PLEASE HAND IN

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UNIVERSITY OF TORONTO
FACULTY OF ARTS AND SCIENCE

MIDTERM

CSC 165H 2006 SUMMER
DURATION — 50 MINUTES

NO AIDS ALLOWED

STUDENT NUMBER: |_|_|_|_|_|_|_|_|_|_|

LAST NAME: _____

FIRST NAME: _____

Do NOT *turn this page until you have received the signal to start.*
(In the meantime, please fill out the identification section above, and read the instructions below.)

This test consists of 4 questions on 7 pages (including this one). Please DO NOT use red pen or pencil to answer the questions. The back of the paper can be used for rough work, which will not be marked, unless you declare clearly which part(s) should be considered as your solution(s). *When you receive the signal to start, please make sure that your copy of the test is complete.*

Please answer questions in the space provided. You will earn 20% of the relevant marks for any question you leave blank or write "I cannot answer this question," on. You will earn substantial part marks for writing down the outline of a solution and indicating which steps are missing.

Write your student number at the bottom of pages 2-7 of this test.

1: _____/10

2: _____/ 4

3: _____/10

4: _____/16

TOTAL: _____/40

Good Luck!

QUESTION 1. [10 MARKS]

Consider the following statement

(S1) Any dog is tired and is thirsty only when it has run a lot.

PART (A) [3 MARKS]

Represent (S1) symbolically, declare the domain and the predicates you use clearly.

PART (B) [3 MARKS]

Draw a Venn Diagram, and shade the region where (S1) can be true. Declare the meanings of the domain and the sets used in the Diagram clearly.

PART (C) [2 MARKS]

Assume that (S1) is true and we also know that a dog does NOT run a lot, what can be concluded? (Circle the appropriate answer below)

(A) The dog is definitely not tired, but thirsty;

(B) The dog is definitely not thirsty, but tired;

(C) The dog is either not tired, or not thirsty;

(D) There is not enough information to determine anything;

(E) I DON'T KNOW THE ANSWER (FOR 20%).

PART (D) [2 MARKS]

Assume that (S1) is true and we also know the a dog is tired, what can be concluded? (Circle the appropriate answer below)

(A) The dog is not thirsty;

(B) The dog runs a lot;

(C) There is not enough information to determine anything;

(D) I DON'T KNOW THE ANSWER (FOR 20%).

QUESTION 2. [4 MARKS]

Use the logical arithmetic laws to simplify the following formulas as much as possible. Show your work step by step. (Also, remind that $p \wedge \neg p$ is equivalent to $False$, $p \vee \neg p$ is equivalent to $True$ for any statement p).

PART (A) [2 MARKS]

$p \rightarrow (p \wedge q)$

PART (B) [2 MARKS]

$(q \rightarrow p) \vee q$

QUESTION 3. [10 MARKS]

Let S be the set of students.
Let C be the set of courses.
Let $prerequisite(x, y)$ represent the meaning that "course x is a prerequisite of course y".
Let $enroll(x, y)$ represent the meaning that "student x enrolls in course y".
Let $finish(x, y)$ represent the meaning that "student x finishes course y".

Rewrite the following English sentences using precise symbolic notations:

PART (A) [2 MARKS]

Not every course has a prerequisite.

PART (B) [2 MARKS]

Some course has at least two prerequisites.

PART (C) [2 MARKS]

Some course is a prerequisite of every course.

PART (D) [2 MARKS]

No course is a prerequisite of itself.

PART (E) [2 MARKS]

When a student enrolls in a course, he or she must finish all the prerequisites.

QUESTION 4. [16 MARKS]

Let $\mathbf{N} = \{0, 1, 2, \cdots\}$, i.e., the set of natural numbers including zero.
Consider the statement:

(S2) $\exists i \in \mathbf{N}, a_i = a_{i+1} \wedge (\forall j \in \mathbf{N}, j > (i+1) \rightarrow a_j < a_i)$

about the sequence $a_0, a_1, a_2, \cdots$.

PART (A) [2 MARKS]

Expression the negation of (S2), pushing the "not" inside as much as possible.

PART (B) [7 MARKS]

Using the proof structure we learned, prove or disprove (S2) carefully for the following sequence

$1, 3, 9, 27, \cdots$; that is, for each $n = 0, 1, 2, \cdots$, $a_n = 3^n$.

PART (C) [7 MARKS]

Using the proof structure we learned, prove or disprove (S2) carefully for the following sequence

$1, 2, 2, -3, -4, -5, -6, \cdots$; that is, for each $n = 0, 1, 2, \cdots$

$$a_n = \begin{cases} 1 & \text{if } n = 0 \\ 2 & \text{else if } n = 1 \text{ or } 2 \\ -n & \text{otherwise.} \end{cases}$$

Total Marks = 40 END OF EXAM