My Approach to Teaching Math (at the high-school level)

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Mathematical reasoning is a beautiful thing. If properly taught, one can see that there is nothing mysterious about it. We start with some 'entity' and mold it, using all the mathematical tools at our disposal, until we have a form we can recognize and talk about. One such entity is an equation (though we may talk about more abstract entities). Our equation may start out looking complicated, but, through a sequence of transformations (whereby we apply the rules we know), we can transform that equation into one of a type we have seen before and know how to deal with. It is as if we are given a new object, which we can rotate, poke, smell, and observe. After a while, our brain recognizes this object to be a pencil-case, which we know how to deal with and can talk about it: we can open it, store something in it, etc. A crucial property of this analogy is that at no point have we extracted/inserted information from/into our system - the information about the entity was there all along, from the beginning - we have merely found ways, through our manipulations, to discover it. We have also not changed our entity's identity/state in the process. Similarly, when we work with equations, we manipulate them in such a way that we never really change what was there initially - our manipulations are such that an equality stays an equality, a constraint remains a constraint, etc. Thus, a student who understands this will not be puzzled as to why we can do one thing to a mathematical entity and not another. The student will understand which manipulations are legal and maintain the state of our entity, and which are not applicable (and why). When teaching my students, my goal is to make mathematical thinking a very intuitive process, removing any notions of 'math as a rule-book' that may have been sown into the students at their schools (an unfortunate effect of many teachers' own discomfort with mathematics being propogated to further generations). A 'rule-book' approach to math is inherently limited, in that students who get used to following given sequences of manipulations in well-defined cases, are thrown into a panic as soon as a problem is tweaked just enough so that it is no longer crystal clear which sequence of rules applies. At some point, they will run out of rules, and math will become for them a mysterious subject matter, incapable of being explained by any finite set of rule sequences. They will be correct in making this conclusion, and thus it will be no surprise that students raised with this approach to math, will at some point down the road claim that math is abstruse and just not for them. Instead, I advocate an approach to math whereby students learn to explore mathematical entities from a variety of angles - in effect, learning to interact with mathematical

entities in a variety of ways. This is analogous to being able to rotate, poke, smell, and observe that hypothetical pencil-case previously mentioned. What this does for the student is ground them in mathematical concepts, placing in their hands a whole range of tools and giving them a language with which to describe what they are doing. They become fearless, knowing that they will have a plan of attack for any problem thrown their way. At least one of the ways of looking at things will help them tackle a problem. If they can find multiple ways of looking at the problem, then they can be confident that they have understood the problem well. Thus, I do not hide terminology or notation from my students, and I do not attempt to simplify mathematical concepts - I merely make them more intelligible. If introduced to proper notation and terminology early on, things will actually be clearer rather than more complicated, as the logic and clarity of the mathematical language will be at their disposal from the onset. Introducing concepts as they come up (without hiding the details) will prevent sudden surprises later on - which often make students feel misguided or deceived. In fact, I have found that students are happy to learn things that have not been overly simplified - they consider it a compliment to be taught concepts at a more advanced level - and they are eager to surprise their teachers and peers with their newly-acquired mathematical maturity. When exposed to 'real math' earlier on, they form no misconceptions about what math is all about. In fact, none of these things are beyond the students' understanding capabilities - they are more likely to be beyond the teachers' explaining capabilities, in which case that issue is the one to be addressed. After all, a concept taught poorly is better not taught at all (for correcting student's misconceptions and adjusting their attitude towards a subject matter is more difficult than building understanding from scratch). However, I think that we are very lucky with the state of technology these days. Free online teaching materials, full courses, and video and audio lectures are readily available. With enough searching, a student can find material capable of explaining concepts at the right level. I strongly believe that students should take advantage of this, and not allow their high-school math classes to limit their learning or to make them build incorrect notions. I further believe that the more sources used by a student to learn about a concept, the more complete the understanding that will form as a result. Thus, a few mediocre (but correct) explanations can have the effect of a thorough and complete explanation.