Verification of Parameterized Concurrent Programs By Modular Reasoning about Data and Control

Zachary Kincaid Azadeh Farzan

University of Toronto

January 30, 2012

Parameterized concurrent programs

Goal

Compute numerical invariants (e.g. intervals, octagons, polyhedra) for parameterized concurrent programs.
Solution: annotation ι such that if some thread T's program counter is at v, then $\iota(v)$ holds over the globals & locals of T.

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Natural model for device drivers, file systems, client/server-type programs, ...

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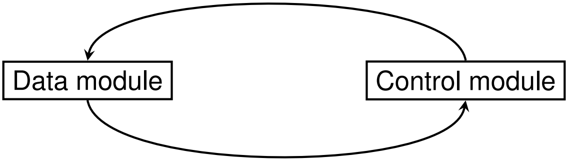
   - **Data module** computes numerical invariants
   - **Control module** computes a program model



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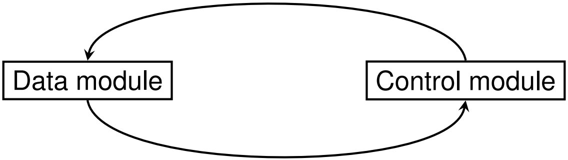

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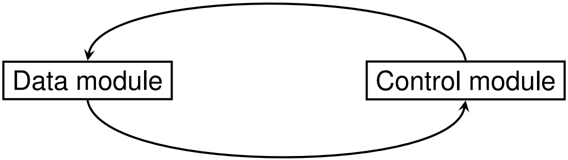

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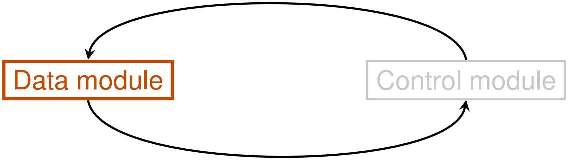

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Sequential program analysis

- Flow analysis: solve a system of equations valued over some abstract domain
- For sequential programs, equations come from the control flow graph:

$$IN(t) = \top$$
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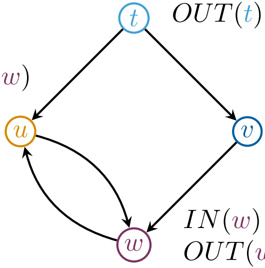

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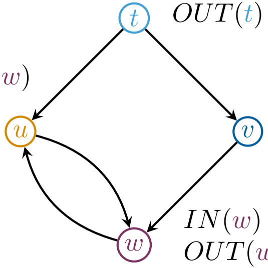

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- How about parameterized programs?

Data flow

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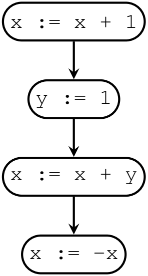

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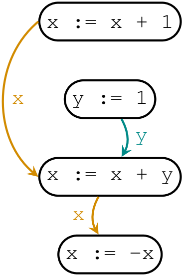

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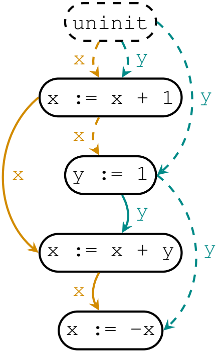
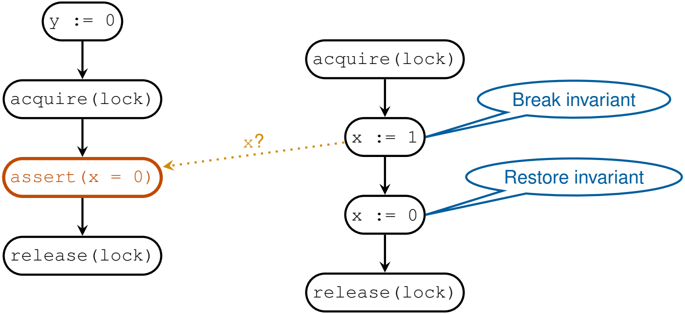


Why data flow?

Invariant: $x = 0$



Data flow graphs

A DFG for a program P is a directed graph $P^\sharp = \langle Loc, \rightarrow \rangle$, where

- $\rightarrow \subseteq Loc \times Vars \times Loc$ is a set of directed edges labeled by program variables

$$\boxed{\texttt{x := x + 1}} \xrightarrow{\texttt{x}} \boxed{\texttt{x := x + y}}$$

- Loc contains a distinguished `uninit` vertex
- Note: # of vertices does not depend on # of threads

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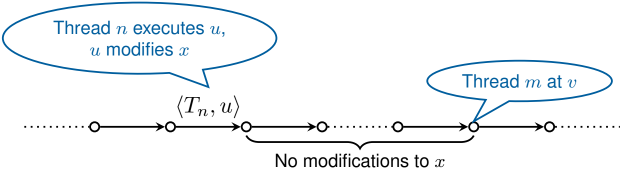

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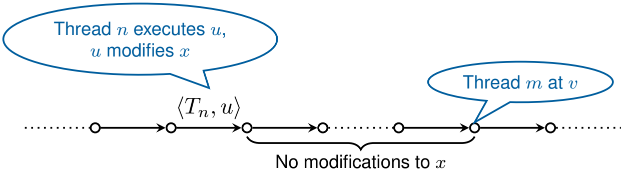

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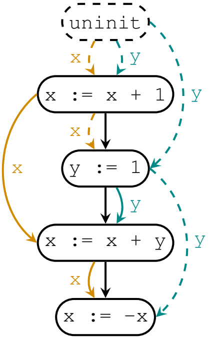

Computing invariants with DFGs

- DFGs induce a set of equations: $$\boxed{\begin{aligned} IN(v)_x &= \bigvee_{u \to^x v} \exists (Vars \setminus \{x\}).OUT(u) \\ IN(v) &= \bigwedge_{x \in Var} IN(v)_x \\ OUT(v) &= [\![v]\!](IN(v)) \end{aligned}}$$
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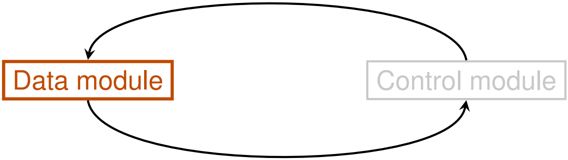
Theorem (DFG soundness)

If σ is a trace represented by a DFG $P^\sharp$, and ι is an inductive annotation for $P^\sharp$, then ι safely approximates the states reached by σ.

Overview



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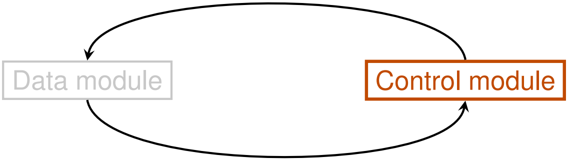

Constructing data flow graphs

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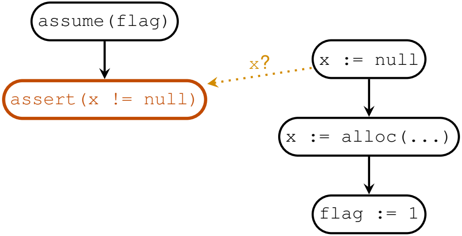
Precise DFG construction needs data

(`flag` is initially 0)



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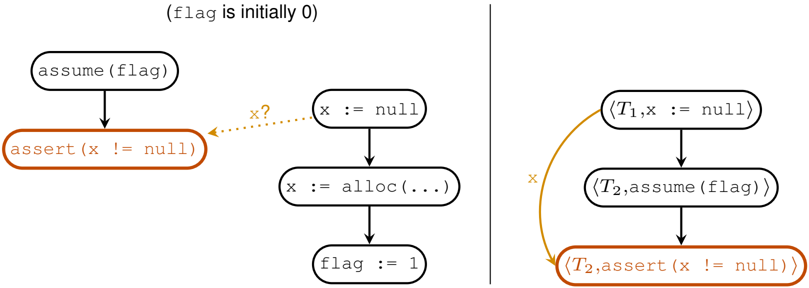

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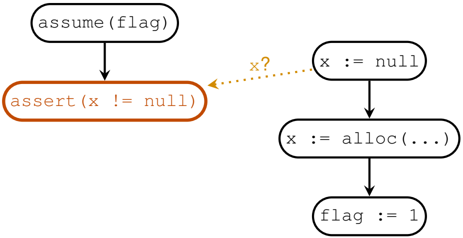

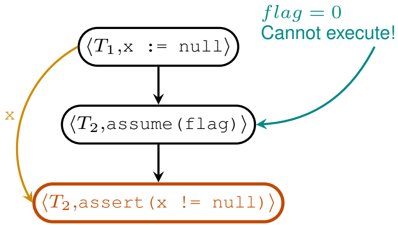

ι-feasible traces

Use an annotation ι to rule out infeasible traces: a trace σ is ι-*in*feasible if there is some subtrace $\sigma'\langle T_n, v\rangle$, some thread m, and some location u such that

- Thread m is at location u after executing σ'
- Thread n may not execute v in any state satisfying $\iota(u)$.

ι-feasible traces: example

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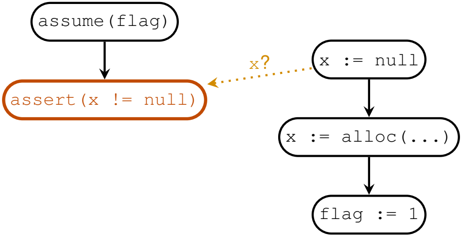

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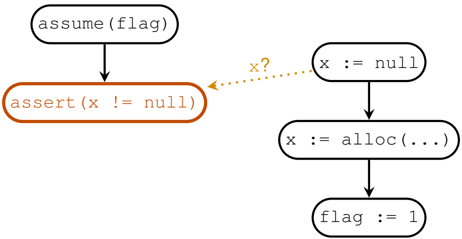

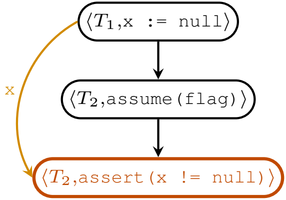

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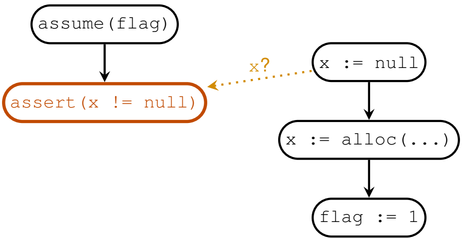

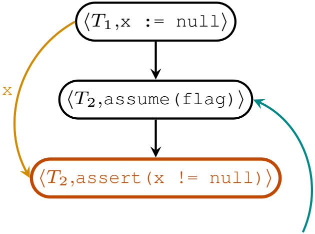

guard: $flag \neq 0$
T_1 at `x := alloc(...)`
T_2 at `assume(flag)`
is $\iota($`x := alloc(...)`$) \wedge flag \neq 0$
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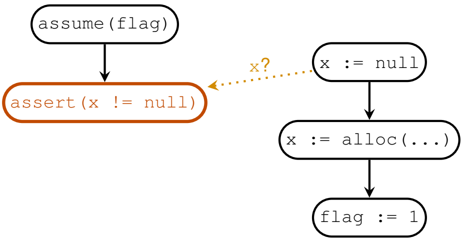

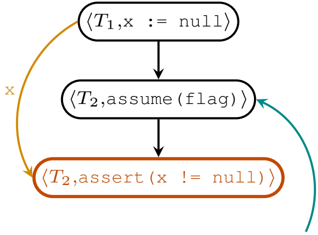

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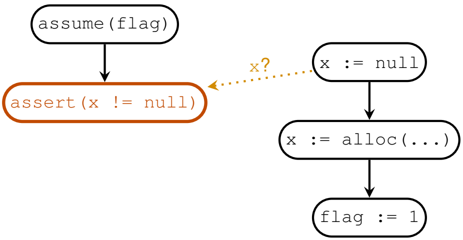

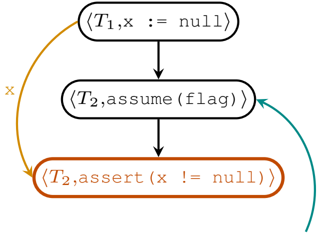

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- $\iota($`x := alloc(...)`$) : flag = 0 \Rightarrow$ infeasible
- $\iota($`x := alloc(...)`$) : true \Rightarrow$ feasible

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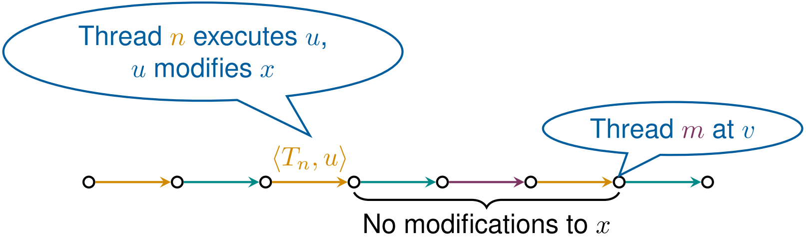
    - Parameterization is still an obstacle
    - Data flow edges for 2-thread ι-feasible witnesses can be computed efficiently

Projection

Lemma (projection)

Let ι be an annotation, let σ be an ι-feasible trace, and let N be a set of threads. Then $\sigma|_N$, the projection of σ onto N, is also ι-feasible.



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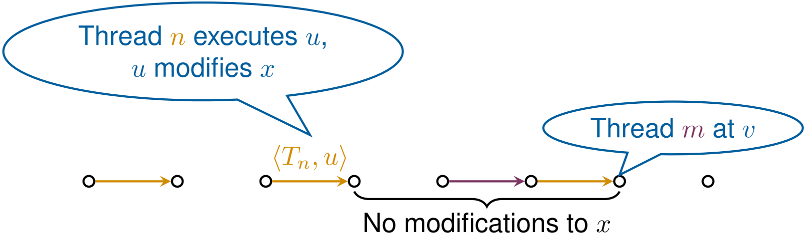

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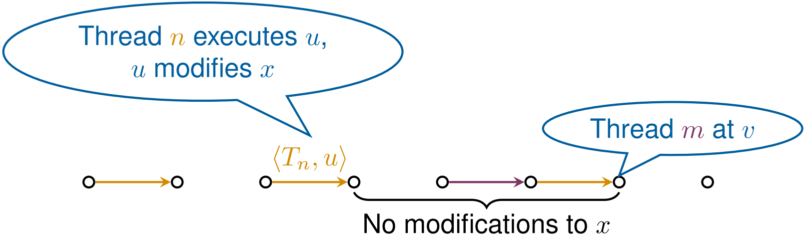

- A data flow edge $u \rightarrow^x v$ has an ι-feasible witness iff it has a 2-thread ι-feasible witness

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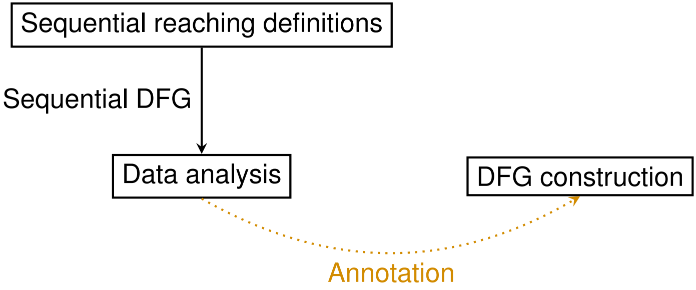

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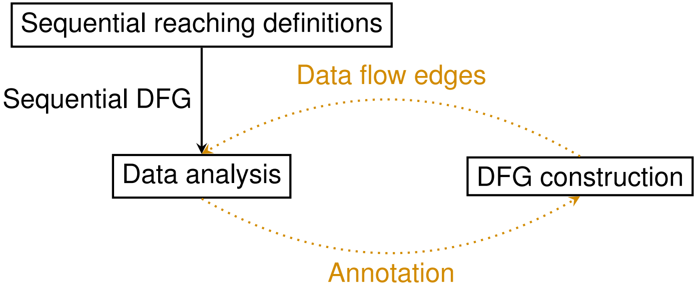

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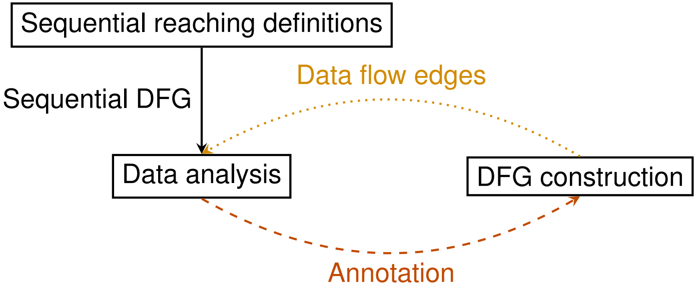

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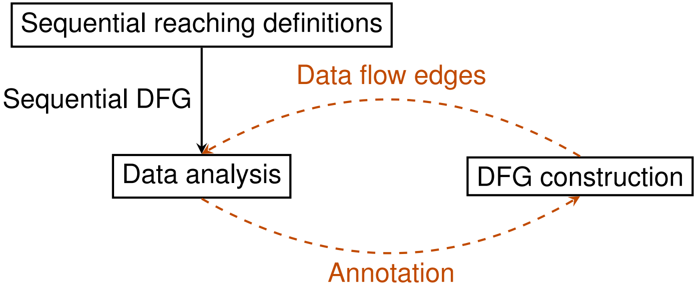

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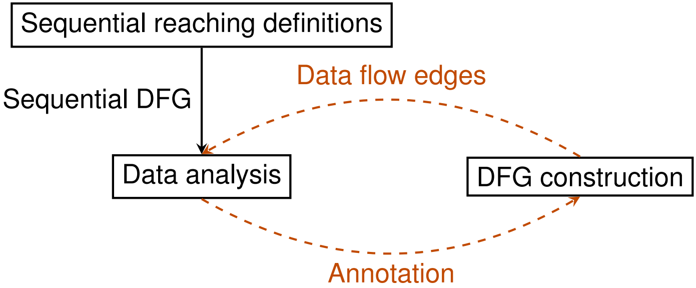

Experimental results

- We implemented our algorithm in a tool, DUET
- Integer overflow & array bounds checks for 15 Linux device drivers
  - DUET proves 1312/1597 (82%) assertions correct in 13m9s

Experimental results: Boolean programs

Boolean abstractions of Linux device drivers:

Suite 1	Duet	Linear interfaces[1]	Improvement
Assertions proved	2503	1382	81% increase
Average time	3.4s	16.9s	5x speedup

Suite 2	Duet	Dynamic cutoff detection[2]	Improvement
Assertions proved	55	19	189% increase
Average time	8.2s	24.9s	3x speedup

[1] S. La Torre, P. Madhusudan, and G. Parlato. Model-checking parameterized concurrent programs using linear interfaces. In CAV, pages 629–644. 2010.

[2] A. Kaiser, D. Kroening, and T. Wahl. Dynamic cutoff detection in parameterized concurrent programs. In CAV, pages 645–659. 2010.

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- Semi-compositional DFG construction algorithm

Questions?

Thank you for your attention.

Bonus slide: future work

- Improved algorithms for inferring groups of related variables to improve DFGs analyses over relational domains (e.g., octagons, polyhedra)
- Extension to handle aliasing