CSC165

September 16, 2014
Announcements

➔ Tutorials start this week, exercise posted on course web page, work on them **before** tutorial.

<table>
<thead>
<tr>
<th>Tutorial section and time</th>
<th>TA, tutorials 1–5</th>
<th>TA, tutorials 5–9</th>
<th>Room</th>
<th>Surnames</th>
</tr>
</thead>
<tbody>
<tr>
<td>L0201, Monday 7:10–8:30</td>
<td>Ekaterina, Gal, Yana, Christina</td>
<td>Ekaterina, Gal, Adam, Nadira</td>
<td>BA2159, BA3008, BA3012, BA3116</td>
<td>A–D, E–Li, Liang–S, T–Z</td>
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</tbody>
</table>

➔ Submit **slogURL.txt** to MarkUs by Friday, include a paragraph on topics from Week 1~2.
Lecture 2.1  Quantifiers (cont.)

Course Notes: Chapter 2
Today’s agenda

➔ Quantifiers, verify / falsify
➔ Sentences, statements
➔ Predicates
➔ Implications
Last week

➔ Universal quantifier: ∀, “for all”, “every”

➔ Existential quantifier: ∃, “there is”, “some”
Fill with “no”, “one”, “example”, “counter-example”

<table>
<thead>
<tr>
<th></th>
<th>Universal</th>
<th>Existential</th>
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<tbody>
<tr>
<td>Verify (prove)</td>
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<td></td>
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<tr>
<td>Falsify (disprove)</td>
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Visualization with Venn Diagram

X: this part must be empty, i.e., with no element

◯: this part must be occupied, i.e., there must be some element in here
Verify $\forall x \in P, x \in Q$
Falsify $\forall x \in P, x \in Q$
Verify $\exists x \in P, x \in Q$
Falsify $\exists x \in P, x \in Q$
Quantifiers as claims about sets

Some symbols (prerequisites in Chapter 1.5):

\[ A \subseteq B \quad A \nsubseteq B \]
\[ A \cap B \quad A \cup B \]
\[ \overline{A} \quad \emptyset \]
Quantifiers as claims about sets

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$E$: set of all employees

$M$: set of male employees

$F$: set of female employees

$O$: set of employees who earn over $42,000$

“All employees earn over $42,000$.”

$\forall x \in E, x \in O$
Quantifiers as claims about sets

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\( E \): set of all employees  
\( M \): set of male employees  
\( F \): set of female employees  
\( O \): set of employees who earn over $42,000

“No male employee earns over $42,000.”

\( \forall x \in M, x \in \overline{O} \)
Quantifiers as claims about sets

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$E$: set of all employees  
$M$: set of male employees  
$F$: set of female employees  
$O$: set of employees who earn over $42,000$

"Some female employee earns over $42,000."

$\exists x \in F, x \in O$
Evaluating quantified claims

\[ E \]: set of all employees

\[ M \]: set of male employees

\[ F \]: set of female employees

\[ O \]: set of employees who earn over $42,000

```
def q2(S1, S2):
    return any({x in S2 for x in S1})
```

```
def q3(S1, S2):
    return all({x in S2 for x in S1})
```

“All employees earn over $42,000.”
Evaluating quantified claims

\[ E: \text{set of all employees} \]
\[ M: \text{set of male employees} \]
\[ F: \text{set of female employees} \]
\[ O: \text{set of employees who earn over $42,000} \]

```
def q2(S1, S2):
    return any({x in S2 for x in S1})
```

```
def q3(S1, S2):
    return all({x in S2 for x in S1})
```

“Some female employee earns over $42,000.”
Evaluating quantified claims

**E**: set of all employees

**M**: set of male employees

**F**: set of female employees

**O**: set of employees who earn over $42,000

```
def q1(S1, S2):
    return not all({x in S2 for x in S1})
```

```
def q4(S1, S2):
    return not any({x in S2 for x in S1})
```

“Some male employee does not earn over $42,000.”
Evaluating quantified claims

\[ E \text{: set of all employees} \]
\[ M \text{: set of male employees} \]
\[ F \text{: set of female employees} \]
\[ O \text{: set of employees who earn over $42,000} \]

“No male employee earns over $42,000.”

```python
def q1(S1, S2):
    return not all({x in S2 for x in S1})

def q4(S1, S2):
    return not any({x in S2 for x in S1})
```
Be able to understand and express quantifications in different ways

- using quantifiers (for all, \( \forall \), \( \exists \), etc.)
- using Venn Diagrams
- using set relations (\( \subseteq \), \( \notin \), \( \cap \), \( \emptyset \), etc.)
- using quantifying functions (q1, q2, etc.)
Lecture 2.2: Sentences, Statements and Implications

Course Notes: Chapter 2
Sentence and Statement

The employee earns less than $55,000.

This is a sentence.

Every employee earns less than $55,000.

This is a statement.
Sentence vs Statement

- A statement is always a sentence.
- A sentence is NOT always a statement.
- A statement is a sentence that is NOT “open”.
- The object in an open sentence is unspecified (unquantified), thus the sentence cannot be evaluated.
- The object in a statement is quantified, thus a statement can be evaluated true or false.
Exercise: Is it a statement?

➔ Roses are red.
➔ Someone ate my sandwich.
➔ The sandwich tastes good.
➔ At least one student is hungry.
Predicate
Predicates

$L$: the employees who earn less than $55,000.

To say: employ $x$ earns less than $55,000.$

We can write: $x \in L$

or we can write: $L(x)$
Predicates

$L\!(\text{Carlos})$

$\neg L\!(\text{Carlos})$

“$\neg$” means “not”: negation
Predicates

$L(x)$ without specifying $x$ is an
Predicate or Set, which to use?

If \( L \) is more like a property, use predicate.

e.g., “\( x \) is a prerequisite of \( y \)”

\( P(x, y) \) (simple and good)

How to do this using set?
Implications
Implication

If an employee is male, then he makes less than $55,000.

If P, then Q.

Antecedent (Assumption)

Consequent (Conclusion)
P \implies Q

“P implies Q”
Verifying / falsify implication

If an employee is male, then he makes less than $55,000.

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Be careful with “if”

Mom: If you eat your vegetables, then you can have dessert.
Be careful with "if"

If it rains yesterday, then the sun rose today.
Converse of implication

\( E \): set of all employees
\( F(x) \): \( x \) is female
\( L(x) \): \( x \) earns less than $55,000

Original: \( \forall x \in E, F(x) \Rightarrow L(x) \)
Converse: \( \forall x \in E, L(x) \Rightarrow F(x) \)

What’s the relation between the two?
Contrapositive

$E$: set of all employees

$F(x)$: $x$ is female

$L(x)$: $x$ earns less than $55,000$

Original: $\forall x \in E, F(x) \Rightarrow L(x)$

Contrapositive: $\forall x \in E, \neg L(x) \Rightarrow \neg F(x)$

What’s the relation between these two?
Summary

→ Sentence vs Statements
→ Predicates
→ Implications
  ◆ Converse
  ◆ Contrapositive
  ◆ ...

Lecture 2.3 Implications cont.

Course Notes: Chapter 2
Numerical example

Define $P(n)$: $n$ is a multiple of 4, and $Q(n)$: $n^2$ is a multiple of 4. We know that

$$\forall n \in \mathbb{N}, P(n) \Rightarrow Q(n)$$

What do the following tell you?

$\Rightarrow$ k is a multiple of 4

$\Rightarrow$ k is not a multiple of 4

$\Rightarrow$ $k^2$ is a multiple of 4

$\Rightarrow$ $k^2$ is not a multiple of 4
Natural language for $P \Rightarrow Q$

If nominated, I will not stand.

If you think I’m lying, then you’re a liar!
Natural language for $P \implies Q$

Whenever I hear that song, I think about ice cream.

I get heartburn whenever I eat supper too late.
Natural language for $P \Rightarrow Q$

Differentiability is sufficient for continuity.

Matching fingerprints and a motive are enough for guilt.
Natural language for $P \Rightarrow Q$

There are no rights without responsibilities.

You can’t stay enrolled in CSC165 without a pulse.
Natural language for $P \Rightarrow Q$

Successful programming requires skill.
Natural language for $P \Rightarrow Q$

To pass CSC165, a student needs to get 40% on the final.
Natural language for $P \Rightarrow Q$

I will go only if you insist.
Vacuous truth

To falsify “P(x) => Q(x)”

➔ find an x such that P(x) is true but Q(x) is false.

∀x ∈ ℝ, x² < 0 ⇒ x > x + 5

➔ All employees earning over $80 trillion are female.

➔ All employees earning over $80 trillion are male.

➔ All employees earning over $80 trillion have pink toenails.
Equivalence

If $P$ then $Q$, and if $Q$ then $P$

$P$ if and only if $Q$

$P$ iff $Q$
Every male employee earns between $25,000 and $45,000.

Every employee earning between $25,000 and $45,000 is male.
Equivalence

\( \forall x \in \mathbb{R}, x^2 < 0 \iff x > x + 5 \)
Idiom

Sometimes there are more than one way to say a thing.

Every D that is a P is also a Q

common: \( \forall x \in D, P(x) \Rightarrow Q(x) \)

less common: \( \forall x \in D \cap P, Q(x) \)
Idiom

Sometimes there are more than one way to say a thing.

**Some D that is a P is also a Q**

common: \( \exists x \in D, P(x) \land Q(x) \)

less common: \( \exists x \in D \cap P, Q(x) \)
Summary
Next week