Generalized Planning: Synthesizing Plans that Work for Multiple Environments

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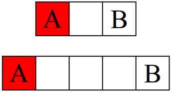
Classical Planning

Classical planning: Given a deterministic dynamic system with a single known initial state and a goal condition, find a plan that achieves the goal.

For example, starting from the leftmost cell A of a linear grid with left and right move actions, act to be in the rightmost cell B.

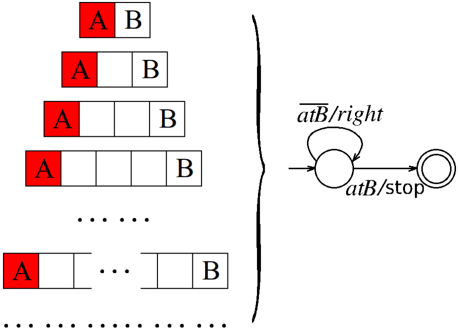


right, right

right, right, right, right

Generalized Planning

Generalized planning: Find a plan that achieves the goal for a number of "similar" planning problems.



Motivation

Existing work:

- [Srivastava, Immerman and Zilberstein 2008; 2011]
  Learning generalized plans using abstract counting.
- [Bonet, Palacios and Geffner 2009]
  Automatic derivation of memoryless policies and fnite-state controllers using classical planners.
- [Hu and Levesque 2010]
  A correctness result for reasoning about one-dimensional planning problems.
- ...

Question: Is there a general definition of generalized planning independent of any specific representation formalism?

Definitions

Two aspects need to be considered for planning:

1. The **agent** $A = \langle Acts, Obs \rangle$ that executes the plan;

   A plan is then a (prefix-closed) partial function $p : Obs^* \to Acts \cup \{\texttt{stop}\}$, often in a less general, finite-state form.

   Plans are independent of the environment.

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2. The **environment** $E = \langle Events, S, s_0, \delta \rangle$ in which the plan is executed.

   A trace is a sequence $t = s_0 e_1 s_1 e_2 \cdots e_n s_n$, where
   - s_0 is the initial state, and
   - $s_i = \delta(s_{i-1}, e_i)$.

   A goal is a specification of desired traces in E. For now: $last(t) \in G \subseteq S$.

   Goals are independent of the agent.

Definitions

To relate agent $A = \langle Acts, Obs \rangle$ and environment $E = \langle Events, S, s_0, \delta \rangle$:

- an observation function $obs : S \to Obs$,
- an execution function $exec : Acts \to Events$.

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A run r of plan p is the trace $trace(p) = s_0 e_1 s_1 e_2 \cdots e_n s_n$, such that

$$\begin{aligned} e_1 &= exec\big(p(obs(s_0))\big), \\ e_2 &= exec\big(p(obs(s_0), obs(s_1))\big), \\ &\cdots \quad \cdots \\ e_i &= exec\big(p(obs(s_0), \cdots, obs(s_{i-1}))\big). \end{aligned}$$

r is complete if $p(obs(s_0), \cdots, obs(s_n)) = \texttt{stop}$.

Definitions

A **basic planning problem** is a tuple

$$P = \langle \mathit{Acts}, \mathit{Obs}, \mathit{Events}, S, s_0, \delta, G, \mathit{obs}, \mathit{exec} \rangle.$$

A plan p is a solution to P iff $r = \mathit{trace}(p)$ is a complete run, and the final state $\mathit{last}(r) \in G$.

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A **generalized planning problem** $\mathcal{P} = \{P_1, P_2, \cdots\}$ is a possibly infinite set of basic planning problems that share the same agent, *i.e.*,

$$P_i = \langle Acts, Obs, Events_i, S_i, s_{i0}, \delta_i, G_i, obs_i, exec_i\rangle.$$

A plan p is a solution to $\mathcal{P}$ iff p is a solution for all $P_i \in \mathcal{P}$.

The Linear Grid Example

Now we can capture the linear grid example as a generalized planning problem

$$\mathcal{P} = \{P_2, P_3, \cdots\},$$

where each $P_i = \langle Acts, Obs, Events_i, S_i, s_{i0}, \delta_i, G_i, obs_i, exec_i \rangle$ is a basic planning problem with i cells

- modeled completely independently
  (*e.g.*, using one proposition per celll or using a planning parameter),
- except that all P_i share the same *Acts* and *Obs*.

Standard Planning

Classical Planning

- Generalized planning with singleton set $\mathcal{P} = \{P\}$.
- The plan does not use the history of observations, except for its size: $p(o_1, \cdots, o_n) = p(n)$. (Sequential plan.)
- PSPACE-complete assuming a logarithmic encoding of the states (Bylander 1994).

Conditional Planning

- Generalized planning with finite set $\mathcal{P} = \{P_1, \cdots, P_N\}$, where $P_i = \langle Acts, Obs, Events, S, s_{i0}, \delta, G, obs, exec \rangle$ are identical except for s_{i0}.
- EXPSPACE-complete assuming a logarithmic encoding of the states (Haslum and Jonsson 1999; Rintanen 2004).

Conformant Planning

- Same as conditional, except $Obs = \{nil\}$ and $obs(s) = nil$ for all s.

Generalized Planning: Finite Case

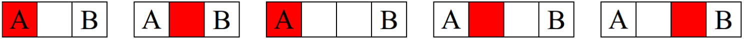
In general, for any finite set $\mathcal{P} = \{P_1, \cdots, P_N\}$, a plan can always be found if one exists.

How? Compile into a classical planning problem $P_{\mathcal{P}}$ with goal $G_{\mathcal{P}}$:
(similar in spirit to [Palacios and Geffner 2009] and [Albore *et al.* 2009])

- the actions are N-vectors of the action alphabet plus no-op;
- the state space is the cross-product of the N state spaces, augmented by
  - an equivalence relation
    (to remember for which problems identical actions must be prescribed),
  - an achievement set
    (to remember for which problems has the goal been achieved);
- the goal is to make every problem in the achievement set;
- no observation;
- the initial state, transition functions and other parts constructed accordingly.

Generalized Planning: Finite Case

An Example



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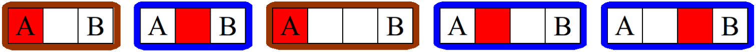

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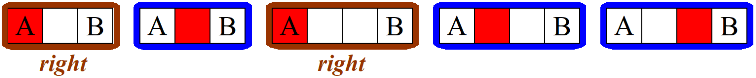

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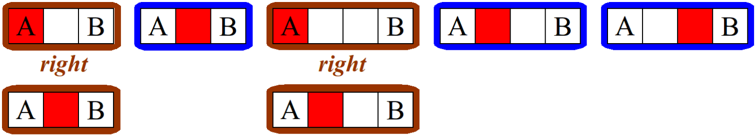

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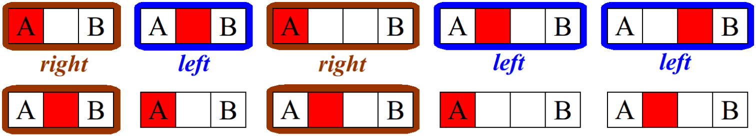

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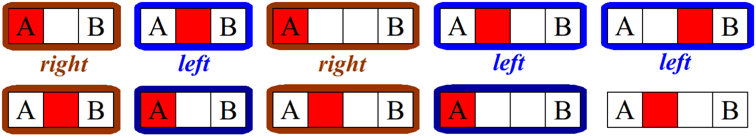

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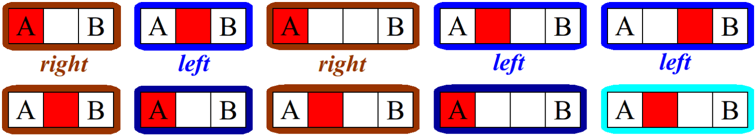

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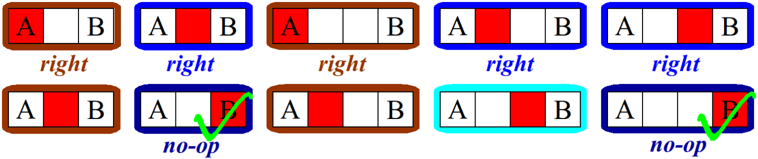

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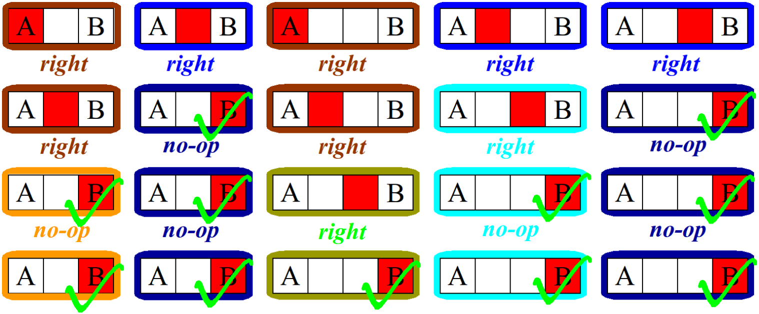

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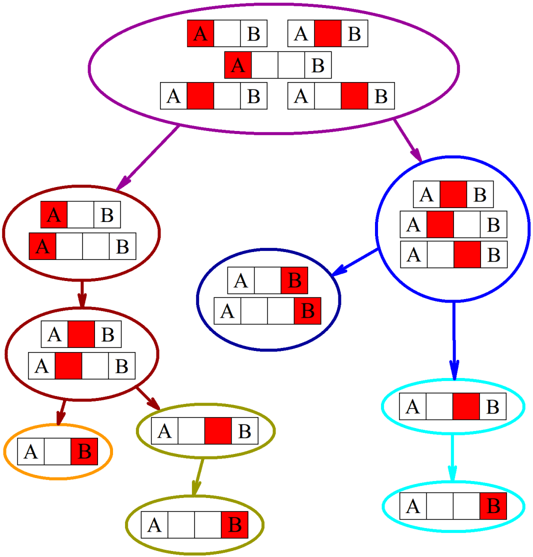

Generalized Planning: Finite Case

Theorem (Soundness and Completeness)

A generalized plan for G in the problem set $\mathcal{P}$ exists iff a classical plan exists for $G_{\mathcal{P}}$ in $P_{\mathcal{P}}$.

Theorem (Complexity)

Generalized planning for finite environments is PSPACE-complete (EXPSPACE-complete assuming a logarithmic encoding of the states).

Generalized Planning: Infinite Case

Existing approaches to generalized planning are essentially planning for infinite environment sets.

To model in our framework

- [Bonet *et al.* 2009]: Create a basic planning problem for each of their problem instances.
- [Hu and Levesque 2010]: Create a basic planning problem for each interpretation of their basic action theory.
- [Srivastava *et al.* 2008]: Create multiple basic planning problems for each of their problem instance, simulating all possible concretization of the abstract actions.

Note that all three approaches use a finite-state plan representation.

Finite Verifiability for One-Dimensional Problems

A reformulation (assuming no sequence functions) of the existing result in [Hu and Levesque 2010]:

A generalized planning problem $\mathcal{P} = \{P_0, P_1, P_2, \cdots\}$ is one-dimensional (simplified), or 1ds, if all $P_i = \langle Acts, Obs, Events, S_i, s_{i0}, \delta_i, G, obs_i, exec\rangle$ satisfy

- $S_i = \{\langle n, \vec{b}\rangle \mid n \in \{0, \cdots, i\}, \vec{b} \in \{\mathrm{true}, \mathrm{false}\}^m\}$;
- $s_{i0} = \langle i, \vec{b}_0\rangle$;
- if $\delta_i(\langle n, \vec{b}\rangle, e) = \langle n - d, \vec{b}'\rangle$ for $n > 0$, then $d \in \{0, 1\}$; furthermore, for all $n' > 0$, $\delta_i(\langle n', \vec{b}\rangle, e) = \langle n' - d, \vec{b}'\rangle$;
- $G \subseteq \{0\} \times \{\mathrm{true}, \mathrm{false}\}^m$;
- $obs_i(\langle n_1, \vec{b}\rangle) = obs_i(\langle n_2, \vec{b}\rangle)$ for all $n_1, n_2 > 0$.

Theorem

Given a 1ds problem $\mathcal{P} = \{P_0, P_1, \cdots\}$ that shares a final-state goal G, and a finite-state plan p with l states, if p is a plan for all of $\{P_0, P_1, \cdots, P_N\}$, where $N = l \cdot 2^m + 1$, then p is a plan for $\mathcal{P}$.

New Insights into One-Dimensional Problems

Is it possible that there exists a generalized plan but no finite-state plan?

Theorem

If a 1ds generalized planning problem has a generalized plan, then it has a finite-state plan.

This justifies the use of finite-state plans, and makes planning for 1ds problems at least recursively enumerable. Even better:

Theorem

A 1ds generalized planning problem $\mathcal{P} = \{P_0, P_1, \cdots\}$ *has a plan, if and only if the finite subset* $\mathcal{P}' = \{P_0, P_1, \cdots, P_N\}$ *has a plan where* $N = 2^m$.

So we can use, *e.g.*, compilation to classical planning, to solve 1ds problems.

Theorem

Generalized planning for 1ds problems is EXPSPACE.

Summary

Contributions

- Proposed a framework to represent and analyze generalized planning; (capturing existing formalisms of standard and generalized planning)
- Proved that generalized planning for finite sets is EXPSPACE- complete;
- Showed 1ds problem is EXPSPACE, and solvable by planning for finite sets.

Future Work

- Use the framework to identify more general problem classes with interesting correctness and computational properties (*e.g.*, full 1d and more);
- Consider more general goals (*e.g.*, temporally-extended and long-running goals);
- Consider unrelated concrete actions with shared abstract actions.