Tutorial classrooms

T0101, Tuesday 9:10am~10:30am:

- BA3102  A-F    (Jason/Jason)
- BA3116  G-L    (Eleni/Eleni)
- BA2185  M-T    (Madina/Madina)
- BA2175  V-Z    (Siamak/Siamak)

T0201: Monday 7:10~8:30pm

- BA2175* A-D    (Ekaterina/Ekaterina)
- BA1240* E-Li   (Gal/Gal)
- BA2185* Liang-S (Yana/Adam)
- BA3116  T-Z    (Christina/Nadira)

T5101: Thursday 7:10~8:30pm

- BA3116  A-F    (Christine/Christine)
- BA2135  G-Li   (Elias/Elias)
- BA1200* Lin-U  (Yiyan/Yiyan)
- GB244*  V-Z    (Natalie/Natalie)
Today’s agenda

More elements of the language of Math

◆ Conjunctions
◆ Disjunctions
◆ Negations
◆ Truth tables
◆ Manipulation laws
Lecture 3.1 Conjunctions, Disjunctions

Course Notes: Chapter 2
Conjunction (AND, $\land$)

*noun*

“the action or an instance of two or more events or things occurring at the same point in time or space.”

*Synonyms*: co-occurrence, coexistence, simultaneity.
Conjunction (AND, ∧)

Combine two statements by claiming they are both true.

\(R(x):\) Car \(x\) is red.
\(F(x):\) Car \(x\) is a Ferrari.

\(R(x) \text{ and } F(x):\) Car \(x\) is red and a Ferrari.
\(R(x) \land F(x)\)
Which ones are $R(x) \land F(x)$
Conjunction (\textbf{AND, } \land)

As sets (instead of predicates):

\( R \): the set of red cars
\( F \): the set of Ferrari cars

\[ x \in R \cap F \]

Intersection
What are \( R, F, R \cap F \)
Using predicates: $R(x) \land F(x)$

Using sets: $R \cap F$
Be careful with English “and”

There is a pen, and a telephone.

\[ O: \text{the set of all objects} \]
\[ P(x): x \text{ is a pen.} \]
\[ T(x): x \text{ is a telephone.} \]

\[ \exists x \in O, P(x) \land T(x) \]
Be careful, even in math

The solutions are $x < 20$ and $x > 10$.

The solutions are $x > 20$ and $x < 10$. 
Disjunction
Disjunction (OR, ∨)

Combine two statements by claiming that at least one of them is true.

\( R(x) \): Car \( x \) is red.
\( F(x) \): Car \( x \) is a Ferrari.

\( R(x) \text{ or } F(x) \): Car \( x \) is red \text{ or} \ a Ferrari.
\( R(x) \lor F(x) \)
Which ones are \( R(x) \lor F(x) \)
Disjunction (OR, \( \lor \))

As sets (instead of predicates):

\( R \): the set of red cars
\( F \): the set of Ferrari cars

\[ x \in R \cup F \]

Union
What are $R$, $F$, $R \cup F$
Using predicates: $R(x) \lor F(x)$

Using sets: $R \cup F$
Be careful with English “or”

Either we play the game my way, or I’m taking my ball and going home.
Summary

➔ Conjunction: **AND, \( \land \), \( \cap \)**

➔ Disjunction: **OR, \( \lor \), \( \cup \)**
Lecture 3.2 Negations

Course Notes: Chapter 2
Negation (NOT, ¬)

All red cars are Ferrari.

\[ \forall x \in C, R(x) \Rightarrow F(x) \]

\(C: \text{set of all cars}\)
Negation (NOT, \( \neg \))

Not all red cars are Ferrari.

\[ \neg \left( \forall x \in C, R(x) \Rightarrow F(x) \right) \]
Exercise: Negate-it!
Exercise: Negate-it!

Rule: the negation sign should apply to the smallest possible part of the expression.

\[ \neg (\forall x \in C, R(x) \Rightarrow F(x)) \] NO GOOD!

\[ \exists x \in C, R(x) \land \neg F(x) \] GOOD!
Exercise: Negate-it!

\[ \forall x \in C, \, R(x) \]

All cars are red.
Exercise: Negate-it!

\[ \exists x \in C, R(x) \]

There exists a car that is red.
Exercise: Negate-it!

\[ \forall x \in C, R(x) \implies F(x) \]

Every red car is a Ferrari.
Exercise: Negate-it!

\[ \exists x \in C, R(x) \land F(x) \]

There exists a car that is red and Ferrari.
Some tips

➔ The negation of a universal quantification is an existential quantification ("not all..." means "there is one that is not...").

➔ The negation of a existential quantification is an universal quantification ("there does not exist..." means "all...are not...").

➔ Push the negation sign inside layer by layer (like peeling an onion).
Exercise: Negate-it!

∀x ∈ X, ∃y ∈ Y, P(x, y)

NEG
Scope
Parentheses are important!

\[ P(x) \lor Q(x) \Rightarrow R(x) \]

NO GOOD!
Scope inside parentheses

\[(\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x < y)\]

\[\Rightarrow (\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x > y)\]
Lecture 3.3  Truth tables

Course Notes: Chapter 2
It’s about visualization...

Venn diagram works pretty well...
... for **TWO** predicates.
What if we have more predicates?
Truth table with 2 predicates

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
<th>$P \lor Q$</th>
<th>$P \implies Q$</th>
<th>$P \iff Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

Enumerate outcomes of all possible combinations of values of $P$ and $Q$.

How many rows are there?
Truth table with **3** predicates

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$R$</th>
<th>$Q \Rightarrow R$</th>
<th>$P \Rightarrow (Q \Rightarrow R)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

How many rows are there?
<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \wedge Q$</th>
<th>$P \wedge \neg P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Satisfiable**

**Unsatisfiable**
<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$\neg (P \lor Q)$</th>
<th>$\neg P \land \neg Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td></td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td></td>
<td>F</td>
</tr>
</tbody>
</table>
De Morgan’s Law

\[ \neg(P \lor Q) \iff \neg P \land \neg Q \]

\[ \neg(P \land Q) \iff \neg P \lor \neg Q \]
Other laws

Commutative laws

\[ P \land Q \iff Q \land P \]

\[ P \lor Q \iff Q \lor P \]
Other laws

Associative laws

\[(P \land Q) \land R \iff P \land (Q \land R)\]

\[(P \lor Q) \lor R \iff P \lor (Q \lor R)\]
Other laws

Distributive laws

\[ P \land (Q \lor R) \iff (P \land Q) \lor (P \land R) \]

\[ P \lor (Q \land R) \iff (P \lor Q) \land (P \lor R) \]
Other laws

Identity laws

\[ P \land (Q \lor \neg Q) \iff P \]

\[ P \lor (Q \land \neg Q) \iff P \]
Other laws

Idempotent laws

\[ P \land P \iff P \]

\[ P \lor P \iff P \]
Other laws

For even more laws, read Chapter 2.17 of Course Notes.
About these laws...

➔ Similar to those for arithmetics.
➔ Only use when you are sure.
➔ **Understand** them, **be able to derive** them, rather than **memorizing** them.
Summary for today

➔ Conjunctions
➔ Disjunctions
➔ Negations
➔ Truth tables
➔ Manipulation laws

➔ We are almost done with learning the language of math.