A Correctness Result for Reasoning about One-Dimensional Planning Problems

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Motivation

- Classical planning produces action sequences in complete worlds.
  - *e.g.*: given, *obj1* at home, *obj2* in office and a *truck*, make *obj1* be in office and *obj2* at home.
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  - *e.g.*: given a truck, *obj1* and *obj2*, location and destination unknown, make both objects be at their destinations.
  - Resulting tree-like plan can handle 16 different cases.
- An even more general form of planning?
  - Given a truck and an unknown number of objects, make them all be at their desired destination!
  - Incomplete knowledge about number results in infinitely many cases.

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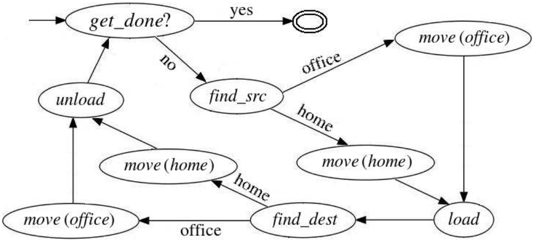
An intuitive plan:



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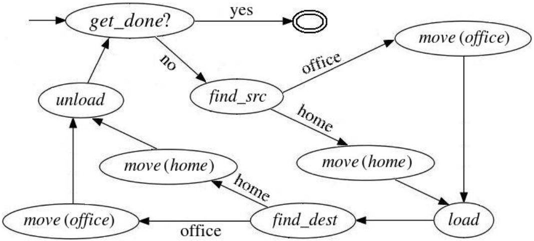

Questions:

1. How do we characterize a planning problem that requires loopy plans?
2. What exactly is a plan with loops?
3. When is a plan "correct" for a problem?
4. ...

Outline of the Talk

Planning with Loops

FSAPLANNER (Hu & Levesque 09) generates plans with loops by

1. generating a plan with loops that works for small instances;
2. testing if the plan also works for other instances.
   (If not, return to Step 1.)

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Plan Verification...

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Needed: finite verification with general correctness guarantee!

Contributions

In this paper, we

1. formally define a representation (FSA plan) for plans with loops;
2. identify a class of (one-dimensional) planning problems whose plan correctness can be finitely verified;
3. show that this verification algorithm enables FSAPLANNER to efficiently generate provably correct plans for this problem class.

The Situation Calculus

The situation calculus is a multi-sorted logic for modeling dynamic environments, with sorts *situation*, *action* and *object*.

- S_0 is the unique initial situation, and $do(a, s)$ is the situation obtained by performing action a in situation s.
- Changing properties modeled by fluents, *i.e.*, functions and predicates whose last argument is a situation term, *e.g.*,

$$loc(S_0) = home \wedge Loaded(do(load, S_0)).$$

- $Poss(a, s)$ is a special relation that holds iff action a is executable in situation s.
- $\mathrm{SR}(a, s)$ denotes the sensing result of action a when performed in situation s.

Problem Representation

The dynamics of a planning problem is axiomatized by a Basic Action Theory (Reiter 01) with sensing (Scherl & Levesque 03)

$$\Sigma = \mathcal{F}\mathcal{A} \cup \Sigma_{una} \cup \Sigma_{pre} \cup \Sigma_{ssa} \cup \Sigma_{sr} \cup \Sigma_0 .$$

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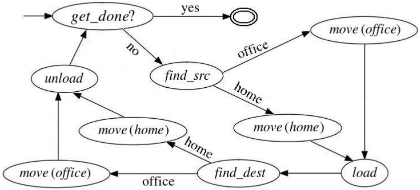
Both infinite domain and incomplete initial state allowed.

Plan Representation

We use a finite-state automaton-like plan representation (called FSA plan), which can be viewed as a directed graph, where

- Each node represents a program state
  - One unique "start state"
  - One unique "final state"
  - Non-final states associated with action



- Each edge labeled with a sensing result (omitted for non-sensing).

Plan Representation

To formalize FSA plans, we introduce a new sort "program states" with Q_0 and Q_F being two constants, and a set of axioms FSA, consisting of

1. domain closure axioms for program states

$$(\forall q).q = Q_0 \vee q = Q_1 \vee \cdots \vee q = Q_n \vee q = Q_F;$$

2. unique names axioms for program states

$$Q_i \neq Q_j \text{ for } i \neq j;$$

3. action association axioms

$$\gamma(Q) = A;$$

4. transition axioms

$$\delta(Q, R) = Q'.$$

Plan Correctness

We use $T(q, s, q', s')$ to denote legal one-step transitions, *i.e.*,

$$T(q, s, q', s') \stackrel{def}{=} \exists a, r.\ \gamma(q) = a \wedge Poss(a, s) \wedge \mathrm{SR}(a, s) = r \wedge \\ \delta(q, r) = q' \wedge s' = do(a, s)$$

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$T^\star(q, s, q', s')$ denotes the reflexive transitive closure of T, *i.e.*, $T^\star(q, s, q', s')$ is true iff starting from program state q and situation s, the FSA plan may reach state q' and situation s'

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Definition

Given a planning problem $\langle \Sigma, G \rangle$, where Σ is an action theory and G is a goal formula, a plan axiomatized by *FSA* is correct iff

$$\Sigma \cup FSA \models \exists s. T^\star(Q_0, S_0, Q_F, s) \wedge G[s].$$

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Need Second-Order Reasoning!

One-Dimensional Planning Problems

A planning problem $\langle \Sigma, G \rangle$ is *one-dimensional* if (intuitively)

- Only one fluent p (called the *planning parameter*) may take unbounded values from natural numbers;
- All functions other than p take values from a finite set, and apart from a possible situation argument
  - either have no argument (finite functions),
  - or have p as its argument (sequence functions);
- Initially, p may be an arbitrary natural number;
- The only effect on p is to decrease it by one, *i.e.*,

$$p(do(a,s)) = x \equiv x = p(s) - 1 \wedge Dec(a) \vee \\ x = p(s) \wedge \neg Dec(a);$$

- The only primitive test involving p in Σ and G is $p = 0$.

Intuitions on Finite Verifiability

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- We have verified that the FSA plan correctly achieves the goal for

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- Can we now conclude that the FSA plan is correct in general??

The Main Theorem

Theorem

Suppose $\langle \Sigma, G \rangle$ is a one-dimensional planning problem with planning parameter p, and FSA axiomatizes an FSA plan. Then there is an N_0 such that

$$\begin{array}{ll} \textit{If} & \Sigma \cup FSA \cup \{p(S_0) \leq N_0\} \models \exists s.\, T^\star(Q_0, S_0, Q_F, s) \wedge G[s], \\ & \qquad \textit{then} \qquad \Sigma \cup FSA \models \exists s.\, T^\star(Q_0, S_0, Q_F, s) \wedge G[s]. \end{array}$$

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In particular, $N_0 = 2 + k \cdot l^m$, where

- m is the number of finite and sequence functions in Σ;
- each such fluent may take at most l different values;
- k is the number of the program states in the FSA plan.

Towards a Tighter Bound

- N_0 is exponential and thus impractical for many cases.
- We proposed an algorithmically obtained bound N_t (Theorem 2), which is usually much smaller than N_0, such that

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With Theorem 2, we can verify that an FSA plan is correct in general by verifying that it is correct for $p(S_0) = 0, 1, 2, \cdots N_t$.

Experimental Results

We used the different bounds in the test phase of FSAPLANNER on four one-dimensional planning problems (`treechop`, `variegg`, `safe` and `logistic`).

Problem	treechop	variegg	safe	logistic
N_{man}*	100	6	4	5
Time (secs)	0.1	0.12	0.09	3.93
N_0	18	345	4098	514
Time (secs)	0.03	>1 day	>1 day	>1 day
N_t	2	3	2	2
Time (secs)	0.01	0.08	0.08	3.56

*: N_{man} is the manually estimated test bound without correctness guarantee.

Conclusion and Future Work

Planning with loops is an interesting and challenging problem.
In this paper, we

- define a generalized plan representation that allows loops;
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Future work:

- Investigate correctness guarantee for more general classes.

Related Work

- Simple problems for KPLANNER (Levesque 2005);
- Goal achievability for rank 1 theories (Lin 2008);
- Extended-LL problems (Srivastava *et al.* 2008);
- Abacus programs (Srivastava *et al.* 2010);
- Deductive approaches (Manna&Waldinger 1987, Magnusson&Doherty 2008);
- Weak guarantee (Winner&Veloso 2007, Bonet *et al.* 2009);
- Model checking (Clarke *et al.* 1999).