CSC263 Winter 2015

Data Structures and Analysis

Section L0301

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Outline for today

- **Why** take CSC263?
- **What** is in CSC263?
- **How** to do well in CSC263?

Why take CSC263?

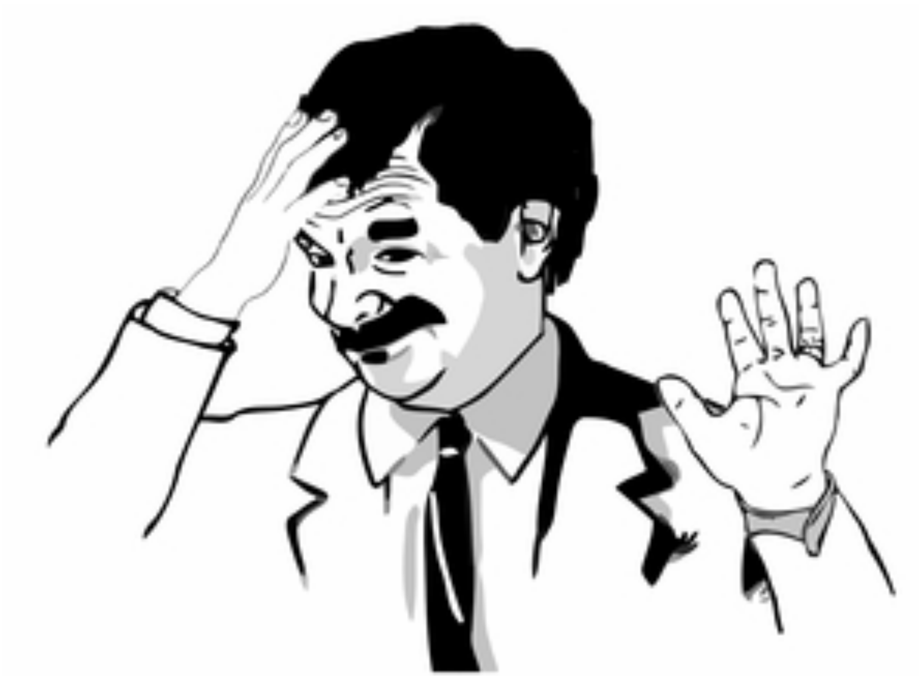
To land a job!

Scenario: The interview

Interviewer: You are given a set of courses, like *CSC165*, *STA247, CSC263, CSC373,* where each course stores a list of prerequisites. Devise an algorithm that returns a valid ordering of taking these courses.

You: (think for 1 minute…) Here is my algorithm:

- ➔ For a valid ordering to exist, there must be a course **X** that has no prerequisite.
- ➔ I choose **X** first, remove **X** from the set of courses, then remove **X** from all other courses' prerequisite list.
- ➔ Find the next course in the set that has no prerequisite.
- ➔ Repeat this until all courses are removed from the set.

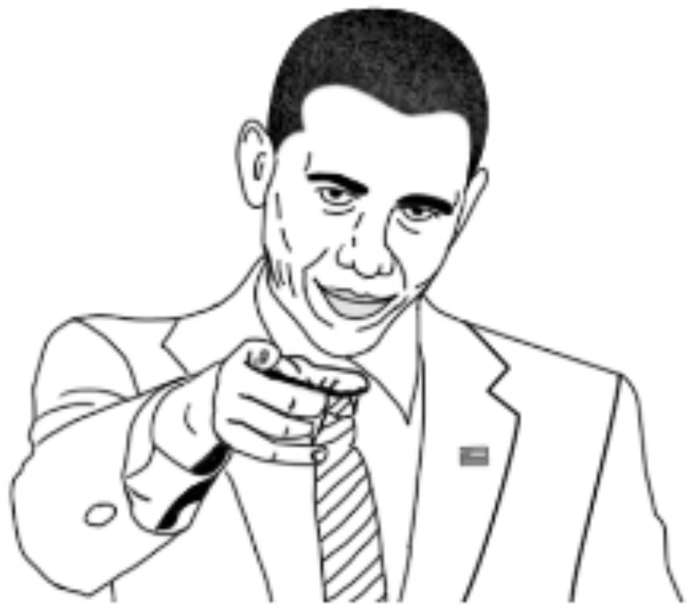


SHUT UP AND GO HOME!

Scenario: The interview, Take 2

Interviewer: You are given a set of courses, like *CSC165*, *STA247, CSC263, CSC373,* where each course stores a list of prerequisites. Devise an algorithm that produces a valid ordering of taking these courses.

You: This is a **topological sort** problem which can be solved using **DFS**.



YOU GOT IT

Data structures are smart ways of organizing data, based on which we can develop efficient algorithms easily, in ways that people who don't take CSC263 can't even imagine.

Design algorithms **like a pro**.

“Bad programmers worry about the code. Good programmers worry about the data structures and their relationships.”

-- Linus Torvalds

What’s in CSC263?

(1) **Data structures**

and

(2) **Analysis**

What data structures

- ➔ Heaps
- ➔ Binary search trees
- ➔ Balanced search trees
- ➔ Hash tables
- ➔ Disjoint set forest
- ➔ Graphs (matrix, lists)
- ➔ ...

What data structures

Ways of storing and organizing data to facilitate access and modifications.

We learn the strength and limitations of each data structure, so that we know which one to use.

(1) **Data structures**

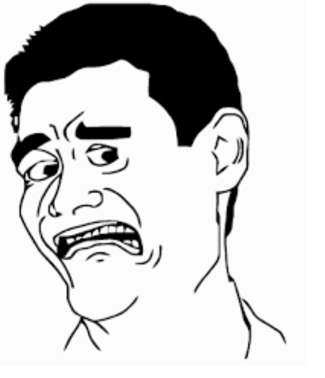
and

(2) **Analysis**

What analyses

- ➔ Worst-case analysis
- ➔ Average-case analysis
- ➔ Amortized analysis
- ➔ Expected worst-case analysis for randomized algorithms
- ➔ ...

Math and proofs



Secret Truth

Data structures are fun to learn,

but **analyses** are more important.

Cooking

- A data structure is like a dish.
- The analysis is to know the effect of each ingredient.

Analyses enable you to invent your own dish.

Background (Required)

- Theory of computation
  - Inductions
  - Recursive functions, Master Theorem
  - Asymptotic notations, “big-Oh”

- Probability theory
  - Probabilities and counting
  - Random variables
  - Distribution
  - Expectations

How to do well in CSC263?

First of all…

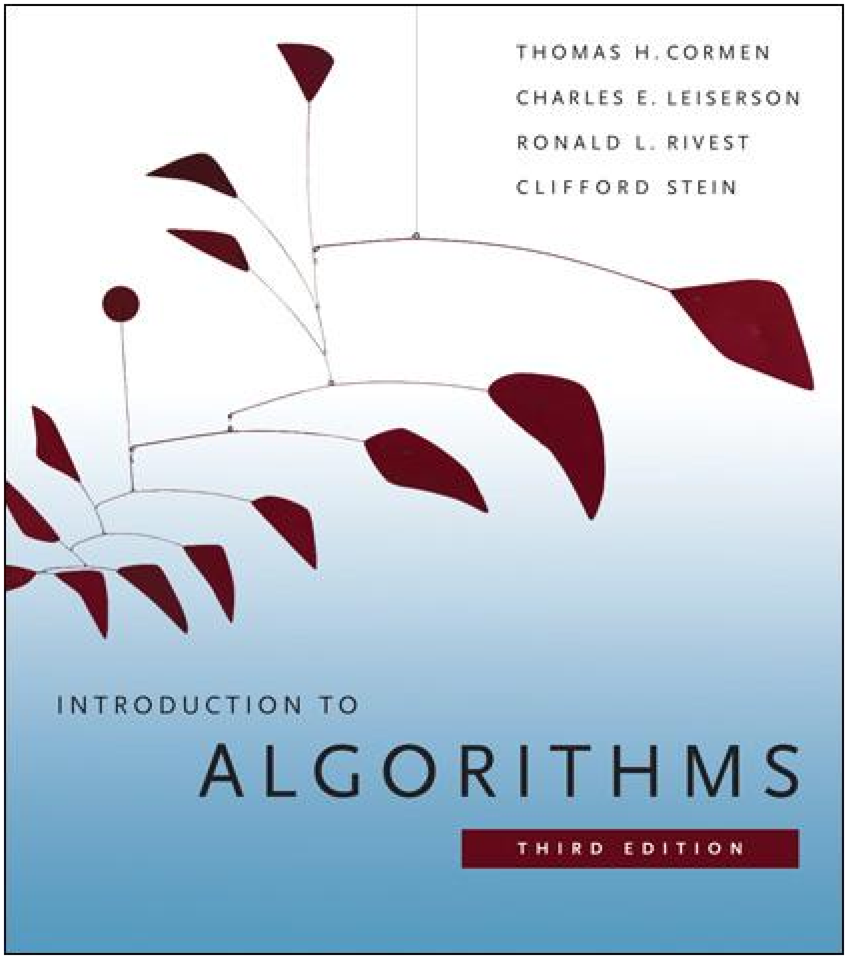
Be interested.

Course Web Page

www.cdf.toronto.edu/~csc263h/winter/

Lecture notes / slides (and everything else) will be posted at the course web page.

Textbook: “CLRS”



Second and third editions are both fine.

Available online at UofT library

Reading for each week is on course info sheet.

Lectures (L0301)

Exception: Thursday, Feb 26, 10am, in RW110

- Tuesdays and Thursdays 10am in SF3202
- Learn stuff

Tutorials (T0301)

Starting this week!

- Fridays 12pm, rooms TBA on web page.
- Practices for homeworks and exams.

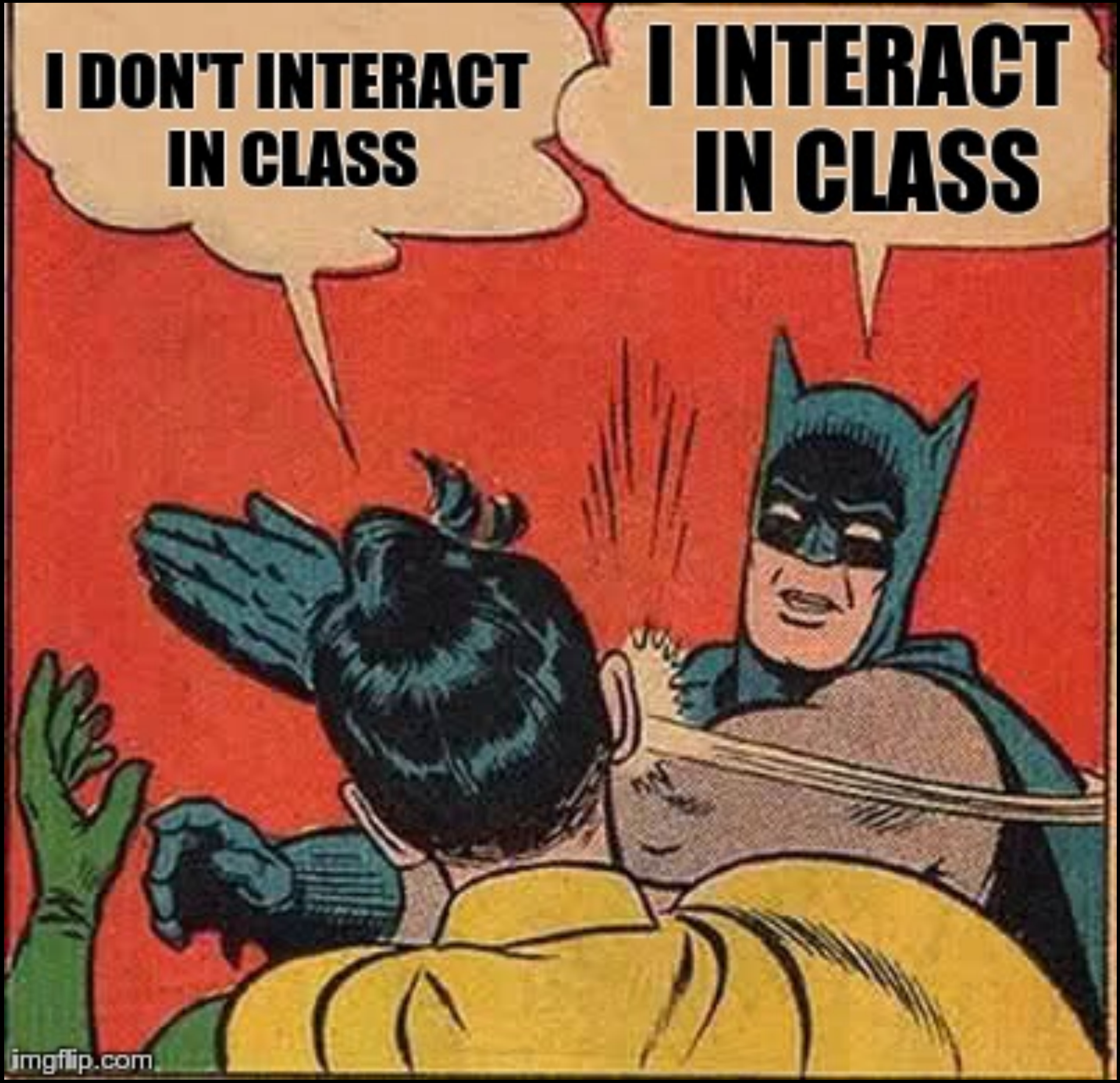
Tutorials are as important as lectures.

A tip for lectures and tutorials...

Get involved in classroom **interactions**

- ➔ answering a question
- ➔ making a guess / bet / vote
- ➔ back-of-the envelope calculations

Emotional involvement makes the brain remember better!


I DON'T INTERACT IN CLASS
I INTERACT IN CLASS
imgflip.com

Course Forum

`piazza.com/utoronto.ca/winter2015/csc263h1`

Use UToronto email to sign-up.

For discussions among students, instructors will be there answering questions, too. Very helpful.

Communicate intelligently!

Don’t discuss homework solutions before due dates.

Office Hours

Thursdays & Fridays 2pm - 4pm
or by appointment

Location: BA5206 (or BA5287)

They are very helpful and NOT scary at all!

Statistically, students who go to office hours get higher grades.

Marking Scheme

- ➔ 8 problem sets: 16% = 2% × 8
- ➔ 2 assignments: 24% = 12% x 2
- ➔ 1 midterm: 24%
- ➔ 1 final exam: 36%
- ➔ **TOTAL** **100%**

Must get at least 40% of the final to pass.

Problem Sets (8 of them)

- Weekly exercises for refreshing what's learned.
- Each problem set has one question, which is supposed to be basic.
- Problem sets are worked **individually**.
- Submissions need to be **typed** into a **PDF** file and submitted to **MarkUS** (link at course web page).
- Due dates are Tuesdays 5:59pm, with 20 hours of lateness allowed with penalty.

about typing

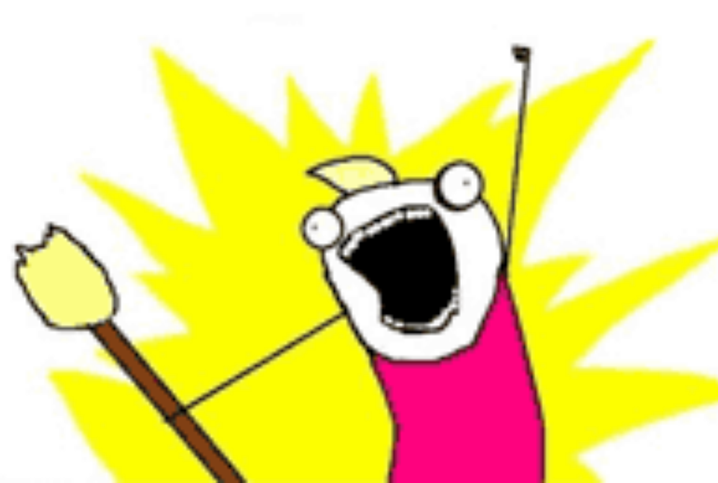
LaTeX is beautiful and strongly recommended

- ➔ We will post our TeX source files, which you can use as templates.
- ➔ Many tutorials online. e.g.,

  http://www.maths.tcd.ie/~dwilkins/LaTeXPrimer/
- ➔ Handy tools that do everything in the browser
  - www.sharelatex.com
  - www.writelatex.com

Problem Set 1 is out today!

due next Tuesday (Jan. 13)



Assignments (2 of them)

- Bigger exercises with more interesting and challenging problems.
- Due dates can be found on course web page.
- Also typed to **PDF** and submitted to **MarkUS**.
- You can form **groups** of up to **4** students.
- **Collaborate intelligently!**
  - Rule of thumb: Collaborate in such a way that **all four of you** can pass the final exam.

Exams

Midterm:

Thursday, February 26, 4 - 6pm.

Fill in the form in the following page if you have a time conflict, by Feb 13th.

http://www.cdf.toronto.edu/~csc263h/winter/tests.shtml

Final exam:

Date to be announced.

Feedback

Feedback at any time is encouraged and appreciated.

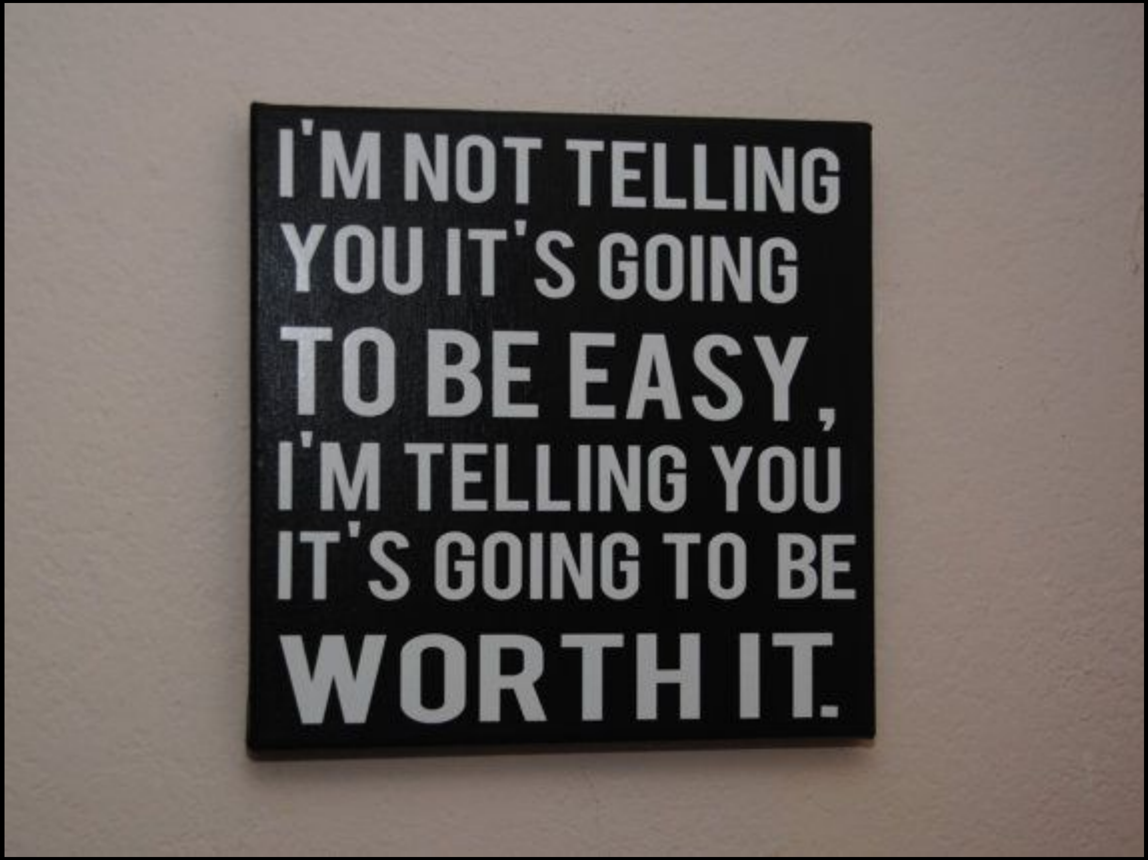
- ➔ Things you like to have more of.
- ➔ Things you want to have less of.
- ➔ A topic that you feel difficult to understand.
- ➔ Anything else related to the course.

You can do it **anonymously** here:

[*http://www.cdf.toronto.edu/~csc263h/winter/feedback.shtml*](http://www.cdf.toronto.edu/~csc263h/winter/feedback.shtml)

Checklist: How to do well

- Be interested.
- Check course web page and email regularly.
- Go to lectures.
- Go to tutorials.
- Read textbook and notes.
- Discuss on Piazza.
- Feel free to go to office hours.
- Work on homeworks, and submit on time.
- Do well in exams.


I'M NOT TELLING
YOU IT'S GOING
TO BE EASY,
I'M TELLING YOU
IT'S GOING TO BE
WORTH IT.

Abstract Data Type (ADT) and Data Structure

Two related but different concepts

In short,

- ADT describes **what** (the **interface**)
  - what data is stored
  - what operation is supported

- Data structure describes **how** (the **implementation**)
  - how the data is store
  - how to perform the operations

Real-life example

ADT

➔ It stores ice cream.

➔ Supported operations:

- start getting ice cream
- stop getting ice cream

Data structures

➔ How ice cream is stored.

➔ How are the start and stop operations implemented.

➔ It's the inside of the machine

A CS example

Stack is an **ADT**

- It stores a list of elements
- supports PUSH(S, v), POP(S), IS_EMPTY(S)

Data structures that can used to implement **Stack**

- Linked list
  - PUSH: insert at head of the list
  - POP: remove at head of the list (if not empty)
  - IS_EMPTY: return “head == None”
- Array with a counter (for size of stack) also works

In CSC263, we will learn many ADTs and many data structures for implementing these ADTs.

Quick announcements

- ➔ Problem Set 1 out
- ➔ Tutorial this Friday (room assignment on course web page)

Review:
Algorithm Complexity

Complexity

Amount of **resource required** by an algorithm, measured as a function of the **input size.**

Time Complexity

➔ Number of **steps** ("running time") executed by an algorithm

In CSC263 we will be dealing with ***time*** *complexity most of the time.*

Space Complexity

➔ Number of units of space required by an algorithm

- e.g., number of elements in a list
- number of nodes in a tree / graph

Example: search a linked list

```
SearchFortyTwo(L):
1.    z = L.head
2.    while z != None and z.key != 42:
3.       z = z.next
4.    return z
```

Let input **L = 41 –> 51 –> 12 –> 42 –> 20 –> 88**

How many times Line #2 will be executed?

4

Now let **L = 41 –> 51 –> 12 –> 24 –> 20 –> 88**

How many times Line #2 will be run?

7 (the last one is z == None)

Note

Running time can be measure by counting the number of times **all lines** are executed, or the number of times **some lines** (such as Line #2 in LinkedSearch) are executed.

It’s up to the problem, or what the question asks.

best-case
worst-case
average-case

Worst-case running time

- ➔ **$t_A(x)$:** the running time of algorithm A with input **x**
- ➔ If it is clear what **A** is, we can simply write **t(x)**
- ➔ The **worst-case running time** **T(n)** is defined as

$$T(n) = \max\{\, t(x) : x \text{ is an input of size } n \,\}$$

"worst-case" is the case with the **longest** running time.

Slow is bad!

Best-case running time

Similarly to worst-case, **best-case** is the case with the **shortest** running time.

$$\min\{\, t(x) : x \text{ is an input of size } n \,\}$$

Best case is not very interesting, and is rarely studied.

Because we should prepare for the worst, not the best!

Example: Search a linked list, again

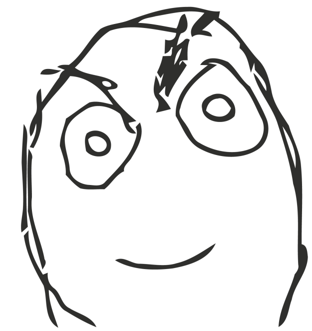
```
SearchFortyTwo(L):
    z = L.head
    while z != None and z.key != 42:
        z = z.next
    return z
```

What is the worst-case running time among all possible **L** with length **n** , i.e., **T(n)**?

T(n) = n + 1

the case where 42 is not in **L** (compare all **n** nodes plus a final *None*)

Average-case running time



In reality, the running time is NOT always the best case, and is NOT always the worst case.

The running time is "**distributed**" between the best and the worst.

For our SearchFortyTwo(L) algorithm the running time is distributed between …

1 and n+1, inclusive

Average-case running time

So, the average-case running time is the **expectation** of the running time which is distributed between 1 and n+1, i.e.,...

Let **t_n** be a random variable whose possible values are between 1 and n+1

$$E[t_n] = \sum_{t=1}^{n+1} t \cdot \Pr(t_n = t)$$

We need to know this!

$$E[t_n] = \sum_{t=1}^{n+1} t \cdot \Pr(t_n = t)$$

Average-case running time

To know **Pr(t_n = t)**, we need to be **given** the **probability distribution** of the inputs, i.e., how inputs are **generated** (following what **distribution**).

Now I give you one:
For each key in the linked list, we pick an integer between **1** and **100** (inclusive), **uniformly at random**.

Figure out Pr(tn = t)

For each key in the linked list, we pick an integer between **1** and **100** (inclusive), **uniformly at random.**

What is ....

$$\Pr(t_n = 1) = 0.01$$

when head is 42

$$\Pr(t_n = 2) = 0.99 \times 0.01$$

head is not 42 and the second one is

$$\Pr(t_n = 3) = (0.99)^2 \times 0.01$$

...

$$\Pr(t_n = n) = (0.99)^{n-1} \times 0.01$$

None of the n keys is 42.

$$\Pr(t_n = n+1) = (0.99)^n$$

the first t keys are not 42 and the t-th is

$$\Pr(t_n = t) = \begin{cases} (0.99)^{t-1} \times 0.01 & 1 \le t \le n \\ (0.99)^n & t = n+1 \end{cases}$$

Now we are ready to compute the average-case running time -- E[tn]

$$\Pr(t_n = t) = \begin{cases} (0.99)^{t-1} \times 0.01 & 1 \le t \le n \\ (0.99)^n & t = n+1 \end{cases}$$

$$E[t_n] = \sum_{t=1}^{n+1} t \cdot \Pr(t_n = t)$$

$$= \sum_{t=1}^{n} t \cdot (0.99)^{t-1} \times 0.01 \quad + (n+1) \cdot (0.99)^n$$

$$= (n+1) \cdot (0.99)^n + 0.01 \cdot \boxed{\sum_{t=1}^{n} t \cdot (0.99)^{t-1}}$$

$$= 100 - 99 \times (0.99)^n$$

This sum needs a bit trick, but can be done!

Calculate the sum (after-class reading)

$$S = \sum_{t=1}^{n} t \cdot 0.99^{t-1} = 1 + 2 \cdot 0.99 + 3 \cdot 0.99^2 + \cdots + n \cdot 0.99^{n-1}$$

$$0.99S = \sum_{t=1}^{n} t \cdot 0.99^{t} = \qquad 1 \cdot 0.99 + 2 \cdot 0.99^2 + \cdots + (n-1) \cdot 0.99^{n-1} + n \cdot 0.99^n$$

take the difference of the above two equations

$$0.01S = \sum_{i=0}^{n-1} 0.99^i - n \cdot 0.99^n = \frac{1 - 0.99^n}{1 - 0.99} - n \cdot 0.99^n$$

$$= 100 - (100 + n) \cdot 0.99^n$$

sum of geometric series

You **should** be comfortable with this type of calculations.

The final result

When the input is generated in a way that for each key in the linked list, we pick an integer between **1** and **100** (inclusive), **uniformly at random.**

The average-case running time of **SearchFortyTwo(L)** (measured by counting Line #2) is:

$$E[t_n] = 100 - 99 \cdot (0.99)^n$$

If n = 0, then $E[t_n]$ = 1, since it's always 1 comparison

If n is very large (e.g., 1000000), $E[t_n]$ is close to 100, i.e., the algorithm is expected to finish within **100** comparisons, even if the worse-case is 1000000 comps.


ONE DOES NOT SIMPLY
TALK ABOUT AVERAGE-CASE RUNNING TIME
WITHOUT TALKING ABOUT INPUT DISTRIBUTION
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asymptotic

upper bound

tight bound

lower bound

Asymptotic notations

O(f(n)): the set of functions that grow **no faster** than **f(n)**

→ if $g \in O(f)$, then we say g is asymptotically **upper bounded** by f

Ω(f(n)): the set of functions that grow **no slower** than **f(n)**

→ if $g \in \Omega(f)$, then we say g is asymptotically **lower bounded** by f

Asymptotic notations

if **g** $\in$ **O(f)** ***and*** **g** $\in$ **Ω(f)**,

then we say **g** $\in$ **Θ(f)**

Θ(f(n)): the set of functions that grow

no slower and no faster than **f(n)**

we call it the **tight** bound.

The ideas behind asymptotic notations

- ➔ We only care about the **rate** of growth, so **constant factors** don't matter.
  - ◆ $100n^2$ and n^2 have the same rate of growth (both are quadrupled when n is doubled)

- ➔ We only care about **large inputs**, so only the **highest-degree** term matters
  - ◆ n^2 and $n^2 + 263n + 3202$ are the nearly the same when n is very large

growth rate ranking of typical functions

$$f(n) = n^n$$

$$f(n) = 2^n$$

$$f(n) = n^3$$

$$f(n) = n^2$$

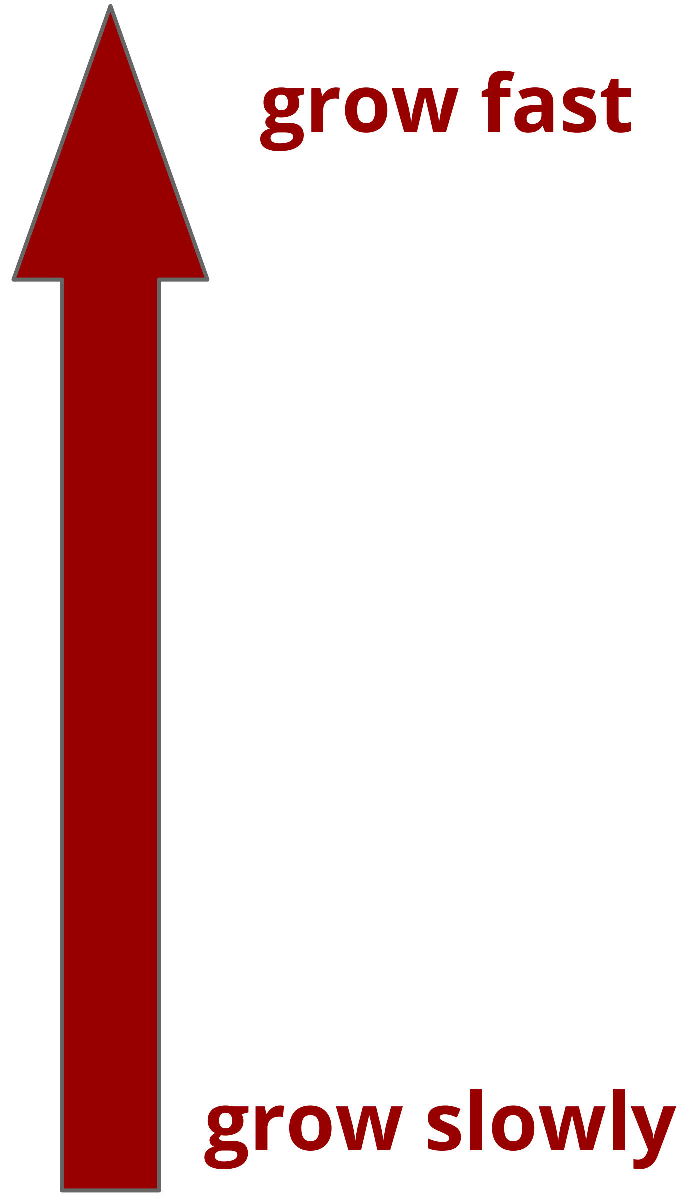
$$f(n) = n \log n$$

$$f(n) = n$$

$$f(n) = \sqrt{n}$$

$$f(n) = \log n$$

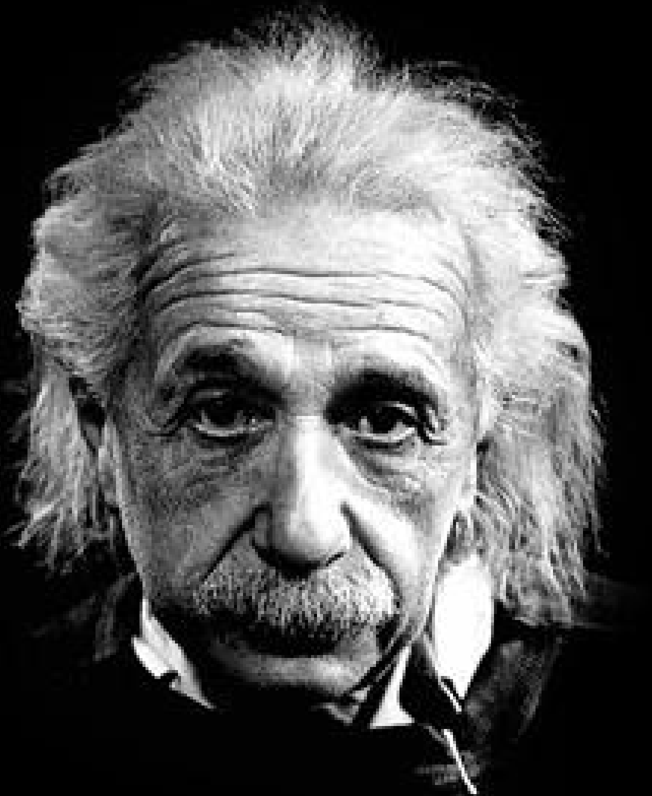
$$f(n) = 1$$



a high-level look at asymptotic notations

*It is a **simplification** of the "real" running time*

- ➔ *it does not tell the whole story about how fast a program runs in real life.*
  - ◆ *in real-world applications, constant factor matters! hardware matters! implementation matters!*
- ➔ *this simplification makes possible the development of the whole **theory of computational complexity**.*
  - ◆ ***HUGE idea!***

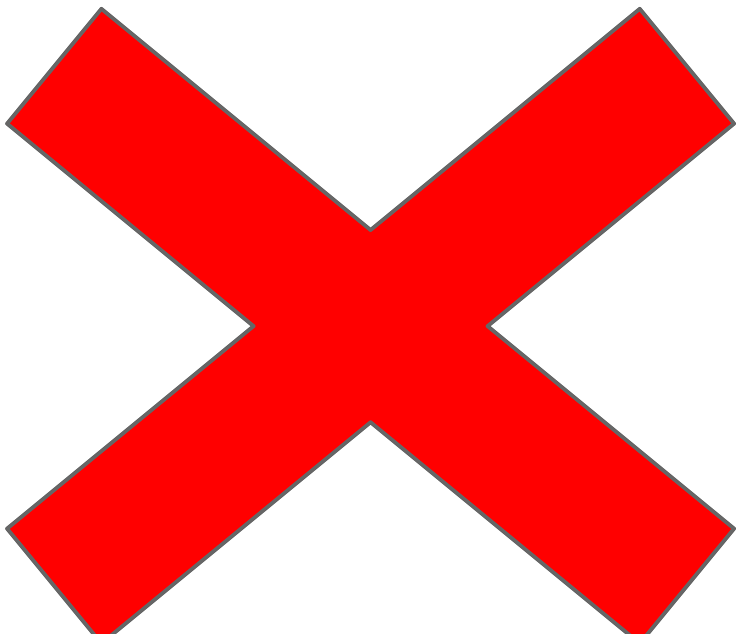


"Make everything as simple as possible, but not simpler."

—Albert Einstein

O is for describing **worst-case** running time

Ω is for describing **best-case** running time



O and **Ω** can **both** be used to upper-bound and lower-bound the **worst-case** running time

O and **Ω** can **both** be used to upper-bound and lower-bound the **best-case** running time

O and **Ω** can **both** be used to upper-bound and lower-bound the **average-case** running time

How to argue algorithm A(x)'s **worst-case** running time is in **O**(n^2)

We need to argue that, ___**for every**___ input x of size n, the running time of A with input x, i.e., t(x) is ___**no larger**___ than **c**$\mathbf{n^2}$, where c > 0 is a constant.

A. for every
B. there exists an
C. no larger
D. no smaller

think about the commuting time from school to home every day

“even the worst day is less than 2 hours”

that means every day is less than 2 hours

How to argue algorithm A(x)'s **worst-case** running time is in $\Omega(n^2)$

We need to argue that, **there exists an** ____________ input x of size n, the running time of A with input x, i.e., t(x) is **no smaller** ____________ than $\mathbf{cn^2}$, where $c > 0$ is a constant.

A. for every
B. there exists an
C. no larger
D. no smaller


“the worst day is more than 2 hours”
“Last Friday it took 4 hours, and it was not even the worst day!”
@视觉古都

How to argue algorithm A(x)'s best-case running time is in O(n^2)

We need to argue that, **there exists an** ________________ input x of size n, the running time of A with input x, i.e., t(x) is **no larger** ________________ than $\mathbf{cn^2}$, where $c > 0$ is a constant.

A. for every
B. there exists an
C. no larger
D. no smaller

How to argue algorithm A(x)'s **best-case** running time is in $\Omega(n^2)$

We need to argue that, **for every** input x of size n, the running time of A with input x, i.e., t(x) is **no smaller** than $\mathbf{cn^2}$, where $c > 0$ is a constant.

A. for every
B. there exists an
C. no larger
D. no smaller

In CSC263

- ➔ Most of the time, we study the upper-bound on worst-case running time.
- ➔ Sometimes we try to get a tight bound Θ if we can
- ➔ Sometimes we study the upper bound on average-case running time.

Note: exact form & asymptotic notations

In CSC263 homework and exam questions, sometimes we ask you to express running time in **exact forms**, and sometime we ask you to express running time in **asymptotic notations**, so always read the question carefully.

If you feel rusty with probabilities, please read the Appendix C of the textbook. It is only about 20 pages, and is highly relevant to what we need for CSC263.

Appendix A and B are also worth reading.

next week

ADT: Priority queue

Data structure: Heap