</td>
<td>yellow</td>
<td>Red Deer Optimistic Chiefs</td>
</tr>
<tr>
<td>3</td>
<td>blue</td>
<td>red</td>
<td>Kenora Stars</td>
</tr>
<tr>
<td>4</td>
<td>blue</td>
<td>yellow</td>
<td>Red Deer Optimistic Chiefs</td>
</tr>
<tr>
<td>5</td>
<td>yellow</td>
<td>red</td>
<td>Red Deer Optimistic Chiefs</td>
</tr>
<tr>
<td>6</td>
<td>yellow</td>
<td>blue</td>
<td>Kenora Stars</td>
</tr>
</tbody>
</table>

Table 2: Games results for Question 4.

However, after collecting all this data, they realized that logic is not their strength, and they do not know how to interpret it. Hence, they are asking for your help to tell them the color of the jerseys that they should wear (or that they should prohibit their opponents from wearing) at the next championship.

(a) Using a sentence of the type “if ... then” and based on the results from Table 2, write (in English) an implication that would tell the “Red Deer Optimistic Chiefs” what color to wear.

(b) Using a sentence of the type “if ... then” and based on the results from Table 2, write (in English) an implication that would tell the “Kenora Stars” what color not to wear.

(c) Using a sentence of the type “if ... then” and based on the results from Table 2, write (in English) an implication that would tell the “Kenora Stars” what color they should not let their opponents wear.

(d) Consider the statement: “If a team wears yellow jerseys, then it loses”. Is this statement true? If not, then give a counterexample.

(e) Consider the statement: “If a team wears red jerseys, then it wins”. Is this statement true? If not, then give a counterexample.

(f) Write an English statement of the type “if ... then” for which game 6 is a counterexample.

(g) Write an English statement of the type “only if” for which both games 4 and 5 are counterexamples.