Short Communication: Mathematics. General

PROCEEDINGS OF THE LATVIAN ACADEMY OF SCIENCES, Section B, 1998, Vol. 52, No. 2 (607), pp. 11–13

THE UNIVERSE OF MATHEMATICS

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To Yury Ivanovich Manin

The paper presents the program of an original course the author has lectured in 1990-1996 in several Latvian educational institutions.

Key words: *Mathematics, culture, human being, mathematicians*

Mathematics Subject Classification (1991): *00A35*

FOREWORD

The text presented below is not a mathematical paper. It is the program of a course the author worked out and lectured at the (former) Riga Higher Pedagogical Seminary and the Imanta Teacher College for school teachers of mathematics and at the University of Latvia for students of 2nd and 4th grades. A special projection of the course was lectured at the Jelgava Technological Lycée.

The course was aimed at considering mathematics in a (as much as possible) wide cultural context. Normally, such a goal is more appropriate for a philosopher rather than a mathematician but the philosopher's treatment should necessarily contain some inherited gaps that can be fulfilled only by a working mathematician.

Of course, for a mathematician it is easier, and more natural, to prove a theorem rather than describe how the idea came about, or how (s)he started to tackle the problem, or why he began to deal with mathematics at all. It is even more difficult to consider such things in a general setting trying to explicate some general laws regulating the life of the mathematical world and mathematicians. It seems that only special circumstances can enforce a mathematician to try to reflect on these topics.

As for the present author, such special circumstances sprang up in Riga, in 1990. That time was a time of common social optimism now known worldwide under the Russian name "Perestroyka". The spirit of reforms was in the air, and, particularly, education was one of the most active spheres. Various new educational and education-oriented institutions were created, and amongst the first was the Riga Higher Pedagogical Seminary (RHPS). The RHPS was organized and conducted by a psychologist, Prof. Gershons Breslavs, and the present author occurred into the start-up team.

The general idea behind RHPS was to make education (i) more close to the student as a human being and (ii) more integral so that any study were encompassing a general cultural context of the subject after which a special technical development should follow. Fortunately, these two goals are consistent (and actually the latter is a way to the former), and can be summarized as *humanizing* and *humanitarizing* education. As any other humanities problem, this one can, and should, be approached in different ways. The RHPS's one was to teach a school teacher to see this goal and give him means to accomplish it.

Since being a participant of the Riga Seminar on Education, the author sometimes used a speculative thesis that "mathematics is, in essence, a humanities discipline", the RHPS's leader proposed him to display the thesis and lecture a corresponding course. No substantial restrictions were imposed and so the course "The universe of mathematics" was born.

THE COURSE PROGRAM

Everything can be regarded from different viewpoints, and mathematics – a universe-in-itself in the space of the human culture – is not an exception. The goal of the course is to draw a multi-dimensional presentation of that complex object and its location in the space. To fulfill this task we will use the classical method of

descriptive geometry.

To wit: we choose in the space of culture a system of coordinates (certainly including a temporal – historical – axis into it) and then observe projections of the object on different coordinate planes: scientific, social, psychological, plane of activities and so on. If it is necessary, we will add to them some special sections, for example, by the plane of historical anecdote. The first approximation to the desired result can be considered achieved, if on the completion of the course there will emerge an integral notion, or at least an image, of the living universe of mathematics.

Themes (projection planes) listed below are to be considered as pointers to problems that deserve reflection rather than headings of topics one needs to learn. The list of literature is patch-work and fragmentary (and is nowhere dense in the continuum of the literature on the subject), however, it allows to support reflection on the subject in the initial stage.[1]

1. Mathematics as a "supernatural" science[2].

The place of mathematics in the system of human knowledge: numbers, figures, ideas; abstractions and formalisms. Methodological scheme: reality $\longrightarrow$ formalism $\longrightarrow$ reality. About "*the unreasonable effectiveness of mathematics in the natural sciences*"[3].

The nature of mathematical objects. Different approaches to foundations of mathematics: Platonism, formalism, intuitionism and constructivism. Bourbaki's program and "bourbakization" of mathematics. Axiomatic method – reduction of mathematics to set theory. Hilbert's thesis and program, Gödel's incompleteness theorem. About philosophy of constructivity (Poincaré, Brouwer, Hermann Weyl, Kolmogorov, Markov, Martin-Löf). And after all, what is continuum?

Activities (functional) methodology of mathematics, absorption of mathematics in the space of culture. Again reality and mathematics. The tree of mathematics.

Mathematics as a language activities. Meanings of mathematical texts: realistic, intuitive, formal. Mathematical text as a mediator between the man and external world, between a human being and a human being, between the left and right cerebral hemispheres.

[1]The list is too long and not included in the paper.

[2]Professor Lev Landau, a famous Soviet physicist and a Nobel laureate in physics, classified sciences into *natural* (physics, chemistry, biology ...), *unnatural* (history, philology, sociology ...) and *supernatural* whose category is the singleton {mathematics}.

[3]This is the title of a famous paper by Eugene Wigner, a well-known specialist in quantum physics and also a Nobel laureate.

2. Mathematics is what mathematicians are dealing with.

Mathematics as activities. Motives of mathematical activities, their historical evolution in the channel of culture.

Panorama of the history of mathematics: the ancient Babylon; the "Greek Miracle"; the Middle Ages and Renaissance – "Dixit Algorizmi", mysticism vs. natural science; the Enlightenment – heroic epoch of analysis; the 19th century – withdrawing mathematics from the natural science and reducing it to the set theory; mathematics of the 20th century – foundational paradoxes and the loss of certainty, boom of mathematics in the present, mathematics and computers.

Dialectics of practice and theory in the history of mathematics, the tree of mathematics.

Emotional foundations of mathematical activities. Mathematics and programming.

Mathematics as a system of comprehension of the world, realistic basis of mathematical activities. Mathematics as a game of mind, intellectually-aesthetic basis of mathematical thinking. Mathematics as a devotedness, dramatic basis of mathematical activities. Mathematics and music. Mathematics as a competition, sport basis of mathematical activities. The great problems of mathematics, Hilbert's problems and mathematics of the 20th century.

Mathematics as a psychotherapy.

Mathematics in the social sense. Social status of mathematics and mathematicians. Mathematics and politics. Mathematical community: historical evolution and contemporary structure. The social nature of the act of proof acceptance, provability and convincingness.

Evolution of mathematics as a drama of ideas. The great men of mathematics. The votaries of mathematics. Mathematics in historical anecdotes.

Aura and myths around mathematics: the most dull science, the most intricate science, the most fanciful science. Poetics of mathematics.

Mathematics – a culture in the human culture: links, interrelations and interaction.

3. Mathematics and thinking.

Thinking: fact-oriented vs. concept-oriented. Mathematics and scientific thinking, hierarchy of reasoning: logical, mathematical, discourse, rational. Mathematics as an instrument and as a system of concepts.

A communicative function of mathematics. Mathematical notions as metaphors, their universal cultural character and the role in mechanisms

of comprehending reality. Mathematics and philosophy.

Examples. *Space: basis, coordinates, multidimensionality, projections. Linearity, differential "microscope", integration. Mechanics as a geometry. Discrete and continuous, algebra and topology. Continuum and metaphors of the descriptive set theory. Algorithms; constructiveness and non-constructiveness, computability and non-computability. Deductive systems and derivations, provability and non-provability. Social systems and dynamic systems theory. Stability, catastrophes, bifurcations, order and chaos.*

Mathematics as a means and a style of thinking. Mathematics and the humanity studies. Mathematics as a humanities discipline.

Thinking of a mathematician: process and result. Vladimir Arnold vs. Nicholas Bourbaki ("bourbakization" or "bourbakery"?). Reality of a mathematician: images and calculi. Functional cerebral asymmetry and mathematics. Art of mathematical symbols setting.

4. The architecture of mathematics.

Is it possible to explain theorems? Mathematics experimental and theoretical, architecture of mathematical theories.

Rotation of scientific programs in the history of mathematics, scientific programs of contemporary mathematics and structuralism. Bourbaki's concept of the mathematical knowledge structure, fundamental role of non-categorical theories. Districts of geometry and labyrinths of number theory. Constructive mathematics – an alternative or a subsystem?

Algebraization of contemporary mathematics, the category theory methodology of design in mathematics. Tendencies of the development of contemporary mathematics.

Mathematics and the computer: reality and prospects. Applied mathematics: peculiarities, methodology, logics. Art of constructing models.

5. Mathematics as a pedagogical task.

Education and mathematics: traditions and the present. Why to study mathematics. Axiomatic method and school mathematics. Contemporary mathematics and the school course.

Psychological problems of school mathematics. *"That time ..."* vs. *"The science proved ..."*.[4]

How to make learning (and, correspondingly, teaching) mathematics interesting. Methodological scheme: question $\longrightarrow$ answer $\longrightarrow$ question. Local deducibility: attractiveness vs. systematics.

6. Mathematical mixture.

Mathematical exotica: monsters of analysis and topology (Peano's curve, Sierpinski's carpet *etc*). Paradoxes of logics and set theory, around the axiom of choice.

Encyclopedia of entertaining mathematics by Martin Gardner. Mathematical puzzles and amusements, puzzle topology. Logic by Smullian. The world of Lewis Carroll.

Mathematics and art. Algebra of harmony and harmony of algebra. Maurice Escher's art. Anatoly Fomenko's art. Fractals.

ACKNOWLEDGEMENTS

First of all, I am indebted to Gershons Breslavs for the very idea and an extremely fruitful environment for its realization at RHPS. I am also grateful to my first students at RHPS, especially to Alexander Gorodinsky and Yury Galitsky, for discussions and stimulating criticism. I also learned much about teaching mathematics during my work at the Jelgava Technological Lycée headed by Alexander Gorodinsky. Ludmila Žalostiba actively assisted me in my efforts there, and I am grateful for her enthusiams and support.

Special thanks go to Jānis Cīrulis who encouraged me to lecture the course at the University of Latvia, and with whom we occasionally exchanged remarks on the subject (it seems it is the only real possible way of discussing humanities aspects of technical disciplines). Of course, I am thankful also to those of my university students who listened me with interest, asked questions (often provocative but stimulating) and participated in seminar discussions.

Another special thanks go to Boris Avramecs, a musician and culturologist, for his sincere interest to the course, encouragement and several (alas, again occasional) discussions of the mysterious relations between music and mathematics.

Thanks are also due to the editor, Mrs. Maruta Dzērve, who suggested to prepare the program for publication.

Most of all I am indebted to Yury Ivanovich Manin, whose books, papers and few personal letters taught me to view mathematics widely, as a unique cultural phenomenon created by humans, not computers. In particular, the influence on the course of two Manin's small books "Provable and Non-provable" and "Computable and Non-computable" is difficult to overestimate. Finally, I'd also like to mention a beautiful book "Men of Mathematics" by Eric Bell, which was very influential too.

Received April 7, 1998

[4]The former means somewhat like "That time, a young army officer and a bright mathematician, in order to develop X was trying to solve Y and, at last, proved Z" whereas the latter is its quotient: "It was proven that Z" or "The science proved Z".