Due: Wednesday, March 5, beginning of lecture

NOTE: Each problem set only counts 5% of your mark, but it is important to do your own work. These assignments will be followed by term tests, each worth 15% of your final grade. You may consult with others concerning the general approach for solving problems on assignments, but you must write up all solutions entirely on your own. Anything else is plagiarism, and is subject to the University’s Code of Behavior.

1. Let $\mathcal{F} = (G, c, s, t)$ be a flow network with $G = (V, E)$ and $c$ being integer valued (i.e. all capacities are non-negative integers). Let $f$ be a maximum flow in $\mathcal{F}$. Now suppose that the capacity of some edge $e \in E$ is increased by 1. Using $f$, give an $O(|V| + |E|)$ time algorithm for computing a maximum flow $f'$ in the modified network. Justify why your algorithm correctly produces the updated max flow.

2. Any string $x \in \{0, 1\}^*$ can be interpreted as the binary representation (ignoring leading zeros) of a non-negative integer, denoted $(x)_2$. (Let the empty string $\lambda$ represent the integer 0.) Specify a Turing machine which on input $x \in \{0, 1\}^*$ computes a $y \in \{0, 1\}^*$ such that $(y)_2 = (x)_2 + 1$.

3. Define the integer maximum flow decision problem MFLOWD as the language
\[
\{< \mathcal{F}, B > | \mathcal{F} \text{ is an integer capacity flow network and } \mathcal{F} \text{ has max flow value } \geq B \}\.
\]
Let MFLOWS be the search/optimization problem (for flow networks with integer capacities) which computes a maximum flow $f : E \rightarrow \mathbb{N}$ where $\mathbb{N} = \{0, 1, 2, \ldots\}$. Say all integer values are represented in binary.

Show that MFLOWS $\xrightarrow{p} MFLOWD$.

Note: You should NOT solve the MFLOWS problem by the Ford Fulkerson algorithm.

4. Prove $\text{PARTITION} \leq_p \text{JSCHEDD}$ where $\text{JSCHEDD}$ is the decision problem corresponding to the unit profit job scheduling problem where each job has a release time, a processing time and a deadline.