CSC165

Larry Zhang, September 23, 2014

Tutorial classrooms

```
T0101, Tuesday 9:10am~10:30am:
        BA3102   A-F       (Jason/Jason)
        BA3116   G-L       (Eleni/Eleni)
        BA2185   M-T       (Madina/Madina)
        BA2175   V-Z       (Siamak/Siamak)

T0201: Monday 7:10~8:30pm
        BA2175*  A-D       (Ekaterina/Ekaterina)
        BA1240*  E-Li      (Gal/Gal)
        BA2185*  Liang-S   (Yana/Adam)
        BA3116   T-Z       (Christina/Nadira)

T5101: Thursday 7:10~8:30pm
        BA3116   A-F       (Christine/Christine)
        BA2135   G-Li      (Elias/Elias)
        BA1200*  Lin-U     (Yiyan/Yiyan)
        GB244*   V-Z       (Natalie/Natalie)
```

slogURL.txt

- **393 / 447** submitted
- can still submit on MarkUS if you haven't.
- can still fix it if you did it wrong
  - a plain **TXT** file: **slogURL.txt**
    - **NOT** slogURL.**pdf**, slogURL.**doc**, slogURL.**txt.pdf**, slogURL.**txt.doc**, or PDF/DOC **renamed** to TXT
  - Submit **individually**
    - If you formed a group with more than **one** person, email Danny and me with **both** of your URLs.

Assignment 1 is out

http://www.cdf.toronto.edu/~heap/165/F14/Assignments/a1.pdf

- ➔ Due on October 3rd, 10:00pm.
- ➔ May work in groups of up to 3 people.
- ➔ Submit on MarkUs: **a1.pdf**
- ➔ Prefer to use **LaTeX**, try the following tools
  - **www.writelatex.com**
  - **www.sharelatex.com**

Today's agenda

- → More elements of the **language of Math**
  - Conjunctions
  - Disjunctions
  - Negations
  - Truth tables
  - Manipulation laws

Lecture 3.1 Conjunctions, Disjunctions

Course Notes: Chapter 2

Conjunction (AND, $\wedge$)

noun

“the action or an instance of two or more events or things occurring at the same point in time or space.”

Synonyms: co-occurrence, coexistence, simultaneity.

Conjunction (AND, $\wedge$)

Combine two statements by claiming they are **both true**.

R(x): Car ***x*** is red.

F(x): Car ***x*** is a Ferrari.

predicates!

R(x) ***and*** ***F(x)***: Car ***x*** is red **and** a Ferrari.

$R(x) \wedge F(x)$

Which ones are $R(x) \wedge F(x)$

Conjunction (AND, $\wedge$)

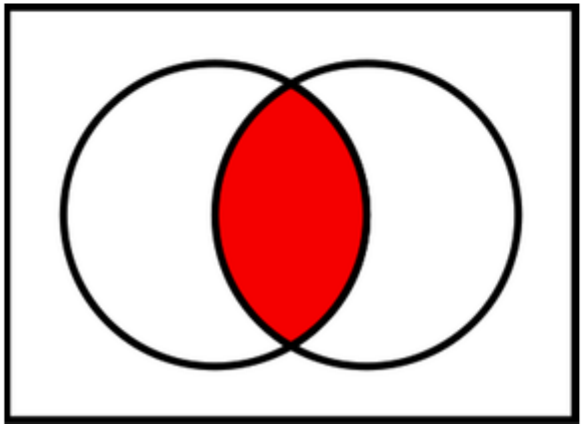
As sets (instead of predicates)**:**

R: the set of red cars

F: the set of Ferrari cars

$$x \in R \cap F$$

Intersection



What are R, F, $R \cap F$

- ➔ Using predicates: $\boldsymbol{R(x) \land F(x)}$
- ➔ Using sets: $\boldsymbol{R \cap F}$

Be careful with English “and”

*There is a pen, **and** a telephone.*

O: the set of all objects

P(x): ***x*** is a pen.

T(x): ***x*** is a telephone.

$$\exists x \in O, P(x) \wedge T(x)$$

There is a pen-phone!

Be careful with English "**and**"

*There is a pen, **and** a telephone.*

O: the set of all objects

P(x): ***x*** is a pen.

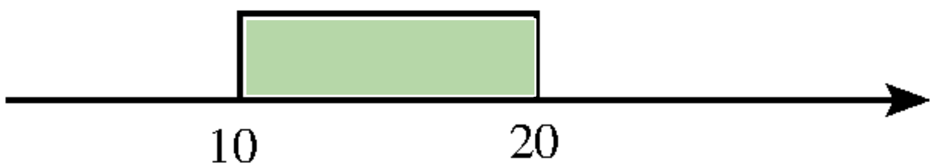
T(x): ***x*** is a telephone.

$$(\exists x \in O, P(x)) \wedge (\exists x \in O, T(x))$$

Be careful, even in math

The solutions are ***x < 20*** *and* ***x > 10****.*

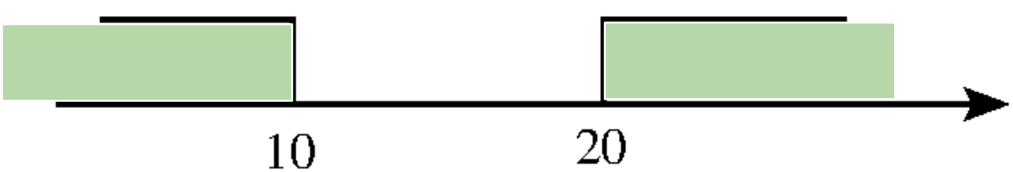
A B



A ∩ B

The solutions are ***x > 20*** *and* ***x < 10****.*

A B



A ∪ B

Disjunction

Disjunction (OR, $\vee$)

Combine two statements by claiming that **at least one of them is true**.

R(x): Car ***x*** is red.

F(x): Car ***x*** is a Ferrari.

R(x) or F(x): Car ***x*** is red **or** a Ferrari.

$R(x) \vee F(x)$

Which ones are $R(x) \lor F(x)$

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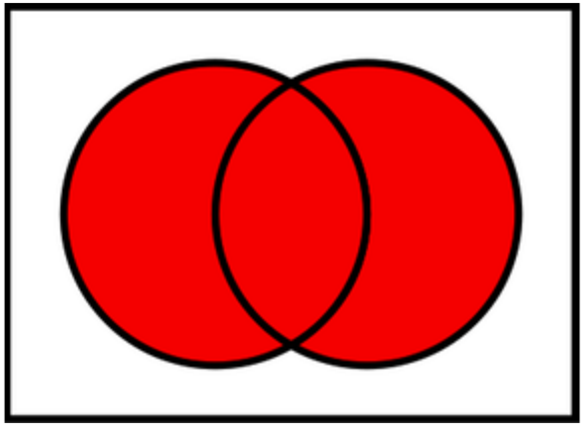
Disjunction (OR, $\vee$)

As sets (instead of predicates)**:**

R: the set of red cars
F: the set of Ferrari cars

$$x \in R \cup F$$



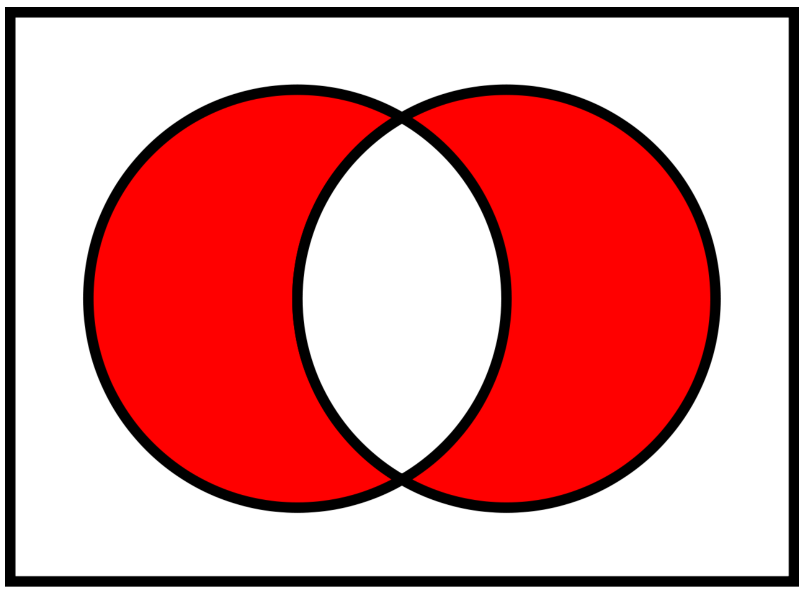
Union

What are R, F, $R \cup F$

- ➔ Using predicates: $\boldsymbol{R(x)} \vee \boldsymbol{F(x)}$
- ➔ Using sets: $\boldsymbol{R} \cup \boldsymbol{F}$

Be careful with English “or”

Either we play the game my way, ***or*** *I’m taking my ball and going home.*



“exclusive or”, not “or”!

Summary

- ➔ Conjunction: **AND,** $\land$, $\cap$
- ➔ Disjunction: **OR,** $\lor$, $\cup$

Quick test

A logician's wife is having a baby. The doctor immediately hands the newborn to the dad.

His wife asks impatiently: "So, is it a boy or a girl?"

*The logician replies: "**Yes.**"*

Source: "21 jokes for super smart people."

http://www.buzzfeed.com/tabathaleggett/jokes-youll-only-get-if-youre-really-smart#1ml5j1x

Lecture 3.2 Negations

Course Notes: Chapter 2

Negation (NOT, ¬)

C: set of all cars

All red cars are Ferrari.

$$\forall x \in C, R(x) \Rightarrow F(x)$$

negation

Not all red cars are Ferrari.

$$\neg(\underline{\forall x \in C, R(x) \Rightarrow F(x)})$$

Negation (NOT, ¬)

Not all red cars are Ferrari.

$$\neg(\forall x \in C, R(x) \Rightarrow F(x))$$

equivalent

There exists a car that is red and **not** Ferrari.

$$\exists x \in C, R(x) \wedge \neg F(x)$$

Exercise: Negate-it!

Exercise: Negate-it!

Rule: the negation sign should apply to the **smallest possible** part of the expression.

$$\neg(\forall x \in C, R(x) \Rightarrow F(x))$$ **NO GOOD!**

$$\exists x \in C, R(x) \wedge \neg F(x)$$ **GOOD!**

Exercise: Negate-it!

$$\forall x \in C, R(x)$$

All cars are red.

NEG

Not all cars are red. $\neg\underline{(\forall x \in C, R(x))}$

There exists a car that is **not** red.

$$\exists x \in C, \neg R(x)$$

Exercise: Negate-it!

$$\exists x \in C, R(x)$$

There exists a car that is red.

NEG

$$\neg\underline{(\exists x \in C, R(x))}$$

There does **not** exists a car that is red.

All cars are **not** red.

$$\forall x \in C, \neg R(x)$$

Exercise: Negate-it!

$$\forall x \in C, R(x) \Rightarrow F(x)$$

Every red car is a Ferrari.

NEG

Not every red car is a Ferrari.

$$\neg\underline{(\forall x \in C, R(x) \Rightarrow F(x))}$$

There is a car that is red and **not** a Ferrari.

$$\exists x \in C, R(x) \wedge \neg F(x)$$

Exercise: Negate-it!

$$\exists x \in C, R(x) \wedge F(x)$$

There exists a car that is red and Ferrari.

NEG

There does **not** exists a car that is red and Ferrari.

$$\neg\underline{(\exists x \in C, R(x) \wedge F(x))}$$

For all cars, if it is red, then it is **not** Ferrari.

$$\forall x \in C, R(x) \Rightarrow \neg F(x)$$

Exercise: Negate-it!

$$\exists x \in C, R(x) \wedge F(x)$$

There exists a car that is red and Ferrari.

There does **not** exists a car that is red and Ferrari.

$$\neg(\exists x \in C, R(x) \wedge F(x))$$

For all cars, it is **red**, then it is not **Ferrari**.
For all cars, it is **Ferrari**, then it is not **red**.

$$\forall x \in C, F(x) \Rightarrow \neg R(x)$$

Some tips

- The negation of a universal quantification is an existential quantification ("**not all...**" means "**there is one that is not...**").
- The negation of a existential quantification is an universal quantification ("**there does not exist...**" means "**all...are not...**")
- Push the negation sign inside **layer by layer** (**like peeling an onion**).

Exercise: Negate-it!

$$\forall x \in X, \exists y \in Y, P(x, y)$$

$$\Big\downarrow \text{NEG}$$

$$\neg\underline{(\forall x \in X, \exists y \in Y, P(x, y))}$$

$$\exists x \in X, \neg\underline{(\exists y \in Y, P(x, y))}$$

$$\exists x \in X, \forall y \in Y, \neg\underline{P(x, y)}$$

Scope

Parentheses are important!

$$P(x) \vee Q(x) \Rightarrow R(x)$$ **NO GOOD!**

$$\underline{(P(x) \vee Q(x))} \Rightarrow R(x)$$ **GOOD!**

$$P(x) \vee \underline{(Q(x) \Rightarrow R(x))}$$ **GOOD!**

Scope inside parentheses

$$(\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x < y)$$
$$\Rightarrow (\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x > y)$$

is the same as

$$(\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x < y)$$
$$\Rightarrow (\forall z \in \mathbb{R}, \exists w \in \mathbb{R}, z > w)$$

Everything happens in parentheses stays in parentheses.

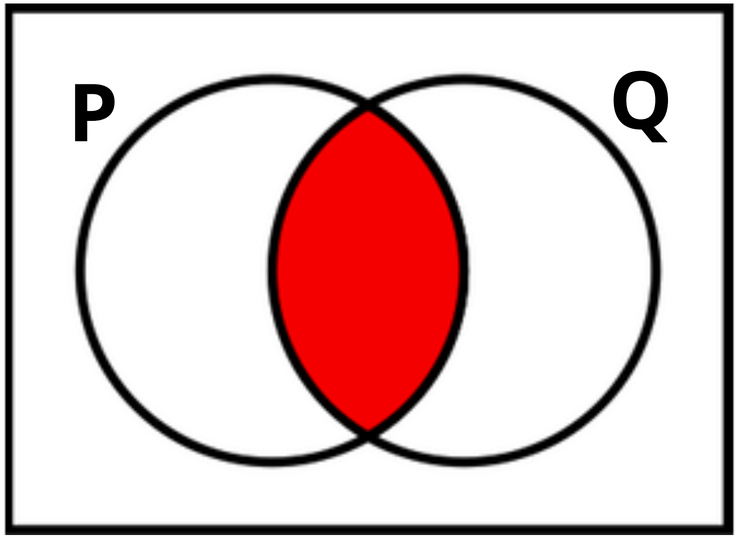
Summary

- Negations
  - understand them in human language
  - **practice** is the key!
- Parentheses
  - use them properly to avoid ambiguity

Lecture 3.3 Truth tables, and some laws

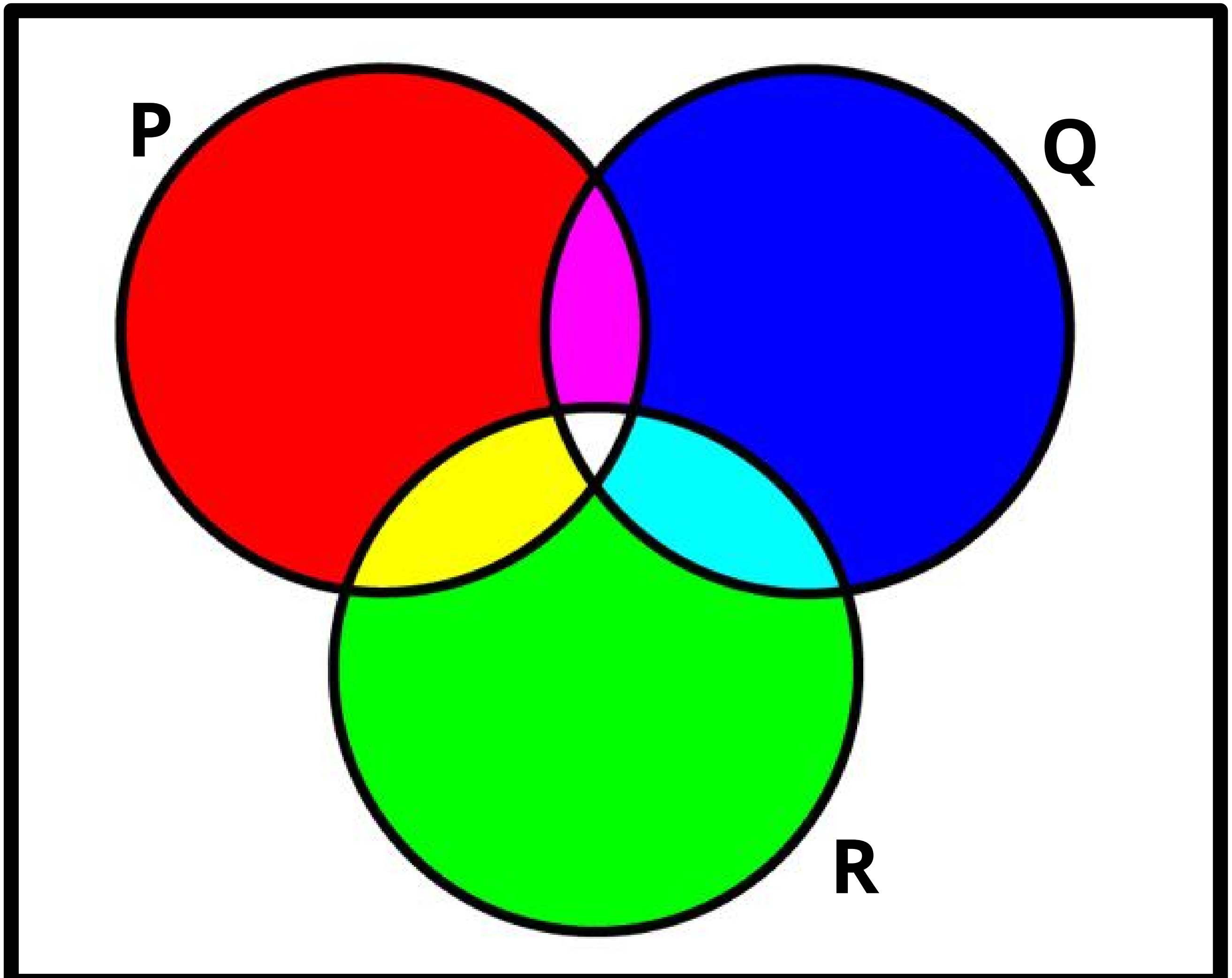
Course Notes: Chapter 2

About visualization...

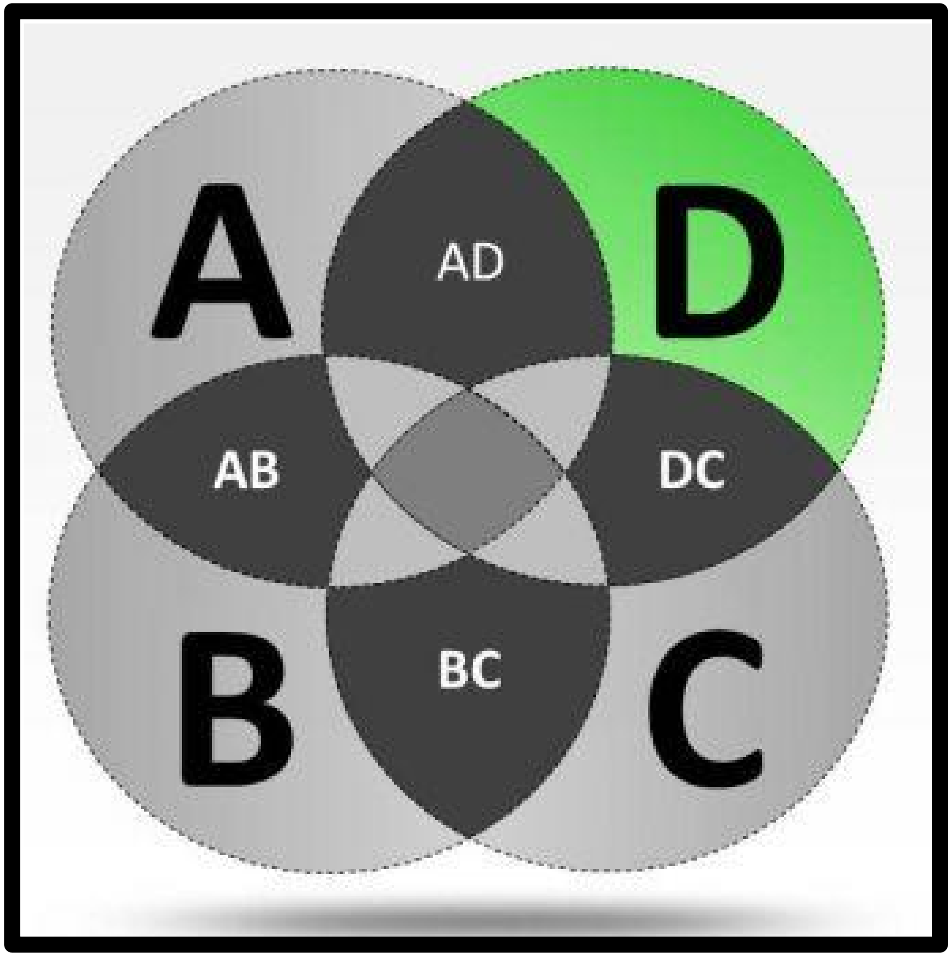


Venn diagram works pretty well...
... for **TWO** predicates.

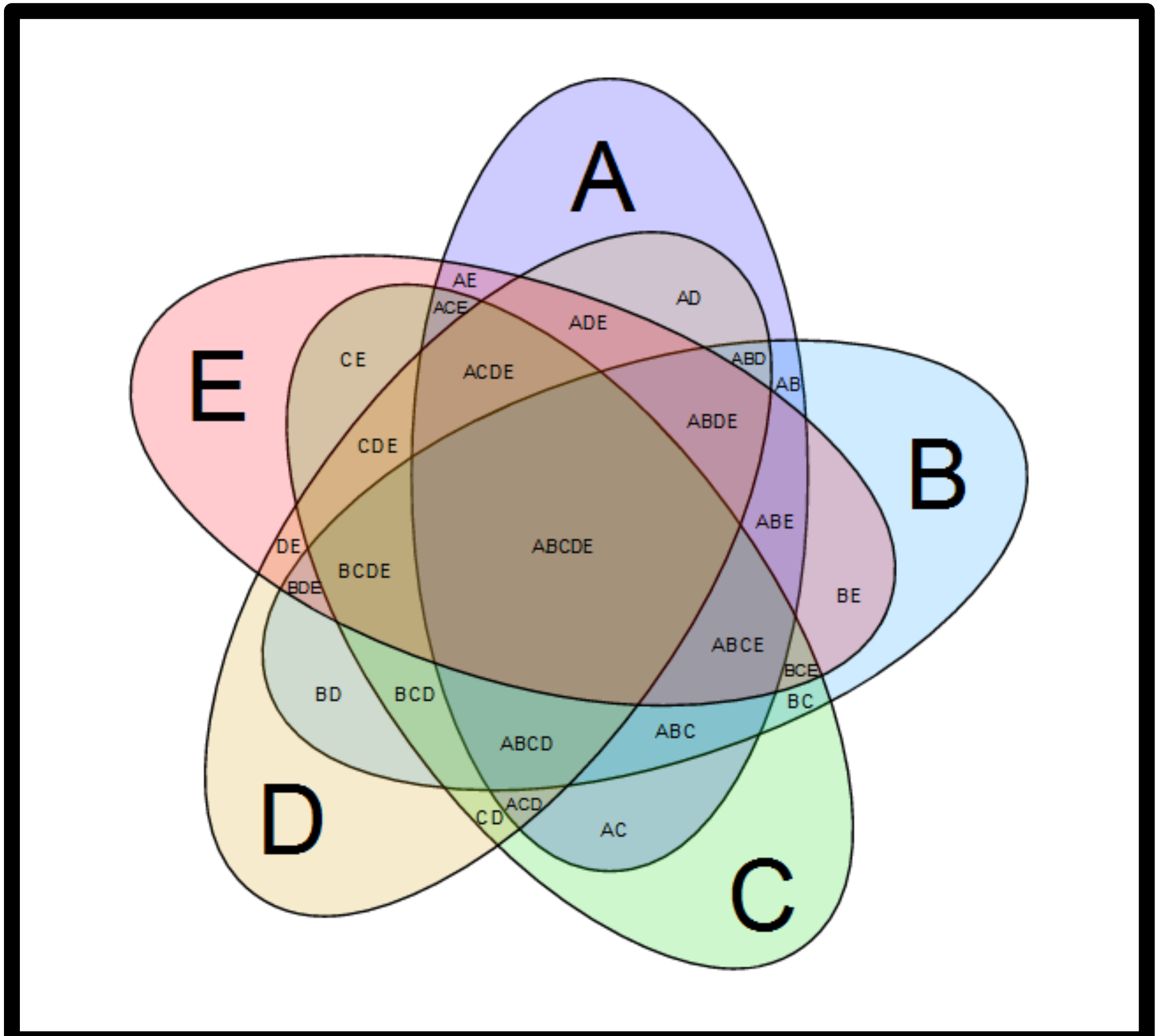
What if we have 3 predicates?



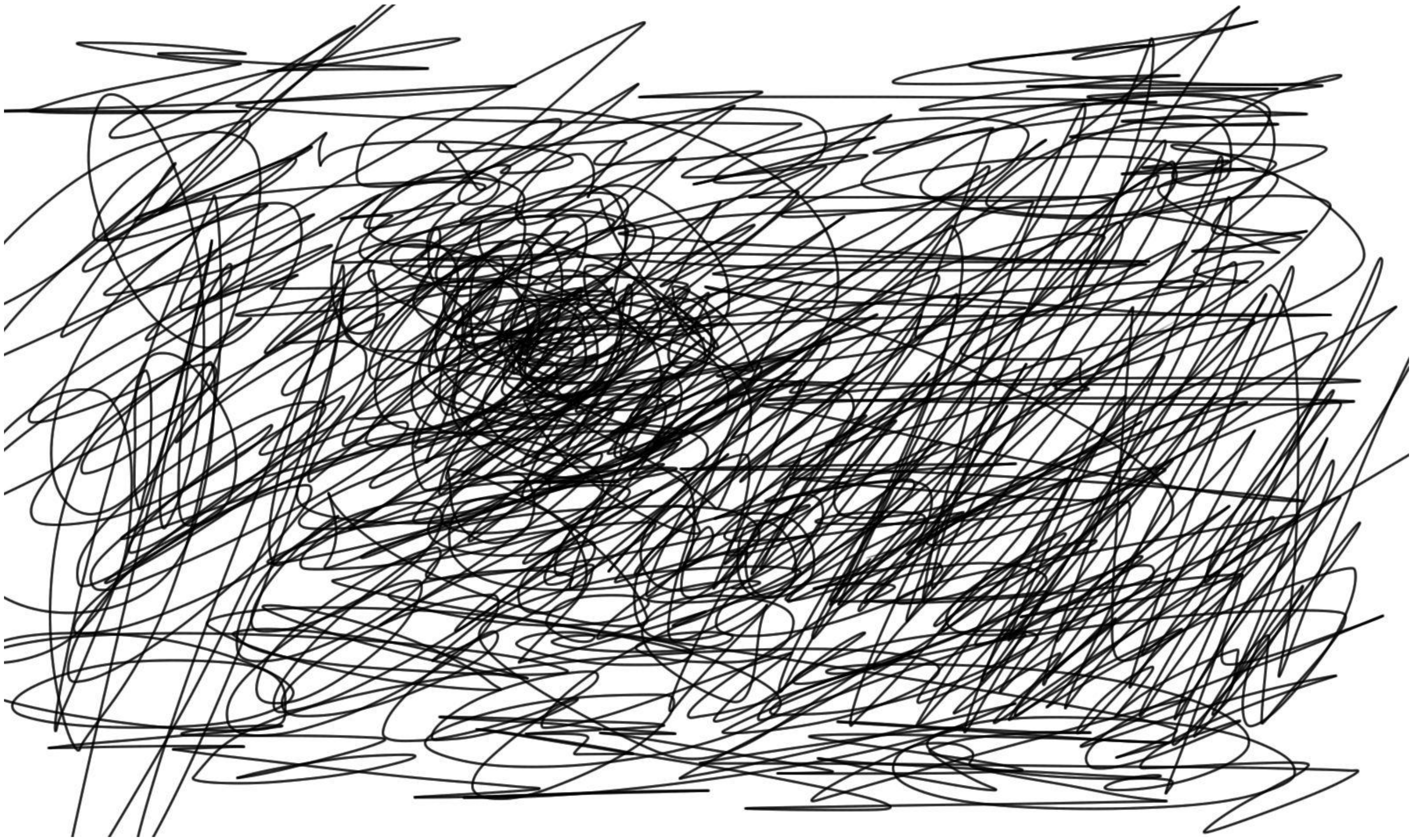
What if we have 4 predicates?



What if we have 5 predicates?



What if we have 20 predicates?



There must be a better way!

It’s called the truth table

Truth table with 2 predicates

P	Q	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

INPUTS

OUTPUTS

Enumerate the **outputs** over **all** possible combinations of **input** values of ***P*** and ***Q***.

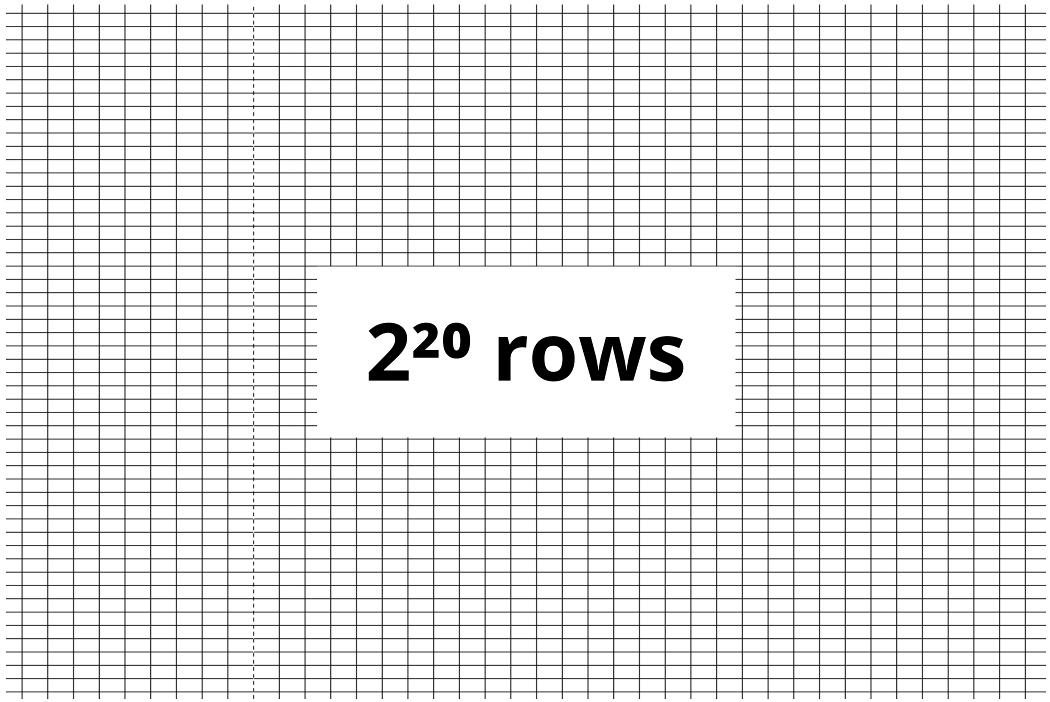
How many rows are there? $\mathbf{2^2 = 4}$

Truth table with 3 predicates

P	Q	R	$Q \Rightarrow R$	$P \Rightarrow (Q \Rightarrow R)$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	T
F	F	T	T	T
F	F	F	T	T

How many rows are there? $2^3 = 8$

Truth table with 20 predicates



It's not a mess, it just a larger table, which computers can process easily!

What can truth tables be used for?

for evaluating expressions

P	Q	R	$Q \Rightarrow R$	$P \Rightarrow (Q \Rightarrow R)$
T	T	T	T	T
T	T	F	F	[illegible]
T	F	T	T	T
T	F	F	T	[illegible]
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	T	T

It's a boy **and** it's not a boy.

It's a boy **or** it's not a boy.

for determining satisfiability

satisfiable

unsatisfiable (contradiction)

tautology (universal truth)

P	Q	$P \land Q$	$P \land \neg P$	$P \lor \neg P$
T	T	**T**	**F**	**T**
T	F	**F**	**F**	**T**
F	T	**F**	**F**	**T**
F	F	**F**	**F**	**T**

for proving equivalence

P Q	$\neg P \lor Q$	$P \Rightarrow Q$
T T	**T**	**T**
T F	**F**	**F**
F T	**T**	**T**
F F	**T**	**T**

$$\neg P \lor Q \Leftrightarrow P \Rightarrow Q$$

for proving equivalence

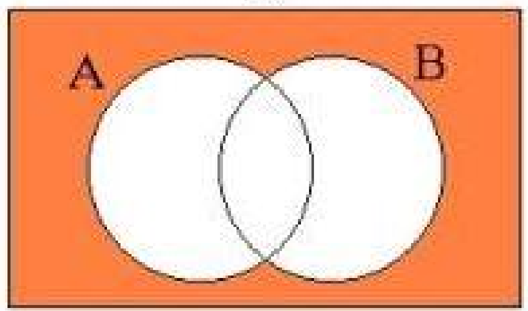
P	Q	$\neg (P \lor Q)$	$\neg P \land \neg Q$
T	T	**F**	**F**
T	F	**F**	**F**
F	T	**F**	**F**
F	F	**T**	**T**

$$\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$$

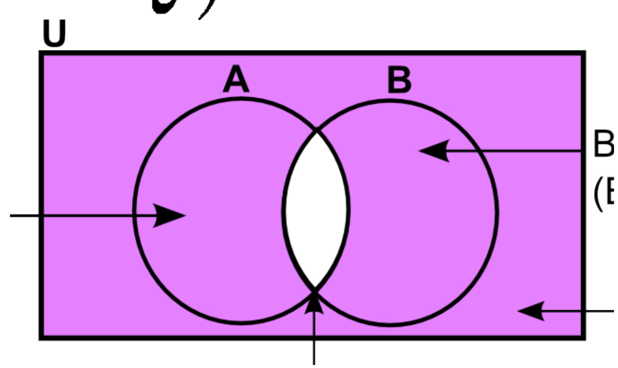
We just proved De Morgan's Law!

De Morgan's Law

$$\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$$



$$\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$$



Augustus De Morgan (1806-1871)

➔ De Morgan’s Law

➔ Mathematical Induction

there are more laws…

Other laws

Commutative laws

$$P \wedge Q \Leftrightarrow Q \wedge P$$

$$P \vee Q \Leftrightarrow Q \vee P$$

Other laws

Associative laws

$$(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$$

$$(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$$

Other laws

Distributive laws

$$P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$$

$$P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$$

Other laws

Identity laws

$$P \wedge \underbrace{(Q \vee \neg Q)}_{\textit{always true}} \Leftrightarrow P$$

$$P \vee \underbrace{(Q \wedge \neg Q)}_{\textit{always false}} \Leftrightarrow P$$

Other laws

Idempotent laws

$$P \wedge P \Leftrightarrow P$$

$$P \vee P \Leftrightarrow P$$

Other laws

For a full list of laws to be used in CSC165, read Chapter 2.17 of Course Notes.

About these laws...

- ➔ Similar to those for arithmetics.
- ➔ Only use when you are sure.
- ➔ **Understand** them, **be able to verify** them, rather than **memorizing** them.
- ➔ **Practice is the key!**

Summary for today

- ➔ Conjunctions
- ➔ Disjunctions
- ➔ Negations
- ➔ Truth tables
- ➔ Manipulation laws
- ➔ **We are almost done with learning the language of math.**

Next week

- ➔ finish learning the **language**
- ➔ start learning **proofs**