A Correctness Result for Reasoning about One-Dimensional Planning Problems

Yuxiao (Toby) Hu   Hector J. Levesque

Department of Computer Science
University of Toronto

{yuxiao,hector}@cs.toronto.edu

May 12, 2010

Motivation

- Classical planning produces action sequence in complete world.
  - *e.g.*: given, *obj1* at home, *obj2* in office and a *truck*, make *obj1* in office and *obj2* at home.
  - Resulting sequential plan only works for this particular setting.

Motivation

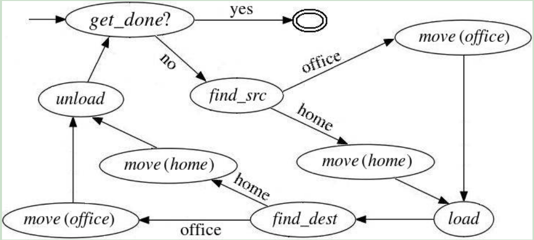
- Classical planning produces action sequence in complete world.
  - *e.g.*: given, *obj1* at home, *obj2* in office and a *truck*, make *obj1* in office and *obj2* at home.
  - Resulting sequential plan only works for this particular setting.
- Conditional planning allow incomplete knowledge by allowing branching on run-time world state.
  - *e.g.*: given a truck, *obj1* and *obj2*, location and destination unknown, make both objects at their destination.
  - Resulting tree-like plan can handle four different cases.

Motivation

- Classical planning produces action sequence in complete world.
  - *e.g.*: given, *obj1* at home, *obj2* in office and a *truck*, make *obj1* in office and *obj2* at home.
  - Resulting sequential plan only works for this particular setting.
- Conditional planning allow incomplete knowledge by allowing branching on run-time world state.
  - *e.g.*: given a truck, *obj1* and *obj2*, location and destination unknown, make both objects at their destination.
  - Resulting tree-like plan can handle four different cases.
- An even more general form of planning?
  - Given a truck and an unknown number of objects, make them all at their desired destination!
  - Incomplete knowledge about number results in infinitly many cases.

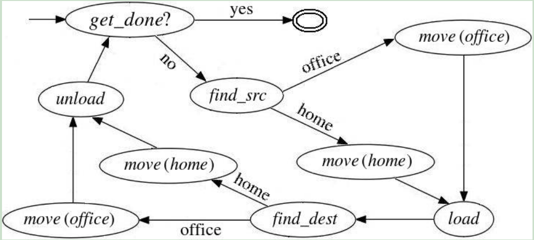
Motivation

An intuitive plan:



Motivation

An intuitive plan:



Moral:

- With this generality, **plans with loops** are needed!

Outline of the Talk

1. Planning with Loops
2. A Formal Notion of Correctness
3. Practical Verification for 1-D Problems
4. Conclusion

Planning with Loops

FSAPLANNER (Hu & Levesque 09) generates plans with loops by

1. generating a plan with loops that works for small instances;
2. testing if the plan also works for other instances.
   (If not, return to Step 1.)

Planning with Loops

FSAPLANNER (Hu & Levesque 09) generates plans with loops by

1. generating a plan with loops that works for small instances;
2. testing if the plan also works for all (?) other instances.
   (If not, return to Step 1.)

Plan Verification...

- Ideally, a candidate plan may pass the testing phase, only if it works for *all* instances of the planning problem.

Planning with Loops

FSAPLANNER (Hu & Levesque 09) generates plans with loops by

1. generating a plan with loops that works for small instances;
2. testing if the plan also works for all (?) other instances.
   (If not, return to Step 1.)

Plan Verification...

- Ideally, a candidate plan may pass the testing phase, only if it works for *all* instances of the planning problem.
- However, this seems impossible with infinitely many cases.

Planning with Loops

FSAPLANNER (Hu & Levesque 09) generates plans with loops by

1. generating a plan with loops that works for small instances;
2. testing if the plan also works for some other instances.
   (If not, return to Step 1.)

Plan Verification...

- Ideally, a candidate plan may pass the testing phase, only if it works for *all* instances of the planning problem.
- However, this seems impossible with infinitely many cases.
- In practice, we only test against *finitely many* larger instances.

Planning with Loops

FSAPLANNER (Hu & Levesque 09) generates plans with loops by

1. generating a plan with loops that works for small instances;
2. testing if the plan also works for some other instances. (If not, return to Step 1.)

Plan Verification...

- Ideally, a candidate plan may pass the testing phase, only if it works for *all* instances of the planning problem.
- However, this seems impossible with infinitely many cases.
- In practice, we only test against *finitely many* larger instances.

Needed: finite verification with general correctness guarantee!

Contributions

In this paper, we

1. formally define a representation (FSA plan) for plans with loops;
2. identify a class of (one-dimensional) planning problems whose plan correctness can be finitely verified;
3. show that this verification algorithm enables FSAPLANNER to efficiently generate provably correct plans for this problem class.

The Situation Calculus

The situation calculus is a multi-sorted logic for modeling dynamic environments, with sorts *situation*, *action* and *object*.

- S_0 is the unique initial situation, and $do(a, s)$ is the situation obtained by performing action a in situation s.
- Changing properties modeled by fluents, *i.e.*, functions and predicates whose last argument is a situation term, *e.g.*,

$$loc(S_0) = home \wedge Loaded(do(load, S_0)).$$

- $Poss(a, s)$ is a special relation that holds iff action a is executable in situation s.
- $\mathrm{SR}(a, s)$ denotes the sensing result of action a when performed in situation s.

Problem Representation

The dynamics of a planning problem is axiomatized by a Basic Action Theory (Reiter 01)

$$\Sigma = \mathcal{F}\mathcal{A} \cup \Sigma_{una} \cup \Sigma_{pre} \cup \Sigma_{ssa} \cup \Sigma_{sr} \cup \Sigma_0,$$

Problem Representation

The dynamics of a planning problem is axiomatized by a Basic Action Theory (Reiter 01)

$$\Sigma = \mathcal{FA} \cup \Sigma_{una} \cup \Sigma_{pre} \cup \Sigma_{ssa} \cup \Sigma_{sr} \cup \Sigma_0,$$

where Σ_{sr} is a set of sensing result axioms (Scherl & Levesque 03):

$$\begin{aligned} \mathrm{SR}(get\_done, s) = r \ \equiv\ & r = yes \wedge parcels\_left(s) = 0 \ \vee \\ & r = no \wedge parcels\_left(s) \neq 0. \end{aligned}$$

Problem Representation

The dynamics of a planning problem is axiomatized by a Basic Action Theory (Reiter 01)

$$\Sigma = \mathcal{FA} \cup \Sigma_{una} \cup \Sigma_{pre} \cup \Sigma_{ssa} \cup \Sigma_{sr} \cup \Sigma_0,$$

where Σ_{sr} is a set of sensing result axioms (Scherl & Levesque 03):

$$\begin{aligned} \mathrm{SR}(get\_done, s) = r \;\equiv\; & r = yes \wedge parcels\_left(s) = 0 \;\vee \\ & r = no \wedge parcels\_left(s) \neq 0. \end{aligned}$$

Definition

A planning problem is a pair $\langle \Sigma, G \rangle$, where Σ is a basic action theory and G is a situation-suppressed goal formula.

Problem Representation

The dynamics of a planning problem is axiomatized by a Basic Action Theory (Reiter 01)

$$\Sigma = \mathcal{F}\mathcal{A} \cup \Sigma_{una} \cup \Sigma_{pre} \cup \Sigma_{ssa} \cup \Sigma_{sr} \cup \Sigma_0,$$

where Σ_{sr} is a set of sensing result axioms (Scherl & Levesque 03):

$$\begin{aligned} \mathrm{SR}(get\_done, s) = r \ \equiv\ & r = yes \wedge parcels\_left(s) = 0 \ \vee \\ & r = no \wedge parcels\_left(s) \neq 0. \end{aligned}$$

Definition

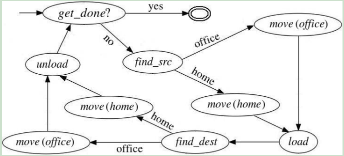
A planning problem is a pair $\langle \Sigma, G \rangle$, where Σ is a basic action theory and G is a situation-suppressed goal formula.

Both infinite domain and incomplete initial state allowed.

Plan Representation

We use a finite-state automaton-like plan representation (called FSA plan), which can be viewed as a directed graph, where

- Each node represents a program state
  - One unique "start state"
  - One unique "final state"
  - Non-final states associated with action



- Each edge labeled with a sensing result (omitted for non-sensing).

Plan Representation

To formalize FSA plans, we introduce a new sort "program states" with Q_0 and Q_F being two constants, and a set of axioms *FSA*, consisting of

1. domain closure axioms for program states

$$(\forall q).q = Q_0 \vee q = Q_1 \vee \cdots \vee q = Q_n \vee q = Q_F;$$

2. unique names axioms for program states

$$Q_i \neq Q_j \text{ for } i \neq j;$$

3. action association axioms

$$\gamma(Q) = A;$$

4. transition axioms

$$\delta(Q, R) = Q'.$$

Plan Correctness

We use $T(q, s, q', s')$ to denote legal one-step transitions, *i.e.*,

$$\begin{aligned} T(q, s, q', s') \stackrel{def}{=} \; & \exists a, r.\, \gamma(q) = a \wedge Poss(a, s) \wedge \mathrm{SR}(a, s) = r \wedge \\ & \delta(q, r) = q' \wedge s' = do(a, s) \end{aligned}$$

Plan Correctness

We use $T(q, s, q', s')$ to denote legal one-step transitions, *i.e.*,

$$T(q, s, q', s') \stackrel{def}{=} \exists a, r.\ \gamma(q) = a \wedge Poss(a, s) \wedge \mathrm{SR}(a, s) = r \wedge \\ \delta(q, r) = q' \wedge s' = do(a, s)$$

$T^{\star}(q, s, q', s')$ denotes the reflexive transitive closure of T, *i.e.*, $T^{\star}(q, s, q', s')$ is true iff starting from program state q and situation s, the FSA plan may reach state q' and situation s'

Plan Correctness

$T^\star(q, s, q', s')$ denotes the reflexive transitive closure of T, *i.e.*, $T^\star(q, s, q', s')$ is true iff starting from program state q and situation s, the FSA plan may reach state q' and situation s', then plan correctness is defined by:

Definition

Given a planning problem $\langle \Sigma, G \rangle$, where Σ is an action theory and G is a goal formula, a plan axiomatized by FSA is correct iff

$$\Sigma \cup FSA \models \exists s. T^\star(Q_0, S_0, Q_F, s) \wedge G[s].$$

Plan Correctness

$T^\star(q, s, q', s')$ denotes the reflexive transitive closure of T, *i.e.*, $T^\star(q, s, q', s')$ is true iff starting from program state q and situation s, the FSA plan may reach state q' and situation s', then plan correctness is defined by:

Definition

Given a planning problem $\langle \Sigma, G \rangle$, where Σ is an action theory and G is a goal formula, a plan axiomatized by FSA is correct iff

$$\Sigma \cup FSA \models \exists s. T^\star(Q_0, S_0, Q_F, s) \wedge G[s].$$

Need Second-Order Reasoning!

One-Dimensional Planning Problems

A planning problem $\langle \Sigma, G \rangle$ is *one-dimensional* if (intuitively)

- Only one fluent p (called the *planning parameter*) may take unbounded values from natural numbers;
- All fluents other than p take values from a finite set V.
- Initially, p may be arbitrary natural number.
- The only effect on p is to decrease it by one, *i.e.*,

$$\begin{aligned} p(do(a,s)) = x \equiv\, & x = p(s) - 1 \wedge Dec(a) \vee \\ & x = p(s) \wedge \neg Dec(a). \end{aligned}$$

- The only primitive test involving p in Σ and G is $p = 0$.

Intuitions on Finite Verifiability

- Suppose we are given a one-dimensional planning problem and a candidate FSA plan.

Intuitions on Finite Verifiability

- Suppose we are given a one-dimensional planning problem and a candidate FSA plan.
- We have verified that the FSA plan correctly achieves the goal for

$$p = 0, 1, 2, \cdots, N.$$

Intuitions on Finite Verifiability

- Suppose we are given a one-dimensional planning problem and a candidate FSA plan.
- We have verified that the FSA plan correctly achieves the goal for

$$p = 0, 1, 2, \cdots, N.$$

- Can we now conclude that the FSA plan is correct in general??

The Main Theorem

Theorem

Suppose $\langle \Sigma, G \rangle$ is a one-dimensional planning problem with planning parameter p, and FSA axiomatize an FSA plan. Then there is an N_0 such that

$$\textit{If} \quad \Sigma \cup FSA \cup \{p(S_0) \leq N_0\} \models \exists s.\, T^\star(Q_0, S_0, Q_F, s) \wedge G[s],$$
$$\textit{then} \quad \Sigma \cup FSA \models \exists s.\, T^\star(Q_0, S_0, Q_F, s) \wedge G[s].$$

The Main Theorem

Theorem

Suppose $\langle \Sigma, G \rangle$ is a one-dimensional planning problem with planning parameter p, and FSA axiomatize an FSA plan. Then there is an N_0 such that

$$\begin{array}{lll} \textit{If} & \Sigma \cup FSA \cup \{p(S_0) \leq N_0\} & \models \exists s.\, T^{\star}(Q_0, S_0, Q_F, s) \wedge G[s], \\ \textit{then} & \Sigma \cup FSA & \models \exists s.\, T^{\star}(Q_0, S_0, Q_F, s) \wedge G[s]. \end{array}$$

The Main Theorem

Theorem

Suppose $\langle \Sigma, G \rangle$ is a one-dimensional planning problem with planning parameter p, and FSA axiomatize an FSA plan. Then there is an N_0 such that

$$\textit{If} \quad \Sigma \cup FSA \cup \{p(S_0) \leq N_0\} \models \exists s.\, T^\star(Q_0, S_0, Q_F, s) \wedge G[s],$$
$$\textit{then} \quad \Sigma \cup FSA \models \exists s.\, T^\star(Q_0, S_0, Q_F, s) \wedge G[s].$$

In particular, $N_0 = 1 + k \cdot m^{|V|}$, where

- m is the number of finite fluents in Σ;
- each such fluent may take at most $|V|$ different values;
- k is the number of the program states in the FSA plan.

Proof Sketch

Suppose, for the sake of contradiction, that there is a smallest $N > N_0$ such that if we start from $p = N$ the FSA plan fails.

$$\left.\begin{array}{ll}
\langle \vec{b}, N, Q_0 \rangle \rightarrow & \langle \vec{b}^{(N_0)}, N, q^{(N_0)} \rangle \rightarrow \\
& \langle \vec{b}^{(N_0-1)}, N-1, q^{(N_0-1)} \rangle \rightarrow \\
& \quad \cdots \qquad \cdots \\
& \langle \vec{b}^{(2)}, N-N_0+2, q^{(2)} \rangle \rightarrow \\
& \langle \vec{b}^{(1)}, N-N_0+1, q^{(1)} \rangle \rightarrow \quad \mathrm{FAIL}
\end{array}\right\} 1 + k \cdot m^{|V|}$$

Proof Sketch

Suppose, for the sake of contradiction, that there is a smallest $N > N_0$ such that if we start from $p = N$ the FSA plan fails.

$$
\langle \vec{b}, N, Q_0 \rangle \rightarrow \quad \left.\begin{array}{l}
\langle \vec{b}^{(N_0)}, N, q^{(N_0)} \rangle \rightarrow \\
\langle \vec{b}^{(N_0-1)}, N-1, q^{(N_0-1)} \rangle \rightarrow \\
\qquad \cdots \qquad \cdots \\
\langle \vec{b}^{\star}, u, q^{\star} \rangle \rightarrow \\
\qquad \cdots \qquad \cdots \\
\langle \vec{b}^{\star}, v, q^{\star} \rangle \rightarrow \\
\qquad \cdots \qquad \cdots \\
\langle \vec{b}^{(2)}, N-N_0+2, q^{(2)} \rangle \rightarrow \\
\langle \vec{b}^{(1)}, N-N_0+1, q^{(1)} \rangle \rightarrow \quad \text{FAIL}
\end{array}\right\} 1 + k \cdot m^{|V|}
$$

Proof Sketch

Suppose, for the sake of contradiction, that there is a smallest $N > N_0$ such that if we start from $p = N$ the FSA plan fails.

$$\langle \vec{b}, N, Q_0 \rangle \rightarrow \left.\begin{array}{ll} \langle \vec{b}^{(N_0)}, N, q^{(N_0)} \rangle \rightarrow & \\ \langle \vec{b}^{(N_0-1)}, N-1, q^{(N_0-1)} \rangle \rightarrow & \\ \quad \ldots \quad \ldots & \\ \langle \vec{b}^{\star}, u, q^{\star} \rangle \rightarrow & \\ \quad \ldots \quad \ldots & \\ \langle \vec{b}^{\star}, v, q^{\star} \rangle \rightarrow & \\ \quad \ldots \quad \ldots & \\ \langle \vec{b}^{(2)}, N - N_0 + 2, q^{(2)} \rangle \rightarrow & \\ \langle \vec{b}^{(1)}, N - N_0 + 1, q^{(1)} \rangle \rightarrow & \text{FAIL} \end{array}\right\} 1 + k \cdot m^{|V|}$$

Then $\langle \vec{b}, N - (u - v), Q_0 \rangle \rightarrow \text{FAIL}$ too!

Proof Sketch

Suppose, for the sake of contradiction, that there is a smallest $N > N_0$ such that if we start from $p = N$ the FSA plan fails.

$$
\langle \vec{b}, N, Q_0 \rangle \rightarrow \left.\begin{array}{ll}
\langle \vec{b}^{(N_0)}, N, q^{(N_0)} \rangle \rightarrow & \\
\langle \vec{b}^{(N_0-1)}, N-1, q^{(N_0-1)} \rangle \rightarrow & \\
\quad \cdots \quad \cdots & \\
\langle \vec{b}^{\star}, u, q^{\star} \rangle \rightarrow & \\
\quad \cdots \quad \cdots & \\
\langle \vec{b}^{\star}, v, q^{\star} \rangle \rightarrow & \\
\quad \cdots \quad \cdots & \\
\langle \vec{b}^{(2)}, N-N_0+2, q^{(2)} \rangle \rightarrow & \\
\langle \vec{b}^{(1)}, N-N_0+1, q^{(1)} \rangle \rightarrow & \mathrm{FAIL}
\end{array}\right\} 1 + k \cdot m^{|V|}
$$

Then $\langle \vec{b}, N-(u-v), Q_0 \rangle \rightarrow \mathrm{FAIL}$ too! Contradiction.

Towards a Tighter Bound

- N_0 is exponential and thus impractical for many cases.
- We proposed an algorithmically obtained bound N_t, which is usually much smaller than N_0:
  - Verify that the FSA plan is correct for $p = 0, 1, 2, \cdots$.
  - Until for some N_t,

$$\langle \vec{b}, N_t, Q_0 \rangle \rightarrow \langle \vec{b}^\star, u, q^\star \rangle \rightarrow \langle \vec{b}^\star, v, q^\star \rangle \rightarrow \mathrm{SUCCESS}.$$

Experimental Results

We used the different bounds in the test phase of FSAPLANNER on four one-dimensional planning problems (`treechop`, `variegg`, `safe` and `logistic`).

Problem	treechop	variegg	safe	logistic
N_{man}*	100	6	4	5
Time (secs)	0.1	0.12	0.09	3.93
N_0	18	345	4098	514
Time (secs)	0.03	>1 day	>1 day	>1 day
N_t	2	3	2	2
Time (secs)	0.01	0.08	0.08	3.56

*: N_{man} is the manually estimated test bound without correctness guarantee.

Conclusion and Future Work

Planning with loops is an interesting and challenging problem.
In this paper, we

- define a generalized plan representation that allows loops;
- give a formal notion of plan correctness under this representation;
- identify the class of one-dimensional problems whose correctness can be finitely verified;
- show that a planner based on this theoretical result efficiently generates provably correct plans for one-dimensional problems.

Conclusion and Future Work

Planning with loops is an interesting and challenging problem.
In this paper, we

- define a generalized plan representation that allows loops;
- give a formal notion of plan correctness under this representation;
- identify the class of one-dimensional problems whose correctness can be finitely verified;
- show that a planner based on this theoretical result efficiently generates provably correct plans for one-dimensional problems.

Future work:

- Investigate correctness guarantee for more general classes.

Related Work

- Simple problems for KPLANNER (Levesque 2005);
- Goal achievability for rank 1 theories (Lin 2008);
- Extended-LL problems (Srivastava *et al.* 2008);
- Abacus programs (Srivastava *et al.* 2010);
- Deductive approaches (Manna&Waldinger 1987, Magnusson&Doherty 2008);
- Weak guarantee (Winner&Veloso 2007, Bonet *et al.* 2009);
- Model checking (Clarke *et al.* 1999).