PLEASE HAND IN

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UNIVERSITY OF TORONTO
FACULTY OF ARTS AND SCIENCE

MIDTERM

CSC 165H 2006 SUMMER
DURATION — 50 MINUTES

NO AIDS ALLOWED

STUDENT NUMBER: |_|_|_|_|_|_|_|_|_|

LAST NAME: ______________________________

FIRST NAME: ______________________________

Do NOT *turn this page until you have received the signal to start.*
(In the meantime, please fill out the identification section above, and read the instructions below.)

This test consists of 4 questions on 6 pages (including this one). Please DO NOT use red pen or pencil to answer the questions. The back of the paper can be used for rough work, which will not be marked, unless you declare clearly which part(s) should be considered as your solution(s). *When you receive the signal to start, please make sure that your copy of the test is complete.*

Please answer questions in the space provided. You will earn 20% of the relevant marks for any question you leave blank or write "I cannot answer this question," on. You will earn substantial part marks for writing down the outline of a solution and indicating which steps are missing.

Write your student number at the bottom of pages 2-6 of this test.

1: ______/10

2: ______/ 4

3: ______/10

4: ______/16

TOTAL: ______/40

Good Luck!

QUESTION 1. [10 MARKS]

Consider the following statement

(S1) Any dog is tired and is thirsty only when it has run a lot.

PART (A) [3 MARKS]

Represent (S1) symbolically, declare the domain and the predicates you use clearly.

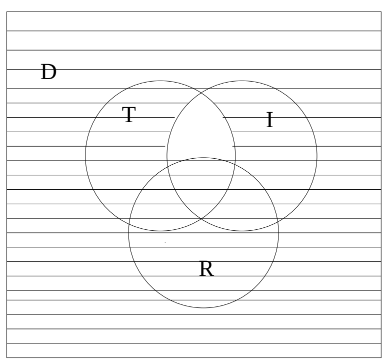
ANSWER:
Symbol d represents a dog. Symbol D represents the set of dogs d. Predicate $tired(d)$ means that a dog d is tired. Predicate $thirsty(d)$ means that a dog d is thirsty. Predicate $run(d)$ means that a dog d has run a lot. Then, (S1) can be represented as:

$$\forall d \in D, (Tired(d) \wedge Thirsty(d)) \rightarrow run(d)$$

PART (B) [3 MARKS]

Draw a Venn Diagram, and shade the region where (S1) can be true. Declare the meanings of the domain and the sets used in the Diagram clearly.

ANSWER:
Let D be the set of dogs, let R be the set of dogs running a lot, let T be the set of dogs that are thirsty, and let I be the set of dogs that are tired.



PART (C) [2 MARKS]

Assume that (S1) is true and we also know that a dog does NOT run a lot, what can be concluded? (Circle the appropriate answer below)

(A) The dog is definitely not tired, but thirsty;

(B) The dog is definitely not thirsty, but tired;

(C) The dog is either not tired, or not thirsty;

(D) There is not enough information to determine anything;

(E) I DON'T KNOW THE ANSWER (FOR 20%).

ANSWER: The solution is (c), think of the contra-positive of (S1).

PART (D) [2 MARKS]

Assume that (S1) is true and we also know the a dog is tired, what can be concluded? (Circle the appropriate answer below)

(A) The dog is not thirsty;

(B) The dog runs a lot;

(C) There is not enough information to determine anything;

(D) I DON'T KNOW THE ANSWER (FOR 20%).

ANSWER: The solution is (c), when the post-condition of the statement is true, the whole sentence is true, no matter what is the truth value of the pre-condition.

QUESTION 2. [4 MARKS]

Use the logical arithmetic laws to simplify the following formulas as much as possible. Show your work step by step. (Also, remind that $p \wedge \neg p$ is equivalent to $False$, $p \vee \neg p$ is equivalent to $True$ for any statement p).

PART (A) [2 MARKS]

$p \rightarrow (p \wedge q)$

ANSWER:

$$
\begin{aligned}
& p \rightarrow (p \wedge q) \\
\equiv\ & \neg p \vee (p \wedge q) \\
\equiv\ & (\neg p \vee p) \wedge (\neg p \vee q) \\
\equiv\ & true \wedge (\neg p \vee q) \\
\equiv\ & \neg p \vee q \text{ (or } p \rightarrow q)
\end{aligned}
$$

PART (B) [2 MARKS]

$(q \rightarrow p) \vee q$

ANSWER:

$$
\begin{aligned}
& (q \rightarrow p) \vee q \\
\equiv\ & (\neg q \vee p) \vee q \\
\equiv\ & (p \vee \neg q) \vee q \\
\equiv\ & p \vee (\neg q \vee q) \\
\equiv\ & p \vee true \\
\equiv\ & true
\end{aligned}
$$

QUESTION 3. [10 MARKS]

Let S be the set of students.
Let C be the set of courses.
Let $prerequisite(x, y)$ represent the meaning that "course x is a prerequisite of course y".
Let $enroll(x, y)$ represent the meaning that "student x enrolls in course y".
Let $finish(x, y)$ represent the meaning that "student x finishes course y".

Rewrite the following English sentences using precise symbolic notations:

PART (A) [2 MARKS]

Not every course has a prerequisite.

ANSWER:
$\exists x \in C, \forall y \in C, \neg prerequisite(y, x)$
or, $\neg \forall x \in C, \exists y \in C, prerequisite(y, x)$

PART (B) [2 MARKS]

Some course has at least two prerequisites.

ANSWER:
$\exists x \in C, \exists y \in C, \exists z \in C, prerequisite(y, x) \wedge prerequisite(z, x) \wedge y \neq z$

PART (C) [2 MARKS]

Some course is a prerequisite of every course.

ANSWER:
$\exists x \in C, \forall y \in C, prerequisite(x, y)$

PART (D) [2 MARKS]

No course is a prerequisite of itself.

ANSWER:
$\forall x \in C, \neg prerequisite(x, x)$
or, $\neg \exists x \in C, prerequisite(x, x)$

PART (E) [2 MARKS]

When a student enrolls in a course, he or she must finish all the prerequisites.

ANSWER:
$\forall s \in S, \forall c \in C, \forall c1 \in C, enroll(s, c) \wedge prerequisite(c1, c) \rightarrow finish(s, c1)$
or, $\forall s \in S, \forall c \in C, enroll(s, c) \rightarrow (\forall c1 \in C, prerequisite(c1, c) \rightarrow finish(s, c1))$

QUESTION 4. [16 MARKS]

Let $\mathbf{N} = \{0, 1, 2, \cdots\}$, i.e., the set of natural numbers including zero.
Consider the statement:

$$\text{(S2)} \quad \exists i \in \mathbf{N}, a_i = a_{i+1} \wedge (\forall j \in \mathbf{N}, j > (i+1) \rightarrow a_j < a_i)$$

about the sequence $a_0, a_1, a_2, \cdots$.

PART (A) [2 MARKS]

Expression the negation of (S2), pushing the "not" inside as much as possible.

ANSWER:

$\neg\exists i \in \mathbf{N}, a_i = a_{i+1} \wedge (\forall j \in \mathbf{N}, j > (i+1) \rightarrow a_j < a_i)$
$\equiv \forall i \in \mathbf{N}, \neg(a_i = a_{i+1} \wedge (\forall j \in \mathbf{N}, j > (i+1) \rightarrow a_j < a_i))$
$\equiv \forall i \in \mathbf{N}, a_i \neq a_{i+1} \vee \neg(\forall j \in \mathbf{N}, j > (i+1) \rightarrow a_j < a_i)$
$\equiv \forall i \in \mathbf{N}, a_i \neq a_{i+1} \vee \neg(\forall j \in \mathbf{N}, j > (i+1) \rightarrow a_j < a_i)$
$\equiv \forall i \in \mathbf{N}, a_i \neq a_{i+1} \vee \neg(\forall j \in \mathbf{N}, \neg(j > (i+1)) \vee a_j < a_i)$
$\equiv \forall i \in \mathbf{N}, a_i \neq a_{i+1} \vee (\exists j \in \mathbf{N}, j > (i+1) \wedge a_j \geq a_i)$

PART (B) [7 MARKS]

Using the proof structure we learned, prove or disprove (S2) carefully for the following sequence

$1, 3, 9, 27, \cdots$; that is, for each $n = 0, 1, 2, \cdots$, $a_n = 3^n$.

ANSWER:

Disprove, that is, to prove $\neg(S2)$, i.e.,
$\forall i \in \mathbf{N}, a_i \neq a_{i+1} \vee (\exists j \in \mathbf{N}, j > (i+1) \wedge a_j \geq a_i)$
is true.

<u>Proof:</u>
Let $i \in \mathbf{N}$,
$a_i = 3^i$ and $a_{i+1} = 3^{i+1}$,
So, $a_i \neq a_{i+1}$ (since $\frac{a_{i+1}}{a_i} = 3 \neq 1$)
So, $a_i \neq a_{i+1} \vee (\forall j \in \mathbf{N}, j > (i+1) \wedge a_j \geq a_i)$
Since i is an arbitrary natural number,
thus, $\forall i \in \mathbf{N}, a_i \neq a_{i+1} \vee (\exists j \in \mathbf{N}, j > (i+1) \wedge a_j \geq a_i)$.

PART (C) [7 MARKS]

Using the proof structure we learned, prove or disprove (S2) carefully for the following sequence

$1, 2, 2, -3, -4, -5, -6, \cdots$; that is, for each $n = 0, 1, 2, \cdots$

$$a_n = \begin{cases} 1 & \text{if } n = 0 \\ 2 & \text{else if } n = 1 \text{ or } 2 \\ -n & \text{otherwise.} \end{cases}$$

ANSWER:
Prove, that is, to prove (S2) is true for this sequence.

Proof:
Let $i = 1$,
Thus, $i \in \mathbf{N}$,
also, $a_1 = a_2 = 2$.
Moreover, let $j \in \mathbf{N}$, such that $j > 2$
hence $a_j = -j$,
hence $a_j < 2 = a_i$.
Thus, $i > 2 \rightarrow a_j < a_i$.
Since j is an arbitrary natural number,
Thus, $\forall j \in \mathbf{N}, j > (i+1) \rightarrow a_j < a_i$.
Thus, $a_i = a_{i+1} \wedge (\forall j \in \mathbf{N}, j > (i+1) \rightarrow a_j < a_i)$.
Since i is a particular natural number,
so, $\exists i \in \mathbf{N}, a_i = a_{i+1} \wedge (\forall j \in \mathbf{N}, j > (i+1) \rightarrow a_j < a_i)$.

Total Marks = 40 END OF EXAM