Packet Switch and Network Architectures

Department of Computer Science, University of Toronto

CSC 2203, Fall 2009	**Date Issued:** Thursday, September 24
Assignment No. 1 – Handout # 5	**Due Date:** Thursday, October 1st by 1pm

This assignment has 5 problems with a total of 15 points. You just need to solve 3 out of the first 4 problems. Depending on which problem you eliminate your total score of 12 or 13 will be scaled down to 10 marks.

1. To Serve or Not to Serve [2 points]. Consider a web server which has a limited capacity, and thus needs a way to control the number of requests that it will serve. The server uses the following admission control mechanism.

Whenever a request **R** arrives at the server, it is kept for ***T*** units of time. If no other request arrives during that time, **R** is admitted and the server will start processing it. If another request arrives during ***T*** time units, **R** is dropped and the new request is hold for ***T*** units of time, and so on.

If arrivals happen according to a Poisson process of rate **λ**, what is the rate of admissions?

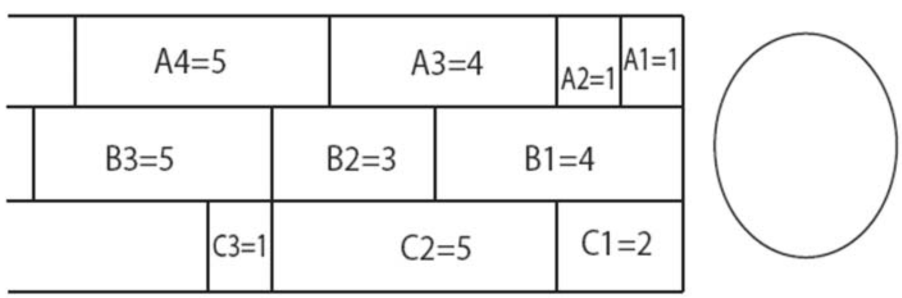

Figure 1. Comparing packet departure times in packetized fair queueing and Deficit Round Robin.

2. Packetized Fair Queueing and Deficit Round Robin [3 points].
(a) There are three queues being served by one server. The packets have arrived at the queues as shown in Figure 1. The packet lengths are in bits, server capacity is 1 bit/sec. Assume the server considers the queues for service in the order A, B, C. Give the departure order of the packets if the server implements (*i*) packetized fair queueing (*ii*) deficit round robin with quantum size Q = 1 bit, and (*iii*) deficit round robin with quantum size Q = 3 bits.

(b) Assume that the deficit round robin server is implemented as follows: The server has an output FIFO, which contains the packet(s) that need to be served immediately. The round robin "scanner" can scan the queues infinitely fast, and always scans from queue 1 to queue N. When the output FIFO becomes empty, the scanner allocates Q bits to each queue in each round, until some packet can depart. Then it finishes the round, dequeuing all packets that can leave and placing them in the output FIFO. The scanner then does not do anything until the output FIFO becomes empty again.

Now assume there are N queues. Queue 1 through queue N - 1 each have a packet of a bits (a > 2). The server starts serving at time 0. At time 1, a packet of length 1 arrives to queue N. No more packets will arrive to the system. When will this new packet be served if the server implements (*i*) packetized fair queueing (*ii*) deficit round robin as described with Q = 1 bit.

3. **Flashing Lights [3 points].** There are 25 old ladies, and 25 rooms each with a light bulb. Initially, all light bulbs are off.

Suppose that there is a fair coin in each room. Each lady goes to each room and flips the coin. If the coin comes up heads, she changes the state of the bulb. Else, she moves to the next room and repeats. What is the average number of bulbs that are on when all 25 ladies are done?

4. Min Geo [2 points]. Let $\boldsymbol{X_1} \sim \text{Geom}(\boldsymbol{p_1})$, $\boldsymbol{X_2} \sim \text{Geom}(\boldsymbol{p_2})$ be two independent geometric random variables. What is the distribution of $\boldsymbol{Y} = \min(\boldsymbol{X_1}, \boldsymbol{X_2})$?

Hint. From the definition of geometric random variable, we know $\Pr(\boldsymbol{X_1} > \boldsymbol{k}) = (1-\boldsymbol{p_1})^{\boldsymbol{k}}$. Find what is the probability $\Pr(\min(\boldsymbol{X_1}, \boldsymbol{X_2}) > \boldsymbol{k})$? Note that $\boldsymbol{X_1}$ and $\boldsymbol{X_2}$ are independent.

5. Max-Min Fairness [5 points]. Let $\boldsymbol{N}$ flows share a link of capacity $\boldsymbol{C}$. These flows want to send at rate $\boldsymbol{f_1} \le \dots \le \boldsymbol{f_N}$. Due to capacity constraint of $\boldsymbol{C}$, these flows need to be assigned rates $\boldsymbol{r_1}, \dots, \boldsymbol{r_N}$ such that it is feasible, that is,

$$\sum_{i=1}^{N} r_i \le C, \qquad f_i \ge r_i \ge 0, \qquad \forall i.$$

(a) **Definition of Max-Min Fairness**: A vector of rates $\boldsymbol{r} = (\boldsymbol{r_1}, \dots, \boldsymbol{r_N})$ is said to be max-min fair if it is feasible; and for any $\boldsymbol{i}$, $\boldsymbol{r_i}$ cannot be increased while maintaining feasibility without decreasing $\boldsymbol{r_j}$ for some flow $\boldsymbol{j} \ne \boldsymbol{i}$, such that $\boldsymbol{r_j} \le \boldsymbol{r_i}$.

(b) **Definition of Max-Min Fairness in terms of allocation policy**: As discussed in the class, a given set of flows can be allocated rates in the max-min fair way in the following manner:

(i) Initially: $S = \{1, \ldots, \boldsymbol{N}\}$, $\boldsymbol{m} = \boldsymbol{N}$ and $\boldsymbol{R} = \boldsymbol{C}$. Here $\boldsymbol{S}$ denotes the remaining flows, $\boldsymbol{m}$ denotes the remaining number of flows and $\boldsymbol{R}$ denotes the remaining capacity.

(ii) Pick the smallest flow $\boldsymbol{i} \in \boldsymbol{S}$. If $\boldsymbol{f_i} < \boldsymbol{R}/\boldsymbol{m}$, set $\boldsymbol{r_i} = \boldsymbol{f_i}$ else set $\boldsymbol{r_i} = \boldsymbol{R}/\boldsymbol{m}$.

(iii) Set $\boldsymbol{m} = \boldsymbol{m} - 1$, $\boldsymbol{R} = \boldsymbol{R} - \boldsymbol{r_i}$. Delete $\boldsymbol{i}$ from $\boldsymbol{S}$.

(iv) If $\boldsymbol{m} > 0$ repeat from (ii).

5.1. Show that any assignment generated by (b) satisfies conditions of (a).

5.2. Show that there is a unique assignment that satisfies (a).

5.3. Using 6.1 and 6.2 show that (a) and (b) are equivalent.