

# Reasoning about Large Taxonomies of Actions in the Situation Calculus



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## Motivation

The situation calculus [Reiter, 2001]

- Initially proposed by J. McCarthy, enriched by R. Reiter et al at Univ. of Toronto
- A high-level first-order logic language
- Representing and reasoning about actions in a natural and compact way

**Problem:** Handling effects of action functions individually and impractical to handle large scales of action functions

### Our Idea

- Using common-sense knowledge: Considering one action as a specialization of another
- E.g., traveling and shipping can be considered as specializations of moving

## Summary of Contributions

- Representing large taxonomies of actions in a hierarchical way
- Representing effects of actions much more compactly based on action hierarchies
- Reasoning problems can be solved exponentially faster (sometimes)
- Bringing possible knowledge engineering advantages

## Background: The Situation Calculus

### Sorts of the language

- Action functions:**  $fry(x, y)$  (frying  $x$  using  $y$ ),  $wash(y)$  (washing  $y$ )
- Situations:**  $S_0$  (the initial situation),  $do(a, s)$  (the situation after doing  $a$  in  $s$ )
- Objects:** persons, locations, food items, ...

### Fluents

- System features whose truth values may vary from situation to situation
- Predicates or functions with the last argument of sort situation  
E.g.,  $FoodReady(x, s)$  – food  $x$  is ready in the situation  $s$   
 $Dirty(x, s)$  –  $x$  is dirty in the situation  $s$

### A Basic Action Theory (BAT) $\mathcal{D}$

- Describing actions and their effects in a dynamic system
- A set of first-order axioms, mainly includes:
  - $\mathcal{D}_{ap}$  (precondition axioms): For each  $A(\vec{x})$ ,  $Poss(A(\vec{x}), s) \equiv \phi_A(\vec{x}, s)$
  - $\mathcal{D}_{ssa}$  (successor state axioms): For each fluent  $F(\vec{y}, s)$ ,  
 $F(\vec{y}, do(a, s)) \equiv \bigvee_i \psi_i^+(\vec{y}, a, s) \vee F(\vec{y}, s) \wedge \neg(\bigvee_j \psi_j^-(\vec{y}, a, s))$
  - $\mathcal{D}_{S_0}$  (an initial theory): All facts hold in the initial situation  $S_0$
  - $\mathcal{D}_{una}$  (unique name axioms for action names) and  $\Sigma$  (foundational axioms)

### An example of kitchen activity domain

- Considering actions involved in a kitchen such as  $cook(x)$ ,  $fry(x, y)$ ,  $wash(x)$ , ..., etc, and fluents such as  $FoodReady(x, s)$ ,  $Dirty(x, s)$ , ..., etc. (See actions in the digraph on the right hand-side)

- An example of the precondition axioms:

$$Poss(wash(x), s) \equiv WashableObject(x) \wedge Dirty(x, s)$$

- An example of the successor state axioms:

$$Dirty(y, do(a, s)) \equiv (\exists x)a = steam(x, y) \vee (\exists x)a = boil(x, y) \vee (\exists x)a = pressureCook(x, y) \vee (\exists x)a = broil(x, y) \vee (\exists x)a = bake(x, y) \vee (\exists x)a = ovenCook(x, y) \vee (\exists x)a = roast(x, y) \vee (\exists x)a = deepFry(x, y) \vee (\exists x)a = stir(x, y) \vee (\exists x)a = fry(x, y) \vee (\exists x)a = stew(x, y) \vee (\exists x)a = makeSalad(x, y) \vee (\exists x)a = prepHotDr(x, y) \vee (\exists x)a = prepColdDr(x, y) \vee Dirty(y, s) \wedge \neg[a = wash(y) \vee a = handWash(y)].$$

- Some examples of the axioms or facts in the initial theory:

$$Food(Egg_0), Vessel(Pot_0), \dots, (\forall x)Vessel(x) \supset \neg Dirty(x, S_0), (\forall x)Food(x) \supset \neg FoodReady(x, S_0), \dots, \text{etc.}$$

### Reasoning using regression operator $R$

- Defined recursively using basic action theories

$$\text{E.g., } R[W_1 \vee W_2] = R[W_1] \vee R[W_2], R[(\exists x)W] = (\exists x)R[W], \dots, R[F(\vec{y}, do(a, s))] = R[\phi_F(\vec{y}, a, s)] \text{ where } F(\vec{y}, do(a, s)) \equiv \phi_F(\vec{y}, a, s) \text{ using } \mathcal{D}_{ssa}$$

- Reiter's regression theorem:  $\mathcal{D} \models W$  iff  $\mathcal{D}_{una} \cup \mathcal{D}_{S_0} \models R[W]$

## Action Hierarchies

### An action diagram $\mathcal{H}$

- $sp(a_1, a_2)$ : Action  $a_1$  is a (direct) specialization of action  $a_2$
- A collection of finitely many axioms:  $sp(A_1(\vec{x}), A_2(\vec{y})) \equiv \phi_{A_1, A_2}(\vec{x}, \vec{y})$   
E.g.,  $sp(fry(x, y), oilyCook(x))$ ,  $sp(oilyCook(x), cook(x))$ , ..., etc
- Acyclic, antisymmetric, multiple inheritance

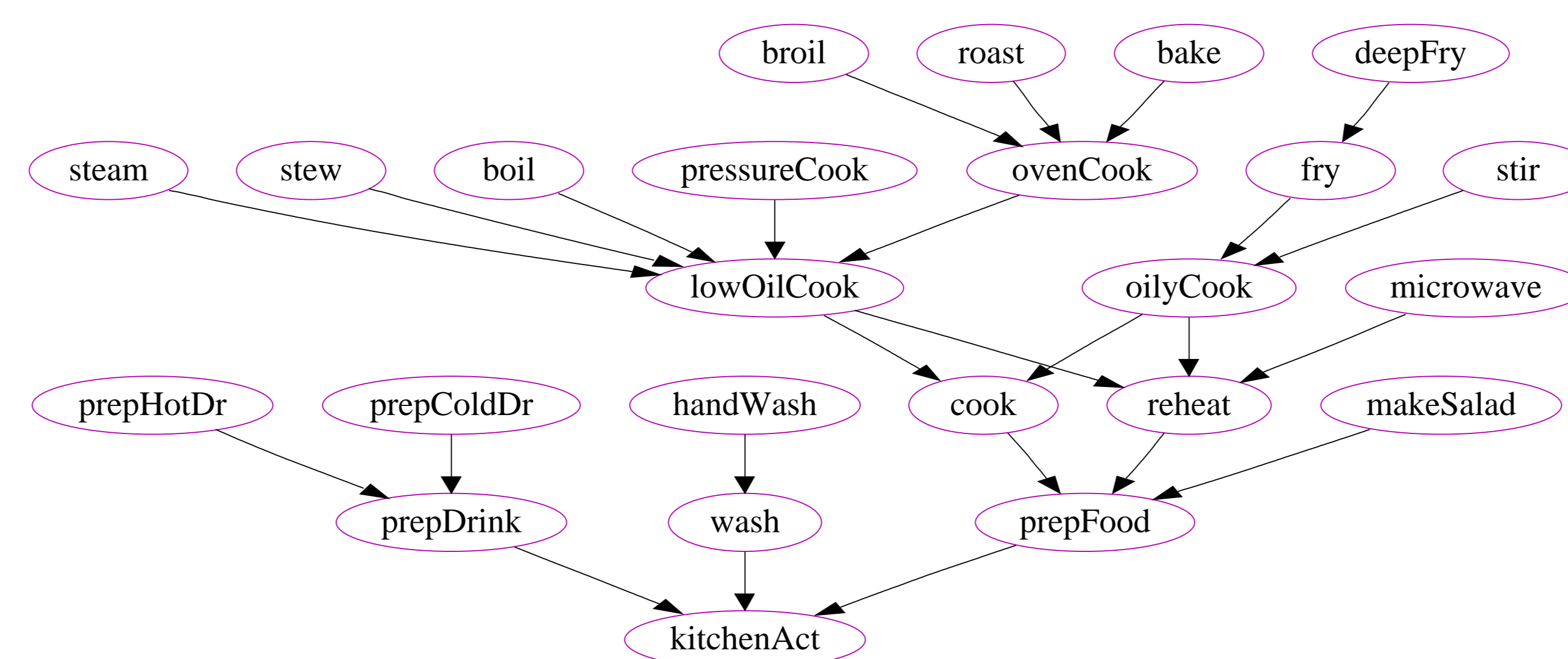
### An action hierarchy $\mathcal{H}^* = \mathcal{H} + \text{the definition of } isA$

- $isA(a_1, a_2)$ : Action  $a_1$  is a (distant) specialization of action  $a_2$
- The reflexive-transitive closure of  $sp$  defined in **second-order** logic
- Theorem:**

For any action functions  $A_1(\vec{x})$  and  $A_2(\vec{y})$ ,  $\mathcal{H}^* \models isA(A_1(\vec{x}), A_2(\vec{y})) \equiv \psi_{A_1, A_2}(\vec{x}, \vec{y})$ , where  $\psi_{A_1, A_2}(\vec{x}, \vec{y})$  is a **first-order** formula and can be found in **finitely many** steps.

### Event slots

- $EventSlot(a, x, l)$ :  $x$  is an event slot  $l$  in action  $a$
- Identifying the relationship between an action and each of its argument  
E.g.,  $EventSlot(fry(x, y), x, ActTarget)$ ,  $EventSlot(fry(x, y), y, Utensil)$ , etc.



## A Modular Basic Action Theory $\mathcal{D}^H$

### Format of axioms

- $\mathcal{D}_{ap}$ ,  $\mathcal{D}_{una}$  and  $\Sigma$  are as usual
- $\mathcal{D}_{S_0}^H$ : includes action hierarchy axioms  $\mathcal{H}^*$  and axioms for event slots in addition
- $\mathcal{D}_{ssa}^H$ : New representation of basic action theories using  $isA$   
E.g.,  $Dirty(y, do(a, s)) \equiv (\exists x)isA(a, prepFood(x)) \wedge EventSlot(a, y, Utensil) \vee (\exists x)isA(a, prepDrink(x)) \wedge EventSlot(a, y, Utensil) \vee Dirty(y, s) \wedge \neg isA(a, wash(y))$

**Regression:** operator is extended to handle  $isA$  and  $EventSlot$

### Correctness of Modular Basic Action Theories

- For any modular BAT  $\mathcal{D}^H$ , there exists an equivalent  $\mathcal{D}$  of Reiter's BAT format, where equivalence means that for any first-order regressable sentence  $W$ ,  $\mathcal{D}^H \models W$  iff  $\mathcal{D} \models W$ .
- A regression theorem similar to Reiter's regression theorem is proved:  $\mathcal{D}^H \models W$  iff  $\mathcal{D}_{S_0}^H \cup \mathcal{D}_{una} \models R[W]$ .
- Although the formal definition of  $isA$  is **second-order**, the reasoning in  $\mathcal{D}^H$  can be reduced to a **first-order** logic reasoning only.

## Advantages of The New Approach

### Computational advantages

When  $\mathcal{H}$  has a tree or forest structure, then the reasoning on  $isA$  is **exponentially faster** than the reasoning on its equivalent clause in Reiter's BAT representation.

### Knowledge engineering advantages

- Allowing taxonomic reasoning about event slot hierarchies and action hierarchies
- Possible applications to action/service retrieval
- Easiness for system update and reuse of action hierarchies and modular BATs

**Implementation (in progress):** E-business domain and kitchen activity domain

### References and Most Related Work

- [Amir, 2000] (De)Composition of Situation Calculus Theories
- [Barker, Porter & Clark, 2001] A Library of Generic Concepts for Composing KBs
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