An Equational Proof

Let’s prove that if \((x, y) = 1\) then \((x + y, xy) = 1\). The proof is mere calculation. It’s given that some integers \(a\) and \(b\) satisfy \(ax + by = 1\). Thus

\[
1 = (ax + by)^2 = a^2 x^2 + b^2 y^2 + 2abxy = a^2(x^2 + xy) + b^2(y^2 + xy) - (a^2 + b^2 - 2ab)xy = (a^2 x + b^2 y)(x + y) - (a - b)^2 xy.
\]

A Boring Proof

Let’s prove that if \((x, y) = 1\) then \((x + y, xy) = 1\). The proof is through the fundamental theorem of arithmetic. Suppose that the prime \(p\) divides \(xy\). It must divide one of \(x\) and \(y\), say \(x\). As \((x, y) = 1\), it cannot divide \(y\), hence cannot divide \(x + y\).