CSC165

Larry Zhang, September 30, 2014

Announcements

- **Assignment 1** due this Friday
- **Term test 1** next Tuesday in class
  - Time: 6:10pm -- 7pm
  - Location: BA1130
- Old exam repository
  - https://exams-library-utoronto-ca.myaccess.library.utoronto.ca/simple-search?query=csc165*&submit=%EF%BF%BD%EF%8F%A5%E9%8A%B5%EF%BF%BD

Today’s agenda

➔ Bi-implications
➔ Transitivity
➔ Mix quantifiers

➔ Proofs

➔ Problem solving session

Lecture 4.1
bi-implication, transitivity, mixed quantifiers

Course Notes: Chapter 2

Review

$P \Rightarrow Q$ is equivalent to

A. $\neg P \vee Q$

B. $P \vee \neg Q$

C. $P \wedge \neg Q$

D. $P \wedge Q$

E. None of the above

Bi-implication

$$P \Leftrightarrow Q$$

Translate this into the conjunction of two disjunctions.

$$(? \vee ?) \wedge (? \vee ?)$$

Bi-implication

$$P \Leftrightarrow Q \equiv (\neg P \lor Q) \land (\neg Q \lor P)$$

Translate this into the **disjunction** of two **conjunctions**.

$$(? \land ?) \lor (? \land ?)$$

Negation of bi-implication

$$\neg(P \Leftrightarrow Q)$$

transitivity

Transitivity

$$\forall x \in X, (P(x) \Rightarrow Q(x)) \wedge (Q(x) \Rightarrow R(x))$$

implies...

Transitivity

$$((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$$

NEG

mixed quantifiers

Different?

$$\forall x \in X, \exists y \in Y, P(x, y)$$

$$\exists y \in Y, \forall x \in X, P(x, y)$$

Different?

$$\forall x \in X, \forall y \in Y, P(x, y)$$

$$\forall y \in Y, \forall x \in X, P(x, y)$$

Different?

$$\exists x \in X, \exists y \in Y, P(x, y)$$

$$\exists y \in Y, \exists x \in X, P(x, y)$$

A more mathematical example

$$\forall\epsilon \in \mathbb{R}^+, \exists\delta \in \mathbb{R}^+, \forall x \in \mathbb{R}, |x-2| < \delta \Rightarrow |x^2-4| < \epsilon$$

Another mathematical example

$$\exists \delta \in \mathbb{R}^+, \forall \epsilon \in \mathbb{R}^+, \forall x \in \mathbb{R}, x > \delta \Rightarrow x^2 > \epsilon$$

Summary

Language of math (logical notations)

- ➔ quantifiers, statements, predicates
- ➔ implications, equivalence
- ➔ conjunctions, disjunctions, negation
- ➔ Venn diagrams, truth tables
- ➔ manipulating laws

It's **not** about using **symbols**, it is about **understanding** and **expressing** in a **logical** way.

Lecture 4.2 Proofs

Course Notes: Chapter 3

Why proofs?

Proofs are important for science.

➔ A mathematician / computer scientist believes nothing until it is proven.

What is a proof

- ➔ A **proof** is an argument that convinces someone who is logical, careful and precise.
- ➔ You first **understand why** something is true, then you use a proof to **share your understanding** with others, to save them time and effort.
- ➔ **no understanding => no proof**
- ➔ **proof => understanding**

How to prove (high level)

1. **Find a proof**

   → **understand** why you believe the thing is true
   → requires **creativity** and multiple attempts
   → **lenient attitude**: discover, investigate

2. **Write up the proof**

   → **express** why you believe the thing is true
   → requires **carefulness** and **precision**
   → **skeptical** attitude: poke holes in the argument
   → sometimes need to go back to Step 1

What we will learn in CSC165

Learn several different **structures** for proofs, so that you can have more ways to try when being creative to find the proof.

Be able to write up proofs in **structured** manners.

We learn **structures**.

direct proof of
universally quantified implications

Find a proof for $\forall x \in X, P(x) \Rightarrow Q(x)$

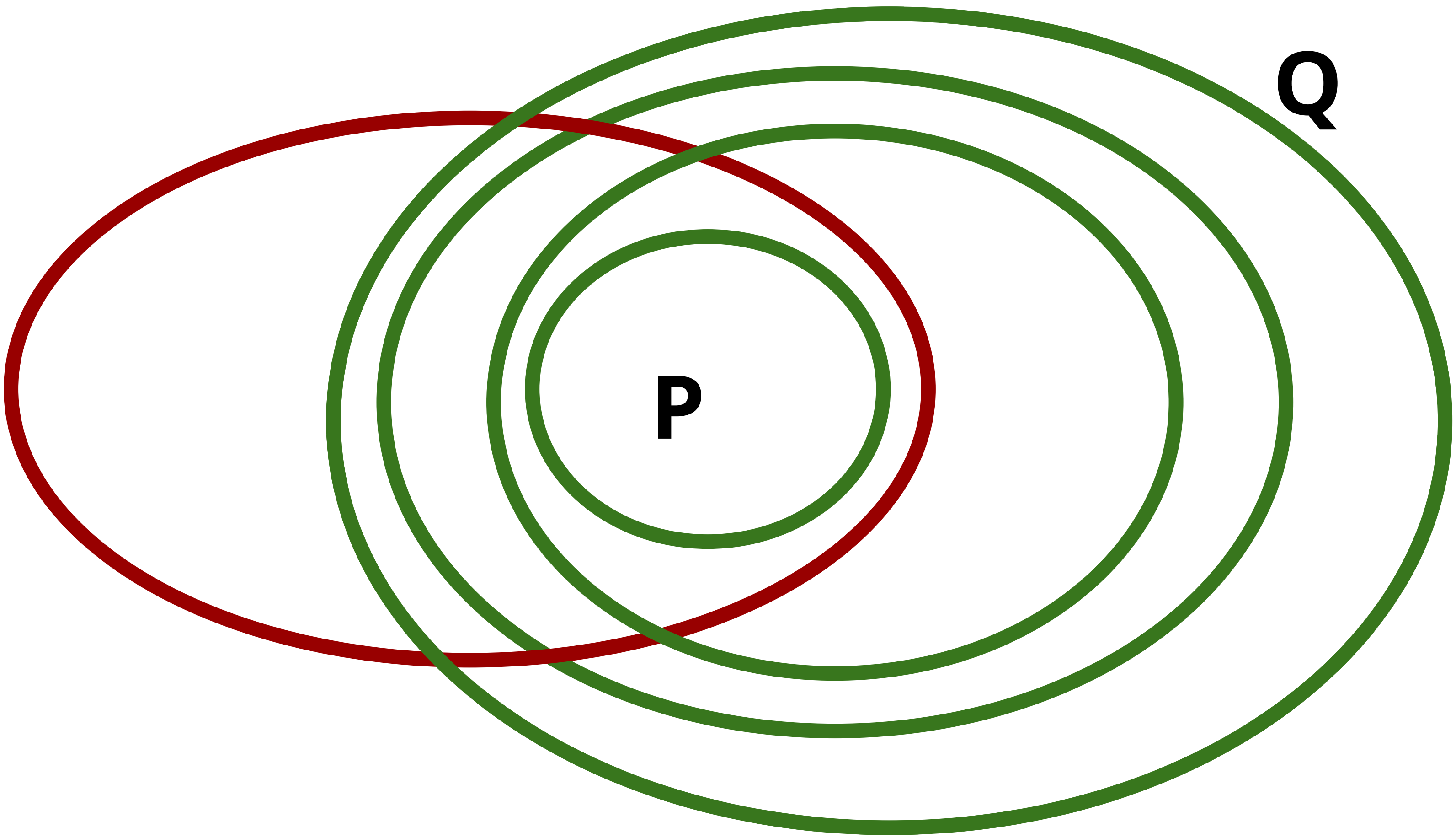
Ideally we would like to have the following.

$$\begin{aligned} \forall x \in X, P(x) &\Rightarrow R_1(x) \\ R_1(x) &\Rightarrow R_2(x) \\ &\dots \\ R_{n-1}(x) &\Rightarrow R_n(x) \\ R_n(x) &\Rightarrow Q(x) \end{aligned}$$

Key: finding the "chain"

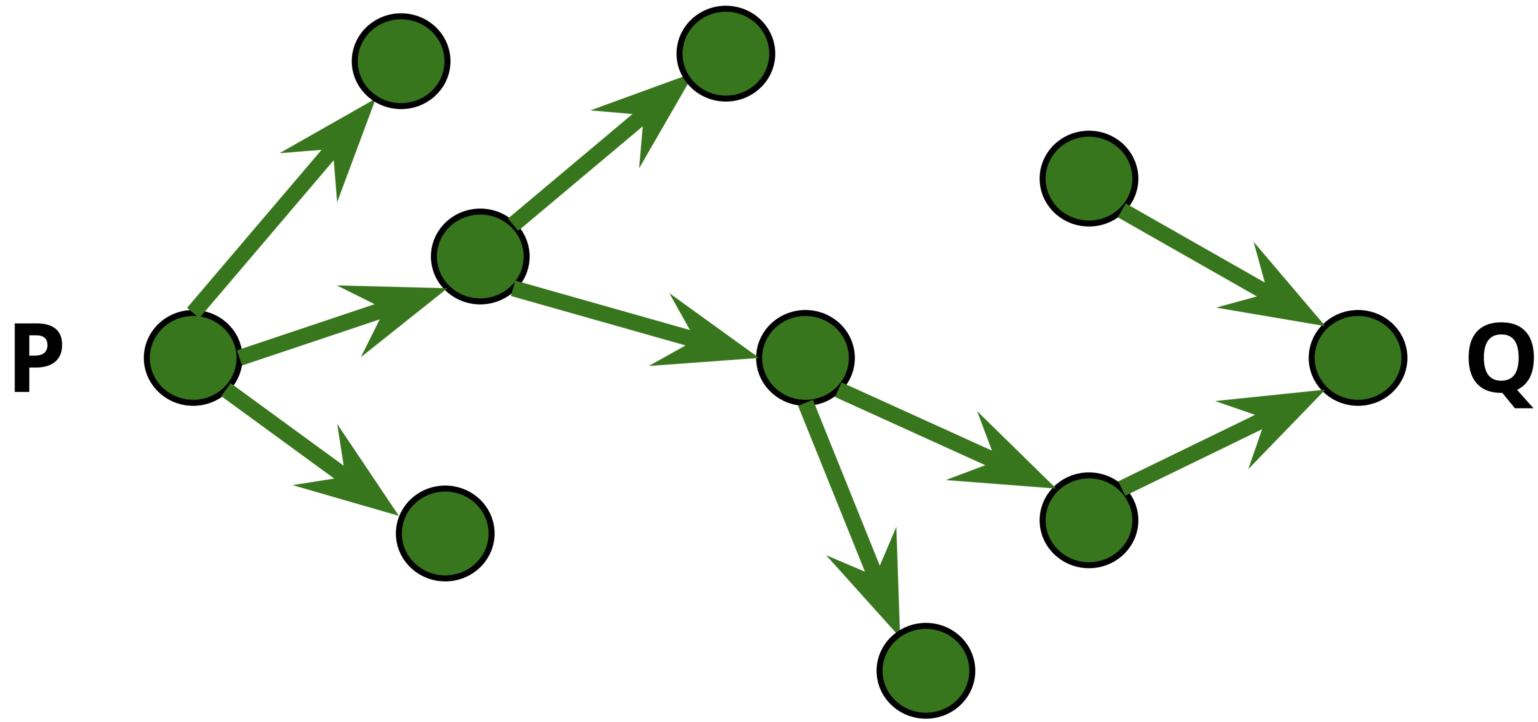
Find the “chain”

$$P \Rightarrow R_1 \Rightarrow R_2 \Rightarrow \cdots \Rightarrow R_{n-1} \Rightarrow R_n \Rightarrow Q$$



Find the “chain”

$$P \Rightarrow R_1 \Rightarrow R_2 \Rightarrow \cdots \Rightarrow R_{n-1} \Rightarrow R_n \Rightarrow Q$$



Search can go both forwards and backwards

Chains with $\land$ and $\lor$

$$(P \Rightarrow R_1) \land (P \Rightarrow R_2)$$

$$(R_1 \Rightarrow Q) \land (R_2 \Rightarrow Q)$$

Write the proof $\forall x \in X, P(x) \Rightarrow Q(x)$

Assume $x \in X$

  Assume $P(x)$

    Then $R_1(x)$

    Then $R_2(x)$

    …

    Then $R_n(x)$

    Then $Q(x)$

  Then $P(x) \Rightarrow Q(x)$

Then $\forall x \in X, P(x) \Rightarrow Q(x)$

Use indentation to present the scope of the assumption.

exercise time

Prove

$$\forall n \in \mathbb{N}, n \text{ is odd} \Rightarrow n^2 \text{ is odd}$$

Prove that for every pair of non-negative real numbers (x, y), if x is greather than y, then the geometric mean, $\sqrt{xy}$ is less than the arithmetic mean, $(x + y)/2$.

Lecture 4.3 problem solving session