CSC 263 Homework 4

Due August 10, 2004

1. (15 points) Let $G = (V, E)$ be an undirected graph that is connected. Assume that we have run BFS on $G$ starting at some vertex $s$.

   (a) Show that the following set of edges constitutes a spanning tree of $G$:

   $$T = \{ \{v, p[v]\} \mid v \in V \}.$$

   (b) Let each edge $e = \{u, v\} \in E$ have the following weight:

   $$w(e) = d[u] + d[v].$$

   Is $T$ a minimum cost spanning tree for $G, w$? Prove it.

2. (15 points)

   (a) Describe how to implement DECREASE-PRIORITY $(v, k)$ on a heap.

   (b) Prove that any heap on $n$ nodes has at most $\lceil n/2^h + 1 \rceil$ nodes of height $h$.

3. (20 points)

   (a) Give the pseudocode for an algorithm that takes an array $A$ of $n$ natural numbers and outputs them in (increasing) sorted order using the priority queue ADT.

   (b) What is the best worst-case running time (in $O$-notation) you can achieve for this algorithm if you implement the priority queue using a heap? Explain.

   (c) Give the pseudocode for an algorithm that takes an array $A$ of $n$ natural numbers and outputs the median number using the priority queue ADT.

   (d) What is the best worst-case running time (in $O$-notation) you can achieve for this algorithm if you implement the priority queue using a heap? Explain.

   (e) Assume you have a function $\text{FindMedian}(A, n)$ that implements part (c) in the amount of time in part (d). Using this function as a black box (that is, just calling it without modifying its content) show how to improve the worst-case running time of Quicksort. How fast does it become (in $O$-notation)? Explain.