HOP-MAP: Efficient Message Passing with High Order Potentials

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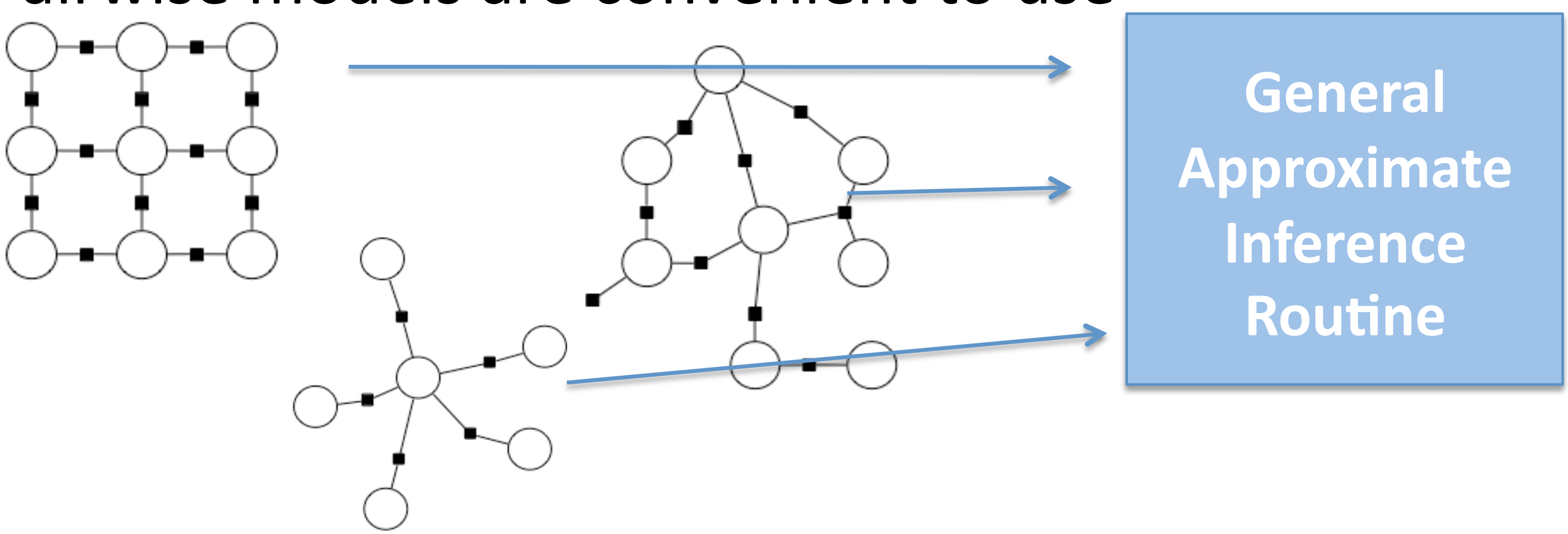
Motivation

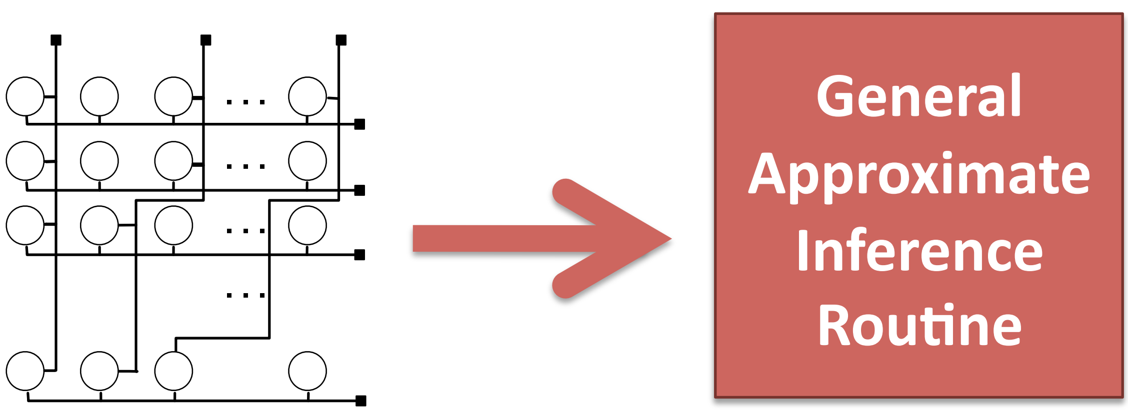
- Modeling interactions among variables in undirected graphical models over discrete variables
- Pairwise models are convenient to use



- But the pairwise models are not good enough
  - Even when optimum can be found, results are often not satisfying

Goal

- We need better models and convenient tools
  - Capture high order interactions
- In the general case – intractable, but useful classes exist where messages can be computed in poly-time.
- Setup - factor graphs over binary variables
  - Easy 1-of-N transformation from discrete variables



High order factor graph

Max Product Belief Propagation

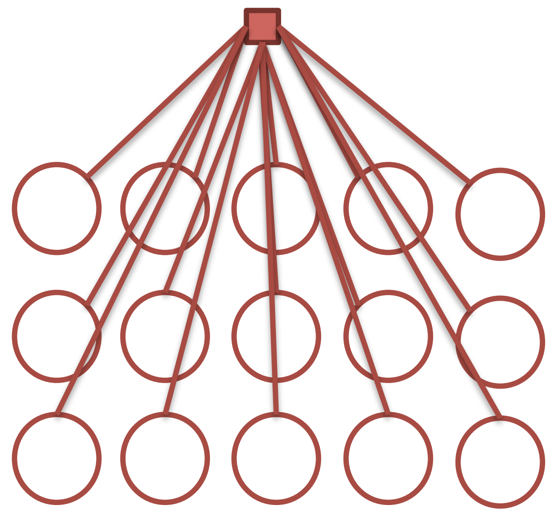
In this work…

- Develop new class of **order-based** potentials.
- Improve computation for existing class of **cardinality** potentials.
- Show how to easily create **compositions** of HOPs.
- **Code** available

Cardinality Potentials

$$\theta(\mathbf{h}) = f\left(\sum_{h_j \in \mathbf{h}} h_j\right)$$

Function value based on number of on variables



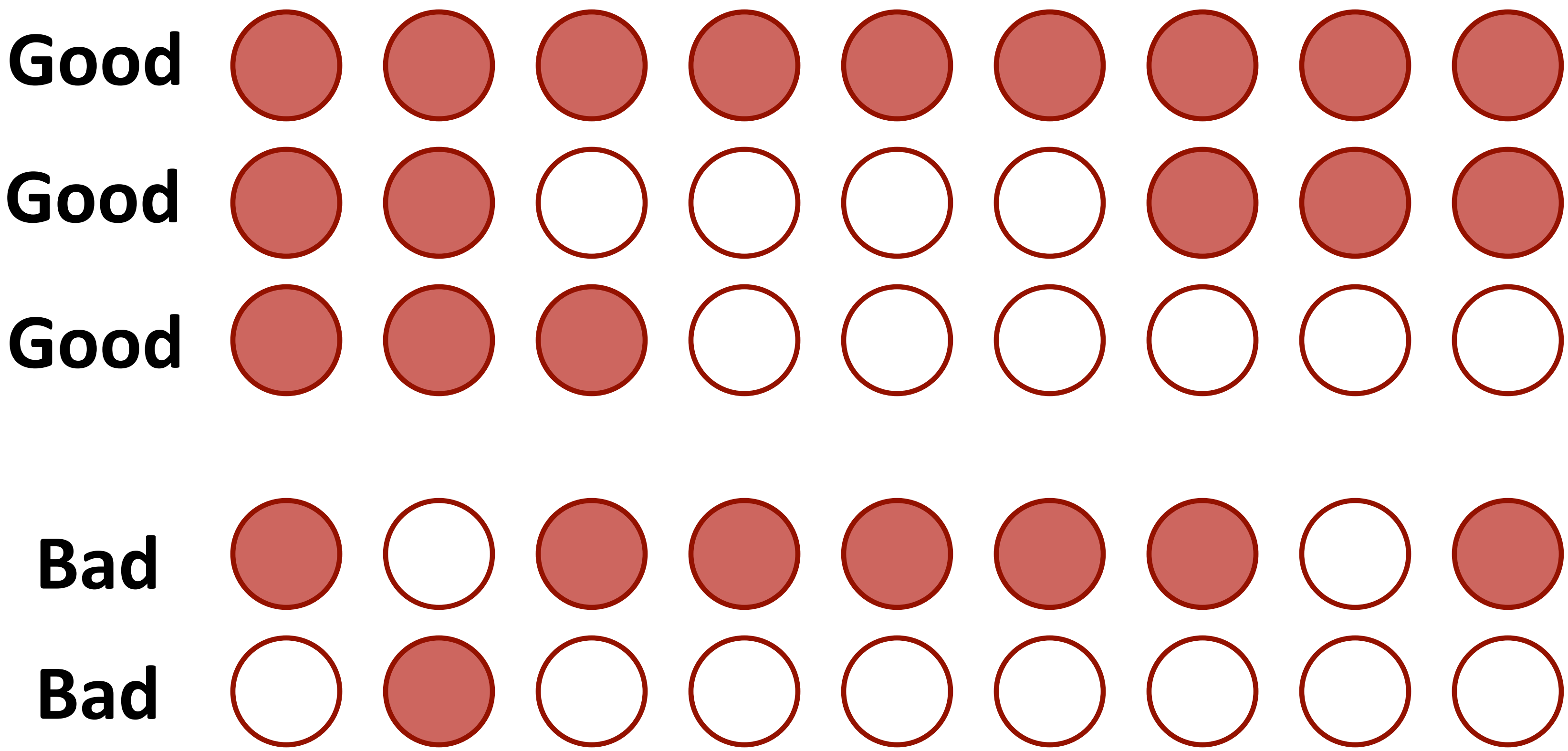
Related Work for General Case	
Gupta et al. (2007)	single maximum in O(N log N).
Potetz et al. (2008)	single approximate message in O(N).
Tarlow et al. (2008)	N exact messages in $O(N^2)$
This work	**N exact messages in O(N log N)**

Specialized Cardinality Potentials

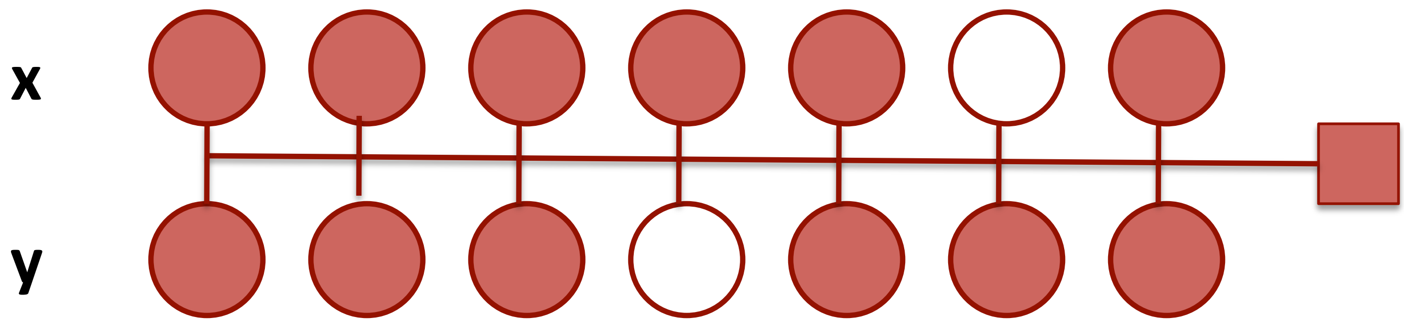
- Pattern-based potentials: generalized Potts model (Kohli et al. 07).
- b-of-N potential (Huang & Jebara 07).
- Distributional priors, e.g., Dirichlet process, Pitman-Yor (Tarlow et al. 08).

Order-Based Potentials 1 – Convex Set

$$f(h_1,\ldots,h_N) = \begin{cases} 0 & \text{if } h_i = 1 \wedge h_k = 1 \Rightarrow h_j = 1 \;\forall \mathrm{i} < \mathrm{j} < \mathrm{k} \\ -\alpha & \text{otherwise} \end{cases}$$



Order Based Potentials 2- Before-After



$$f(x_1,\ldots,x_N,y_1,\ldots,y_N)=\begin{cases}0 & \text{if } \min_{i:x_i=1} i > \max_{j:y_j=1} j\\ -\alpha & \text{otherwise}\end{cases}$$

Composition of Potentials

- If some variables act as logical switches

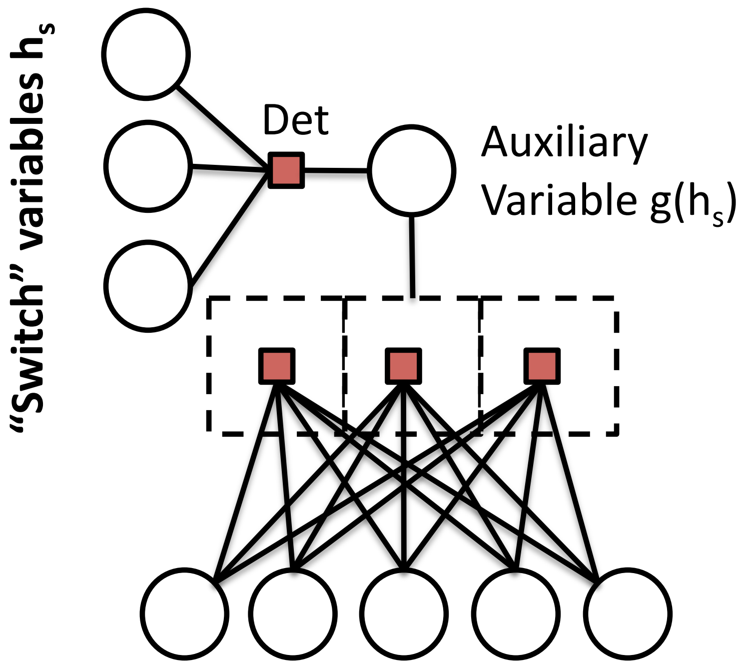
$$f(h_1,\ldots,h_N) = \begin{cases} g_0(h_2,\ldots,h_N) & \text{if } h_1 = 0 \\ g_1(h_2,\ldots,h_N) & \text{if } h_1 = 1 \end{cases}$$

- And if g_1,g_2 are tractable HOPs

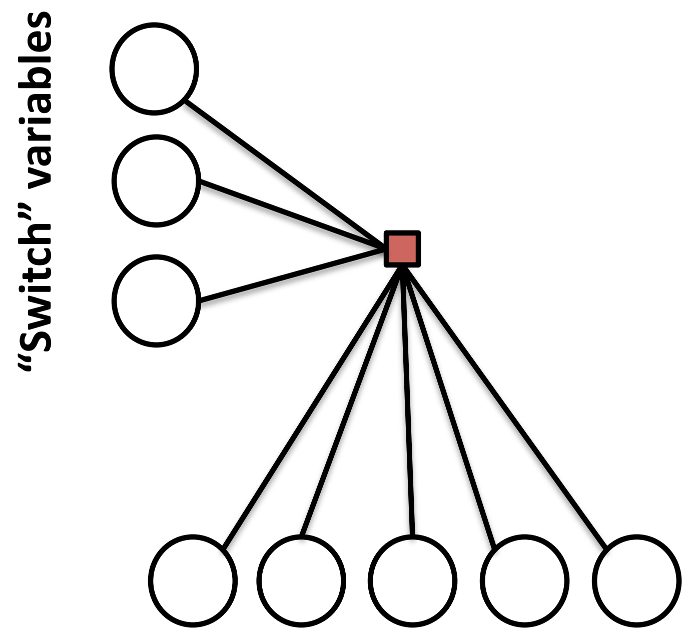
↓

- Can efficiently compute messages based on known messages for tractable HOPs
- No need to re-derive updates
- Generalizes to >1 subset of switching variables h_s switching among K HOPs of (say) at most log*N* cost with

$$O(2^{|h_s|} + K \log N)$$



Gates Representation



Composite Factor

(Minka & Winn 08)

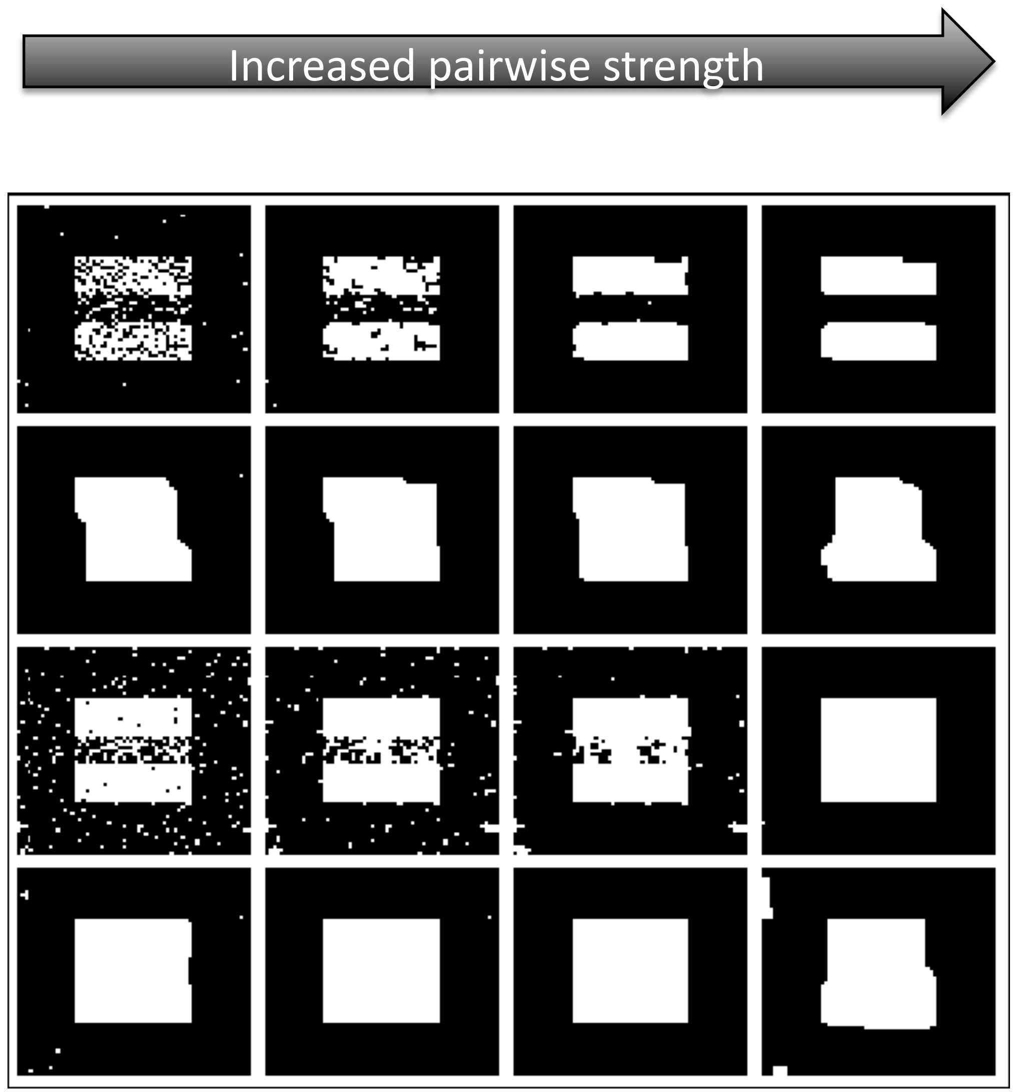
What are these HOPs good for?

- Many existing and novel models can be constructed by mixing and matching of HOPs.
- Interactive Poster! Can you figure out which?
- Priors for image segmentation tasks:
  – Over segments size (multilabel)
  – **Encourage % of pixels to be on (bg/fg)**
  – Convex image parts
- Bipartite matching
- Affinity Propagation/Facility location
- **User preferences of document *i* over *j* in ranking**
- …

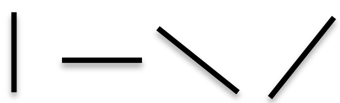
Related Work

- **Context Specific Independence** (Boutilier et al. 96)
  - Original motivation: more structure than just graph
  - HOP-MAP generalizes tree-based CPT representation
- **COMPOSE** (Duchi et al. 07)
  - Same big computational idea: special purpose algorithms for computing max-product messages
  - HOP-MAP is finer-grained ➔ broadly applicable.
  - Can be used together (code for COMPOSE pending)
- **Gates** (Minka & Winn 08)
  - Composite factors essentially implement max-product for Gates over high order potentials.

Experiments – Image segmentation



unary, pairwise

unary, pairwise,
convexity along 

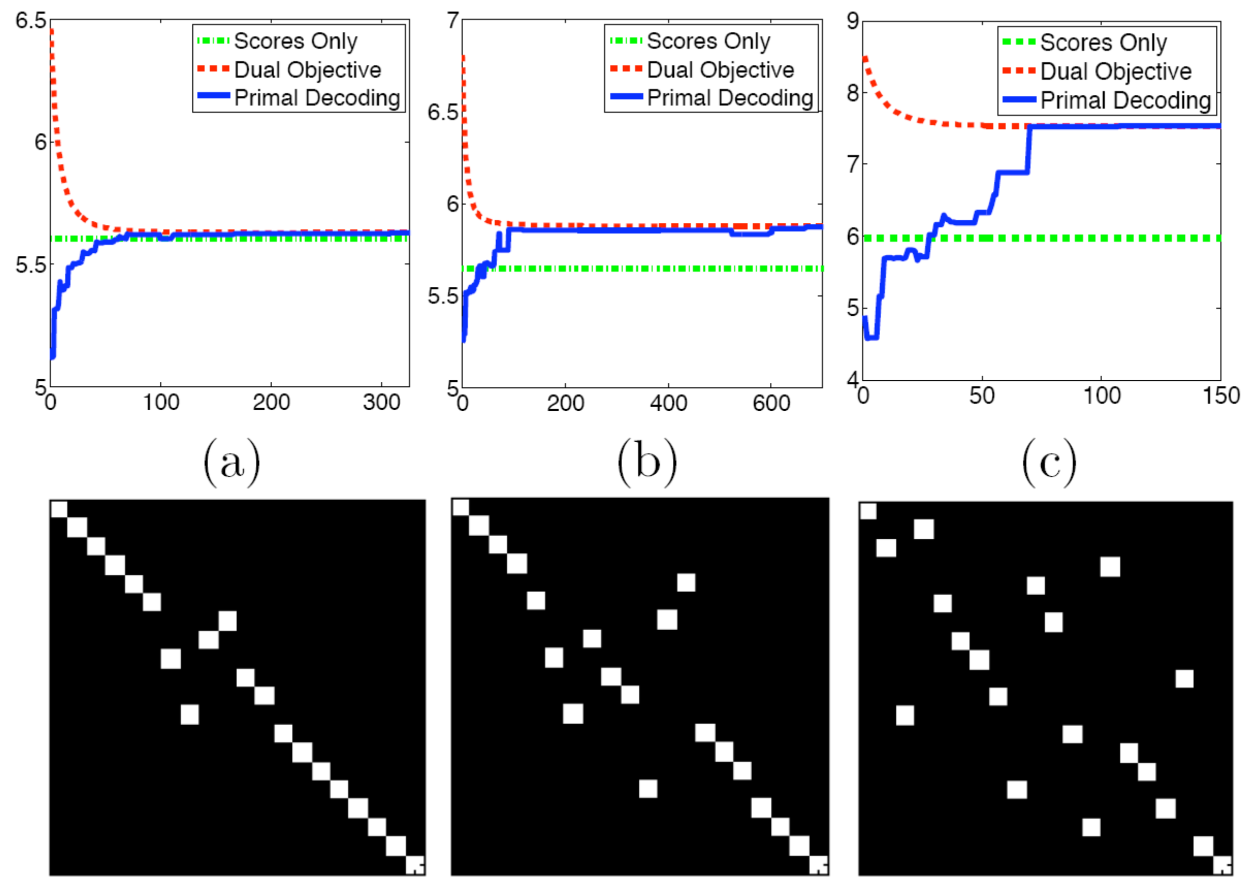
unary,pairwise,
cardinlaity $f(k) = -\left|\frac{1}{4}d^2 - k\right|$

unary,pairwise,
cardinlaity,convexity

Experiments – Rank Aggregation

- Document ranking for IR
- Goal – learn score for document-query pairs, then rank using scores.
- users are better at saying "I prefer doc *i* over doc *j*" than "I give it a score of 8.5" with respect to some query.
- Can be incorporated using before-after potentials.
- Simulated data
  - Unary potentials – high score -> high rank
  - λ : before-after potential strength
  - ρ: fraction of satisfied constraints

Experiments – Rank Aggregation



$\lambda = 0.01$ $\rho = 13/18$ | $\lambda = 0.05$ $\rho = 15/18$ | $\lambda = 0.25$ $\rho = 18/18$

Conclusions

- HOPs of interest in real-world problems are often structured, and therefore are often tractable
- We are building a vocabulary to expand beyond pairwise interactions, *conveniently*.
- Present efficient/novel components for “inner loop” of message passing
- Can use “outer loop” of choice
  - Max-Product, GEMPLP (Globerson and Jaakkola, 2008) High-Order TRW due to Komodakis and Paragois, 2009)
- Code and Demos at http://www.cs.toronto.edu/~dtarlow/hops/