CSC165

Larry Zhang, September 23, 2014

Tutorial classrooms

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T0101, Tuesday 9:10am~10:30am:
        BA3102   A-F       (Jason/Jason)
        BA3116   G-L       (Eleni/Eleni)
        BA2185   M-T       (Madina/Madina)
        BA2175   V-Z       (Siamak/Siamak)

T0201: Monday 7:10~8:30pm
        BA2175*  A-D       (Ekaterina/Ekaterina)
        BA1240*  E-Li      (Gal/Gal)
        BA2185*  Liang-S   (Yana/Adam)
        BA3116   T-Z       (Christina/Nadira)

T5101: Thursday 7:10~8:30pm
        BA3116   A-F       (Christine/Christine)
        BA2135   G-Li      (Elias/Elias)
        BA1200*  Lin-U     (Yiyan/Yiyan)
        GB244*   V-Z       (Natalie/Natalie)
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Today's agenda

- More elements of the **language of Math**
  - Conjunctions
  - Disjunctions
  - Negations
  - Truth tables
  - Manipulation laws

Lecture 3.1 Conjunctions, Disjunctions

Course Notes: Chapter 2

Conjunction (AND, $\wedge$)

noun

"the action or an instance of two or more events or things occurring at the same point in time or space."

Synonyms: co-occurrence, coexistence, simultaneity.

Conjunction (AND, $\wedge$)

Combine two statements by claiming they are **both true**.

R(x): Car ***x*** is red.

F(x): Car ***x*** is a Ferrari.

R(x) and F(x): Car ***x*** is red **and** a Ferrari.

$R(x) \wedge F(x)$

Which ones are $R(x) \land F(x)$

Conjunction (AND, $\wedge$)

As sets (instead of predicates)**:**

R: the set of red cars

F: the set of Ferrari cars

$$x \in R \cap F$$

Intersection

What are R, F, $R \cap F$

- ➔ Using predicates: $\boldsymbol{R(x) \wedge F(x)}$

- ➔ Using sets: $\boldsymbol{R \cap F}$

Be careful with English "and"

*There is a pen, **and** a telephone.*

O: the set of all objects

P(x): ***x*** is a pen.

T(x): ***x*** is a telephone.

$$\exists x \in O, P(x) \wedge T(x)$$

Be careful, even in math

The solutions are $x < 20$ *and* $x > 10$.

The solutions are $x > 20$ *and* $x < 10$.

Disjunction

Disjunction (OR, $\vee$)

Combine two statements by claiming that **at least one of them is true**.

R(x): Car ***x*** is red.

F(x): Car ***x*** is a Ferrari.

R(x) or F(x): Car ***x*** is red **or** a Ferrari.

$R(x) \vee F(x)$

Which ones are *R(x)* ∨ *F(x)*

Disjunction (OR, $\vee$)

As sets (instead of predicates)**:**

R: the set of red cars

F: the set of Ferrari cars

$$x \in R \cup F$$

Union

What are R, F, $R \cup F$

- ➔ Using predicates: $\boldsymbol{R(x) \vee F(x)}$
- ➔ Using sets: $\boldsymbol{R \cup F}$

Be careful with English "or"

Either we play the game my way, ***or*** *I'm taking my ball and going home.*

Summary

- ➔ Conjunction: **AND,** $\wedge$, $\cap$
- ➔ Disjunction: **OR,** $\vee$, $\cup$

Lecture 3.2 Negations

Course Notes: Chapter 2

Negation (NOT, ¬)

C: set of all cars

All red cars are Ferrari.

$$\forall x \in C, R(x) \Rightarrow F(x)$$

$\Downarrow$ **negation**

Negation (NOT, ¬)

Not all red cars are Ferrari.

$$\neg(\forall x \in C, R(x) \Rightarrow F(x))$$

equivalent

Exercise: Negate-it!

Exercise: Negate-it!

Rule: the negation sign should apply to the **smallest possible** part of the expression.

$$\neg\underline{(\forall x \in C, R(x) \Rightarrow F(x))}$$ **NO GOOD!**

$$\exists x \in C, R(x) \wedge \neg\underline{F(x)}$$ **GOOD!**

Exercise: Negate-it!

$$\forall x \in C, R(x)$$

All cars are red.

NEG

Exercise: Negate-it!

$$\exists x \in C, R(x)$$

There exists a car that is red.

NEG

Exercise: Negate-it!

$$\forall x \in C, R(x) \Rightarrow F(x)$$

Every red car is a Ferrari.

NEG

Exercise: Negate-it!

$$\exists x \in C, R(x) \wedge F(x)$$

There exists a car that is red and Ferrari.

Some tips

- ➔ The negation of a universal quantification is an existential quantification ("**not all…**" means "**there is one that is not…**").
- ➔ The negation of a existential quantification is an universal quantification ("**there does not exist…**" means "**all…are not…**")
- ➔ Push the negation sign inside **layer by layer** (**like peeling an onion**).

Exercise: Negate-it!

$$\forall x \in X, \exists y \in Y, P(x, y)$$

NEG

Scope

Parentheses are important!

$$P(x) \vee Q(x) \Rightarrow R(x)$$ **NO GOOD!**

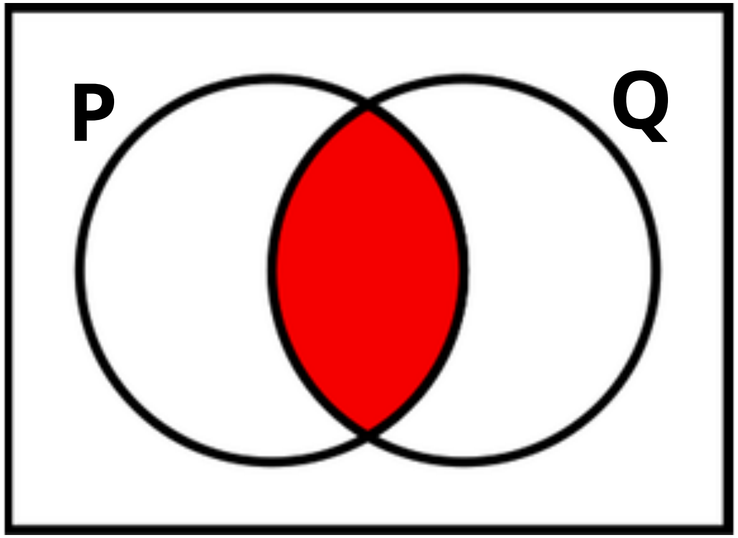
Scope inside parentheses

$$(\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x < y)$$
$$\Rightarrow (\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x > y)$$

Lecture 3.3 Truth tables

Course Notes: Chapter 2

It's about visualization...



Venn diagram works pretty well...
... for **TWO** predicates.

What if we have more predicates?

Truth table with 2 predicates

P	Q	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

Enumerate outcomes of all possible combinations of values of ***P*** and ***Q***.

How many rows are there?

Truth table with 3 predicates

P	Q	R	$Q \Rightarrow R$	$P \Rightarrow (Q \Rightarrow R)$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	T
F	F	T	T	T
F	F	F	T	T

How many rows are there?

Satisfiable

Unsatisfiable

P	***Q***	$P \land Q$	$P \land \neg P$
T	T		
T	F		
F	T		
F	F		

P Q	$\neg (P \vee Q)$	$\neg P \wedge \neg Q$
T T		
T F		
F T		
F F		

De Morgan's Law

$$\neg(P \lor Q) \Leftrightarrow \neg P \land \neg Q$$

$$\neg(P \land Q) \Leftrightarrow \neg P \lor \neg Q$$

Other laws

Commutative laws

$$P \wedge Q \Leftrightarrow Q \wedge P$$

$$P \vee Q \Leftrightarrow Q \vee P$$

Other laws

Associative laws

$$(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$$

$$(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$$

Other laws

Distributive laws

$$P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$$

$$P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$$

Other laws

Identity laws

$$P \wedge (Q \vee \neg Q) \Leftrightarrow P$$

$$P \vee (Q \wedge \neg Q) \Leftrightarrow P$$

Other laws

Idempotent laws

$$P \wedge P \Leftrightarrow P$$

$$P \vee P \Leftrightarrow P$$

Other laws

For even more laws, read Chapter 2.17 of Course Notes.

About these laws...

- → Similar to those for arithmetics.
- → Only use when you are sure.
- → **Understand** them, **be able to derive** them, rather than **memorizing** them.

Summary for today

- ➔ Conjunctions
- ➔ Disjunctions
- ➔ Negations
- ➔ Truth tables
- ➔ Manipulation laws
- ➔ **We are almost done with learning the language of math.**